# INTERPRETATION OF GEODETIC OBSERVATIONS OF THE HIGH-RISE BUILDINGS DISPLACEMENTS 

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#### Abstract

In the article presents a new approach to processing results of geodetic observations of the highrise buildings displacements. As a result of three cycles of observations of the spatial position of deformation marks on the facade of high-rise buildings have been obtained displacements, the nature of which does not allow to correctly determine the nature of the deformation of the building. The obtained values of displacements may correspond to the roll, the bend or twist of the building, as well as combinations of these types of deformations. In this paper, we propose a method for analyzing displacements of a high-rise building with the approximation of the results of observations by spatial curves. The equations of a line, parabolic curve, helix and conical helix were investigated. The technique of construction of displacements approximation models was gave. The approach evaluating the quality of construction of the proposed models was considered. The results showed the effectiveness of the proposed methodic. On the example of results processing of the experimental observations performed construction of displacements models. The optimal solution is obtained using a helix equation. This equation can be used for calculating the twist and bend of structures and for predicting future displacements.


Keywords: high-rise building, displacement, roll, temperature, bend, twist, helix.

## Introduction

Today, high-rise buildings are an essential part of modern urban environmental. In the last century the construction of high-rise buildings had an experimental character. Now the building of 200 m high are the norm and are being built in all major cities, including in a tight housing. Most buildings are built as a set of $4-8$ buildings. In such circumstances, high-rise buildings are affected by many factors leading to the displacement of the spatial axis of the building. Such factors may include the reaction of soil foundation, the load of the surrounding buildings, the dynamic load of the vehicles (Biktashev 2006). In addition, for high-rise buildings are characterized by of specific types of loads, which significantly affect their position, such as temperature and wind load. Naturally, in this case the carrying out geodetic monitoring is essential for successful exploitation of high-rise building.

## 1. Main problem and task

Modern geodetic monitoring tools allow to solve the challenges and define all the necessary deformation characteristics. For high-rise buildings is the main characteristic of the building is roll. However, in the case of high-rise buildings due to the impact of wind and temperature loads in addition there are specific types of deformations such as bend and twist.

Traditional methods for rolls determining by geodetic methods are based on the comparison of the measurement results in the lower and upper parts of the building. Among these methods have proved well method of angles, method of directions, coordinate method, the projection method and so forth. (Biktashev 2006). At the same time for the unique determination of the roll necessary condition is to perform leveling of the foundation of the building to determine the tilt of the foundation, which in theory is uniquely associated with a roll of the building. Today the most
widespread and at the same time the most precise, and easy to implementation is coordinate method for determining the spatial displacements of high-rise buildings with using of total station (Mi et al. 2008).

Thus the observation is performed from the points of spatial angular-linear or GNSS networks, which may be located on the neighboring buildings. This method is very flexible and provides the necessary accuracy of observations. For a rough estimate of the accuracy of this method, we can use the expression from (Kala 2009). In the following we will analyze of high-rise buildings displacements which is obtained by this method.

Due to the factors noted above for high-rise buildings roll in pure form is the exception rather than the norm. Usually the deformation is more complex. This is the sum of displacement of the building and deformation of the building structure itself. For simplicity we consider the case of moving the buildings in one plane. Figure $1(\mathrm{a}, \mathrm{b}, \mathrm{c})$ shows the possible deformations of building with definition of displacement in several sections.

In the case of building simple roll $\gamma$, partial rolls in the various sections of the building are calculated by the known formulas:

$$
\gamma_{1}=\frac{\Delta_{1}}{H_{1}} \rho ; \quad \gamma_{2}=\frac{\Delta_{2}}{H_{2}} \rho ; \quad \gamma_{3}=\frac{\Delta_{3}}{H_{3}} \rho .
$$

