EXOTIC OPTIONS: A CHOOSER OPTION AND ITS PRICING

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Abstract. Financial instruments traded in the markets and investors’ situation in such markets are getting more and more complex. This leads to more complex derivative structures used for hedging that are harder to analyze and which risk is harder managed. Because of the complexity of these instruments, the basic characteristics of many exotic options may sometimes be not clearly understood. Most scientific studies have been focused on developing models for pricing various types of exotic options, but it is important to study their unique characteristics and to understand them correctly in order to use them in proper market situations. The paper examines main aspects of options, emphasizing the variety of exotic options and their place in financial markets and risk management process. As the exact valuation of exotic options is quite difficult, the article deals with the theoretical and practical aspects of pricing of chooser options that suggest a broad range of usage and application in different market conditions. The calculations made in the article showed that the price of the chooser is closely correlated with the choice time and low correlated with its strike price. So the first mentioned factor should be taken into consideration when making appropriate hedging and investing decisions.

Keywords: call option, put option, exotic option, price, value, chooser, time to expiration, strike price.


JEL classification: G100, G130.

1. Introduction

The development of financial markets and growing uncertainty of its participants are the main incentives to look for the new, flexible financial instruments. The level of investment risk is increasing simultaneously, so everyone operating in financial markets has to react suddenly to market changes and to correct his investment strategy in time. For this reason investors are looking for new investment possibilities that could fit changing situation in the market and generate income from the investment.

Derivative securities, if they are used correctly, can help investors to increase their expected returns and minimize their exposure to risk. For the first time exchange listed
options were traded in 1973. Since then, the volumes of their trade had risen sharply all over the world. This development was determined by the specific features that options include. Empirical researches presented in financial literature (Friedentag 2000; Kotze 2011; Whaley 2006; Cuthbertson, Nitzsche 2003) illustrated that options can be used in many ways to create various investment opportunities. These financial means offer leverage and insurance for investors that are risk-averse. Leverage in options trading means that a change in the underlying will result in a greater percentage change in the option, everything else being equal (Krawiec 1998). More risky investors can use these instruments as a mean of speculation.

Standard types of options are traded actively but new types of options arise as investors try to create a hedging for their investment portfolios. Over the last decade the size of the market of exotic options in well-developed financial markets has expanded. A large variety of such complex instruments is available to investors. Exotic options are a generic name given to derivative securities that have more complex cash-flow structures than standard put and call options. The principal motivation for trading exotic options is that they permit a much more precise views on future market behavior than those offered by “plain vanilla” options. Some exotic options provide high leverage because they can focus on the payoff structure very precisely. Exotics are usually traded over the counter and are marketed to sophisticated corporate investors or hedge funds (Avellaneda, Laurence 2000; Whaley 2006). They have almost unlimited flexibility and can be adapted to the specific needs of any investor. These options are playing a significant hedging role and, thus, they meet the hedgers’ needs in cost effective ways. Exotic options are usually less expensive and more efficient than standard instruments. Exotic options can be used as attractive investments and trading opportunities.

According to the situation in the markets it could be stated that more complex relationships between investors and financial intermediaries, growing uncertainty will increase the use of these derivative securities, so it is purposeful to analyze deeper the abundance of suggested exotics and one of the main aspects of trading in any security, i.e. their proper evaluation. In order to use all kinds of options investors meet with a problem of determining the price of a product which depends on the performance of another security. Because of their specific features not all standard methods of option pricing are adequate to price exotics.

The goal of research is to examine and analyze the valuation of chosen complex derivative security.

Logical analysis and synthesis of scientific literature, comparative analysis and graphical modeling were used for the research.

2. Theoretical aspects of exotic options

An option can be described as an instrument giving its owner the right but not the obligation to buy or sell something at in advance fixed price. Options are available on
a wide range of products, beginning from grain, raw materials and ending in financial assets, gold or real estate (Hull 2000; Friedentag 2000). In this article the main attention is paid on stock options, because exchange traded stock options probably have the longest trading history of any derivative since their successful launch on the Chicago Board of Exchange in 1973. These options are spread across both exchange-traded and over-the-counter markets (Martin 2001).

There are two types of stock options – calls and puts. A call option gives the holder the right to buy specified quantity of the stock at the strike price on or before expiration date. The writer of the option, however, has the obligation to sell the underlying asset if the buyer of the call option decides to exercise his right to buy. A put option gives the holder the right to sell specified quantity of the underlying stock at the strike price on or before an expiry date. The writer of the put option has the obligation to buy the agreed stock at the strike price if the buyer decides to exercise his right to sell. The option holder is the person who buys the right conveyed by the option. The option writer or seller is obliged to perform according to the terms of the option. Strike price or exercise price is the price at which the option holder has the right either to purchase or to sell the underlying asset (Jarrow, Rudd 1983).

The style of an option refers to when that option is exercisable. According to Options Clearing Corporation (OCC) there may be three different styles of options: American style, European style and capped. An American style option may be exercised at any time prior to its expiration. European style option may be exercised only during a specified period before the option expires. Usually such an option is exercised on its expiration day (Hull 2000).

All mentioned aspects actual to plain options are the same for the exotics as well. Exotic options may have uncertain exercise prices, expiration time and several underlying assets, which may not follow lognormal or normal diffusion processes. Exotics are mainly over the counter instruments.

According to Mark Rubinstein and Eric Reiner (1992), exotic options can be divided into eleven classes:

1) packages – options which are equivalent to a portfolio containing only standard European calls and possibly cash and the underlying asset itself;
2) forward-start options - options which are paid for in the present but which are only received at a prespecified future date;
3) compound options – options whose underlying assets are themselves options;
4) chooser options - options which are paid for in the present but which at some prespecified future date are chosen to be either a put or a call;
5) barrier options – options whose payoff depends not only on the price at expiration of the underlying asset but also on whether or not the underlying asset has previously reached some other “barrier” price;