In the case of pure roll, in all sections of the building the condition $\gamma_{1}=\gamma_{2}=\gamma_{3}=\gamma$ are saved. In the case of the building axis bend, or a combination of bend
and building roll, partial rolls are not equal each other $\gamma_{1} \leq \gamma_{2} \leq \gamma_{3} \leq \gamma$ in the sections. Actually in this case, the value $\gamma_{i}$ can not be called a roll. Thus it is possible to make two very important conclusions. First, and as it indicated in this work (Biktashev 2006) to perform observation in the traditional scheme of the upper and lower parts of the building impossible. Displacement must be determined, at least in four or five sections of the building. This condition is the main constraint on the use of GNSS observations. Despite the significant popularity and effectiveness of GNSS observations, as noted in the works (Akib et al. 2004; Ogaja et al. 2008; Khoo et al. 2010; Ting-Hua et al. 2013), this technology can only be used in combination with other measuring devices, such as inclinometers (Ozer et al. 2010) or total stations (Hostinová, Kopáčik 2008). Secondly, in the analysis of the building axis spatial displacement the determination of deformation type only on the values of the measured displacements is difficult. There are many approaches to the construction of displacement prediction models (Henriques et al. 2013). But they are only bad or a good approximation of the deformation process from a mathematical point of view. What is the physical meaning of the measured displacements? It is a well-known problem of geodesy. What do we measure deformation or displacement, and what kind of deformation we have fixed: roll, bend or twist?

If the first problem is now solved simply using total stations, which work in reflectorless mode, with the decision of the second problem, there are certain


Fig. 1(a). Variants of axis displacement of the building - building pure roll (displacement)
Fig. 1(b). Variants of axis displacement of the building - building pure bend (deformation)
Fig. 1(c). Variants of axis displacement of the building - the total effect of building roll and bend (a combination of displacement and deformation)
difficulties. Theoretically determination of the deformation type is possible if performed construction of the building three-dimensional model, for example, by using terrestrial laser scanning method. However, this approach may be suitable only for buildings of low height, and from the organizational point of view the implementation of such method of observation extremely difficult. If the observations are made by total station, the problem has a solution for performing measurements in either of two mutually perpendicular axes, or when observations are performed in the alignment of one of the axes of the building that as mentioned above in a tight housing is almost impossible to implement. Thus, monitoring is carried out for the available points of the building, determination of coordinates of which is possible to perform with the required accuracy. In this case, the question arises logically. How on the results of measurements in different asymmetrical points of the building to determine the type of the deformation process?

The aim of this paper is to investigate the spatial displacement the axis of the high-rise building with a new approach, which is based on an approximation of the measurements by space curve, the parameters of which are calculated and deformation characteristics established deformation behavior of high-rise buildings. This paper presents the construction of the approximate curve using the least squares method.

## 2. Methodology of investigation

Before going to the presentation of research methods we consider the most common nowadays the mathematical model to describe the deformation characteristics of high-rise buildings. Widespread use of GNSS technologies, which are characterized by a large redundancy of data has led to the use for processing of measurements results the Kalman filtering method. In many papers (Ehigiator-Irughe et al. 2014, Gourine et al. 2015) the attempts to use a Kalman filter to predict the deformation of high-rise buildings are made. While the Kalman filter on the output provides a mathematical model of the building displacement kinematics, it has some disadvantages. As we said earlier measurements refer to the highest point of the building, where the GNSS receiver installed. In this case, the resulting model describes the displacement is not just a building, but the top point. Second, the physical meaning of parameters derived by prediction model is unknown.

A wide class of prediction models discussed in (Henriques et al. 2013). Here, questions of application
the periodic, exponential, rational and exponential functions for the simulation and prediction of deformation in engineering projects are considered. In recent years, are highly popular advanced mathematical prediction models, which are based on the application: sequential filtering (Gandolfi et al. 2015), Fast Fourier Transform (FFT) (Ozer et al. 2010), the method of analysis of the singular spectrum (Khelifa et al. 2010), the trend analysis (Hostinová, Kopáčik 2008), neural networks (Gourine et al. 2012). As in previous cases, the physical content the parameters of these models are often unknown and may even mislead the investigator.

We offer to investigate the models that have in our view a clear physical meaning. We consider our proposed theoretical models that best describe the types deformation discussed above, namely the roll, bend and twist, as well as their combination. For ease of computation, all models are represented in parametric form.

Earlier, it was stated that the simplest type of displacement the axis of a high-rise building is actually roll. In this case, the position of the building axis in space is described by the parametric equations of straight line in the space:

$$
\left.\begin{array}{l}
X(t)_{i}=X_{0}+a t_{i} ;  \tag{1}\\
Y(t)_{i}=Y_{0}+b t_{i} \\
Z(t)_{i}=Z_{0}+c t_{i}
\end{array}\right\}
$$

where $X_{0}, Y_{0}, Z_{0}$ - the coordinates of the initial point on the line; $a, b, c$ - the direction cosines of the unit vector on the line; $t$ - parameter.

The values of the direction cosines determine the position of a straight line and it can be found the deviation of the building axis from the vertical and its inclination with respect to the coordinate axes in the plane.

The value of bend and even more twist impossible to determine from straight line equation. Therefore, we proposed to describe the deformations of twist using an approximation equation of a helix. Below two variants of the helix parametric Equations (2) are given:

$$
\left.\left.\begin{array}{l}
X(t)_{i}=X_{0}+a \cos \omega t_{i}  \tag{2}\\
Y(t)_{i}=Y_{0}+a \sin \omega t_{i} \\
Z(t)_{i}=Z_{0}+v t_{i}
\end{array}\right\} \Rightarrow \begin{array}{l}
X(t)_{i}=X_{0}+a \cos \varphi_{i} ; \\
Y(t)_{i}=Y_{0}+a \sin \varphi_{i} ; \\
Z(t)_{i}=Z_{0}+b \varphi_{i} .
\end{array}\right\}
$$

In the Equations (2), the following symbols are used: $\varphi_{i}=\omega t_{i}$ - angle of rotation of the straight line connecting the center of the helix and a specific point; $b=\frac{v}{\omega}=\frac{h}{2 \pi}$ - the coefficient of proportionality; $a-$ the
radius of the cylinder on which the helix is located; $h=2 \pi b$ - helix step; $\operatorname{tg} \alpha=\frac{h}{2 \pi a}=\frac{b}{a}$ - the rise of helix, $\alpha$ - the angle between the tangent to the curve and the plane XY.

Due to geometric properties the helix describes the deformation of a more complicated character. Using helix parameters the curvature at any point may be calculated:

$$
\begin{equation*}
\chi=\frac{a}{\left(a^{2}+b^{2}\right)} \tag{3}
\end{equation*}
$$

and helix twist:

$$
\begin{equation*}
\kappa=\frac{b}{\left(a^{2}+b^{2}\right)} \tag{4}
\end{equation*}
$$

If the twist and bend of the building caused by the simultaneous effect of temperature and wind loads, the overall deformation of the building will be more complicated than a simple twist. The deformation of the building will be uneven and vary depending on the altitude. For a description of this deformation conical helix can be used.

$$
\left.\left.\begin{array}{l}
X(t)_{i}=X_{0}+a t_{i} \cos \omega t_{i}  \tag{5}\\
Y(t)_{i}=Y_{0}+a t_{i} \sin \omega t_{i} \\
Z(t)_{i}=Z_{0}+v t_{i}
\end{array}\right\} \Rightarrow \begin{array}{l}
X(t)_{i}=X_{0}+a t_{i} \cos \varphi_{i} ; \\
Y(t)_{i}=Y_{0}+a t_{i} \sin \varphi_{i} \\
Z(t)_{i}=Z_{0}+b \varphi_{i}
\end{array}\right\}
$$

In the Equations (2), the following symbols are used: $\varphi_{i}=\omega t_{i}$ - angle of rotation of the straight line connecting the center of the helix and a specific point; $b=\frac{v}{\omega}=\frac{h}{2 \pi}$ - the coefficient of proportionality.

For comparison of the proposed models, we consider the use of polynomial models. As in all engineering applications, more complicated types of deformations are described by polynomial models. We take into account the fact that the bend of high-rise buildings due to the temperature effect is described by the following expression (Biktashev 2006) $\Delta=\frac{\alpha_{t} \Delta t H^{2}}{2 D}$, where $\alpha_{t}$ - thermal expansion coefficient of the material, $\Delta t$ - temperature difference, $H$ - the section highs of the building, $D$ - the average diameter of buildings. That is the dependence on temperature effect has a parabolic form. Then, for our purposes it is sufficient to use the quadratic model (6), which will describe the most difficult type of deformation, at the same time roll, bend and twist possible.

$$
\left.\begin{array}{l}
X(t)_{i}=X_{0}+a t_{i}+d t_{i}^{2} ;  \tag{6}\\
Y(t)_{i}=Y_{0}+b t_{i}+e t_{i}^{2} ; \\
Z(t)_{i}=Z_{0}+c t_{i}+f t_{i}^{2} ;
\end{array}\right\},
$$

where $a, b, c, d, e, f$ - the coefficients of the curve, which in the case of polynomial models do not have particular physical content.

In this case, the proximity of the equations obtained for the measured values will be indirectly indicate the presence of bend axis of the building. Just note that the use of polynomial models of higher order is unreasonable, because it is obvious that the accuracy of approximation in this case would rise, but to make the certain conclusion about the nature of the deformation process, such models do not allow.

## 3. The results of monitoring

As the object of investigation was chosen high-rise building, located in the central part of Kiev. The total height of the building is 161 m (Fig. 2). According to their functional purpose this building is a shopping and office complex.

Structurally, the building model is a frame-monolithic scheme. In the plan the building has a rectangular shape with rounded corners. For observations was established linear-angular network which carried out the observation points the prism reflector mounted on the facade of the building. The position of the building in a thigh housing allows to observe only the


Fig. 2. Image of the research object - a building in the center of Kyiv
north-eastern and south-eastern part of the building. It is in these parts of the facade were fixed by four prismatic reflectors. The reflectors have been installed at the following heights: SC1-SL1 - 25,8 m, SC2-SL2 $57,0 \mathrm{~m}, \mathrm{SC} 3-\mathrm{SL} 3$ - $91,1 \mathrm{~m}, \mathrm{SC} 4-\mathrm{SL} 4-127,2 \mathrm{~m}$. The geometry of the building and its position relative to the accepted coordinate system is shown in Figure 3.

It is obvious that on the results of observations at these points is very difficult to establish an overall picture of moving the building. Observations of the reflectors on the facade of the building were carried out with an interval of six months. The first cycle of observations was made in the summer of 2012. In this paper we present the results of observations after four cycles. In Table 1 below are shown the displacements which were calculated by determination of the coordinates of the mark.

Analysis of Table 1 shows that was fixed the actual displacement of the building axis, but the reason for this displacement is difficult to determine. Here we note that in the foundation of the building have been installed the deformation marks, which each year is laid geometric leveling course, in order to determine foundation roll. During the reporting period were fixed little values of sediments at $3-5 \mathrm{~mm}$. The irregular sediments as a source of building roll, were not fixed.

To perform the analysis of displacement using the considered by us above models of approximation perform the transformation of the measured displacements so that the direction of displacement coincided with the main axes of the building. The transformations displacements from XOY coordinate system to pon coordinate system is executed. The transformation is performed using the azimuth angle equal $324^{\circ}$ of the longitudinal axis of the building. Results converted displacements are shown in Table 2.

As a result of the transformation displacements, the nature of the deformation is not made clear, however, more clearly shows the direction in which the building is moved (Figs 4, 5). Before carrying out mathematical modeling displacements, let's look at represented in graphical form displacements of the last cycle in the direction of the cross axis.

Remarkable are the values of displacements obtained in the last cycle of observations. Values of displacement in cross axis direction several times higher than in the direction of the longitudinal axis, which indicates a possible bend of the building. At the same time values of these displacements to the south-eastern


Fig. 3. The geometry of the building and its position relative to the coordinate system accepted

Table 1. The calculated displacement

| Marks | Cycles |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 29.08 .2012 |  | 24.12 .2012 |  | 21.06 .2013 |  |
| $\Delta, \mathrm{~mm}$ | $\Delta X$ | $\Delta Y$ | $\Delta X$ | $\Delta Y$ | $\Delta X$ | $\Delta Y$ |
| SC1 | -2 | 0 | 0 | -3 | 1 | 5 |
| SC2 | -11 | 2 | -3 | -6 | -4 | 2 |
| SC3 | -4 | -1 | 1 | -6 | -4 | 2 |
| SC4 | -4 | 0,5 | -3 | -3 | -12 | 0 |
| SL1 | 0 | 0 | 0 | 0 | 0 | -2 |
| SL2 | -3 | -7 | 0 | 0 | 0 | 2 |
| SL3 | -4 | -12 | -5 | -18 | -10 | -13 |
| SL4 | -2 | -10 | -3 | -17 | -12 | -12 |

Table 2. Displacements, converted to the main axes of the building

| Marks | Cycles |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 29.08 .2012 |  | 24.12 .2012 |  | 21.06 .2013 |  |
| $\Delta, \mathrm{~mm}$ | $\Delta p$ | $\Delta n$ | $\Delta p$ | $\Delta n$ | $\Delta p$ | $\Delta n$ |
| SC1 | -1 | 1 | -2 | -2 | 4 | 4 |
| SC2 | 0 | 2 | -6 | -3 | -3 | 4 |
| SC3 | -4 | 1 | -2 | -6 | -2 | 4 |
| SC4 | -3 | 3 | -4 | -1 | -9 | 7 |
| SL1 | 0 | 0 | 0 | 0 | -1 | -2 |
| SL2 | -6 | -4 | 0 | 0 | 2 | 1 |
| SL3 | -9 | -8 | -14 | -12 | -16 | -5 |
| SL4 | -7 | -7 | -12 | -12 | -17 | -4 |



Fig. 4. The displacements of the reflectors in the direction of the cross axis for the north-east facade.


Fig. 5. The displacements of the reflectors in the direction of the cross axis for the south-east façade
part of the facade is twice higher than those displacements to the north-eastern part, which indicates the presence of the building twist.

## 4. Displacements research

In order to simulate displacements, we use the measurement results in the last cycle of the south-eastern part of the facade, shown in Table 2. We used the models (1), (2), (5) and (6) for the construction of models of a straight line, the helix, conic helix and parabolic curve.

Simulation carried out under the standard scheme of the least squares method. The model takes the form $A x=\Delta$. The solution to this system of equations, assuming that the observations have an equal accuracy are $\mathbf{x}=\left(\mathbf{A}^{T} \mathbf{A}\right)^{-1} \mathbf{A}^{T} \Delta$, where $\Delta$ - the column vector of the measured displacements.

For further calculations we need a coefficient matrix of corrections equations A. Each point gives three corrections equations and in our case the matrix of corrections equations will have 15 lines. Just find three corrections equations and continue to carry out the replacement of the parameter $t$. From Equations (1), (2), (5) and (6) the following matrix of corrections equations for each point will be:
for a straight line in space

$$
\mathbf{A}_{i}=\left[\begin{array}{cccccc}
1 & 0 & 0 & t_{i} & 0 & 0 \\
0 & 1 & 0 & 0 & t_{i} & 0 \\
0 & 0 & 1 & 0 & 0 & t_{i}
\end{array}\right],
$$

for a helix

$$
\mathbf{A}_{i}=\left[\begin{array}{ccccc}
1 & 0 & 0 & \sin \left(\omega t_{i}\right) & a t_{i} \cos \left(\omega t_{i}\right) \\
0 & 1 & 0 & \cos \left(\omega t_{i}\right) & -a t_{i} \sin \left(\omega t_{i}\right) \\
0 & 0 & 1 & 0 & 0
\end{array}\right],
$$

for a conic helix

$$
\mathbf{A}_{i}=\left[\begin{array}{cccccc}
1 & 0 & 0 & t_{i} \sin \left(\omega t_{i}\right) & a\left(1+\frac{\operatorname{tg}\left(\omega t_{i}\right)}{\omega t_{i}}\right) & 0 \\
0 & 1 & 0 & t_{i} \cos \left(\omega t_{i}\right) & a\left(-1+\frac{\operatorname{ctg}\left(\omega t_{i}\right)}{\omega t_{i}}\right) & 0 \\
0 & 0 & 1 & 0 & 0 & t_{i}
\end{array}\right],
$$

for parabolic curve in space

$$
\mathbf{A}_{i}=\left[\begin{array}{ccccccccc}
1 & 0 & 0 & t_{i} & 0 & 0 & t_{i}^{2} & 0 & 0 \\
0 & 1 & 0 & 0 & t_{i} & 0 & 0 & t_{i}^{2} & 0 \\
0 & 0 & 1 & 0 & 0 & t_{i} & 0 & 0 & t_{i}^{2}
\end{array}\right]
$$

Using the solutions of least squares method for various spatial curves were obtained parameters which are shown in Table 3.

Table 3. Parameters of space curves

| Para- <br> meters | Straight <br> Line | Helix | Conic <br> Helix | Parabolic <br> Curve |
| :---: | :---: | :---: | :---: | :---: |
| $X_{0}, \mathrm{~m}$ | $2,2 \times 10^{-3}$ | $-0,2 \times 10^{-3}$ | $-9,2 \times 10^{-4}$ | $0,7 \times 10^{-3}$ |
| $Y_{0}, \mathrm{~m}$ | $1,7 \times 10^{-3}$ | $-6,8 \times 10^{-3}$ | 0,0094 | $0,1 \times 10^{-3}$ |
| $Z_{0}, \mathrm{~m}$ | 0 | 0 | 0 | 0 |
| $a$ | $-0,1 \times 10^{-3}$ | $6,6 \times 10^{-3}$ | $1,1 \times 10^{-4}$ | $-1,1 \times 10^{-5}$ |
| $b$ | $-0,1 \times 10^{-3}$ | $-8,4 \times 10^{-3}$ | $-1,7822$ | $-1,6 \times 10^{-5}$ |
| $c$ | 1 | 1 | 1 | 1 |
| $d$ | - | - | - | $-0,8 \times 10^{-6}$ |
| $e$ | - | - | - | $-0,8 \times 10^{-6}$ |
| $f$ | - | - | - | 0 |

Using these parameters for different equations were calculated corrections and estimated error of approximation.

Table 4. Corrections and the accuracy of the approximation

| Correction | Straight <br> Line | Helix | Conic <br> Helix | Parabolic <br> Curve |
| :---: | :---: | :---: | :---: | :---: |
| $v_{\max }, \mathrm{mm}$ | 4 | 2 | 4 | 5 |
| $v_{\min }, \mathrm{mm}$ | -7 | -7 | -4 | -5 |
| $\sum\|v\|, \mathrm{mm}$ | 25 | 17 | 23 | 23 |
| $\mu, \mathrm{~mm}$ | 3.4 | 2.9 | 2.5 | 3.8 |

We perform analysis of the data presented in Table 3 and Table 4. For this we compare the accuracy of different approximation methods and compare the received corrections and the accuracy of the parameter estimates.

## 5. Discussion

From Table 4 it is clear that the best approximation provides a helix. It provides minimum variance, as well as a minimum sum of corrections by module.

The quality of the approximation can be checked by calculating the accuracy of the parameter estimates and verifying the condition fulfillment:

$$
\begin{equation*}
P_{i} \geq 3 m_{P_{i}}, \tag{7}
\end{equation*}
$$

where $P_{i} \geq 3 m_{P_{i}}$ - curve parameter; $m_{P_{i}}=\mu_{i} \sqrt{Q_{P_{i}}}$ accuracy parameter estimation $P_{i}$.

We are performed calculations for all curves and presented them in Table 5, where the symbol " + " is indicated by compliance with the conditions (7) and the symbol " $x$ " violation of condition (7).

Table 5. Quality estimation of approximation

| Parameter <br> estimation | Straight <br> Line | Helix | Conic <br> Helix | Parabolic <br> Curve |
| :---: | :---: | :---: | :---: | :---: |
| $m_{X_{0}}, \mathrm{~m}$ | x | x | x | x |
| $m_{Y_{0}}, \mathrm{~m}$ | x | + | x | x |
| $m_{Z_{0}}, \mathrm{~m}$ | x | x | x | x |
| $m_{a}$ | x | + | x | x |
| $m_{b}$ | x | + | x | x |
| $m_{c}$ | + | + | + | + |
| $m_{d}$ | - | - | - | x |
| $m_{e}$ | - | - | - | x |
| $m_{f}$ | - | - | - | x |

Note that all the parameters of the spatial line are determined not reliable. This indicates that the displacements character has a nonlinear dependence.

On the other hand, the conical helix and parabolic curve approximation do not provide the required accuracy. This fact can be explained by the nature of the mismatch of the proposed models to the deformation process.

In the case of the helix with the required accuracy defined parameters $a$ and $b$ on which can calculate the curvature and twist of a helix. As we see among all the curves, the parameters of the helix determined most accurately and therefore this curve best describes the deformation of a high-rise building.

## Conclusions

In the work we considered the research of high-rise buildings displacements using spatial curves. Displacements and deformation of high-rise buildings is a process that describes by very complex mathematical expressions. Calculation of possible displacements of high-rise buildings performs by methods of structural mechanics. But the quality of these calculations depends essentially on the initial data and the construction of models which is performed by numerical methods.

If the displacement of construction occurs during operation, then performing displacements simulation using structural mechanics methods is difficult. So, it is necessary to solve the inverse problem. By the measured displacement to determine the parameters of the model. However, if the numerical methods used in finding the model parameters, it is extremely difficult to solve the inverse problem. Therefore, we proposed to carry out the construction of models through the approximation of results of geodetic measurements.

This approach is the optimal solution. Construction of deformation process models was done using the straight line, parabolic curve, helix and conical helix. The methodic of models construction was considered and on the specific example illustrates the effectiveness of the proposed approach. The results allow to perform a detailed analysis of the nature of the deformation of a high-rise building and get the optimal equation for the displacements simulation.

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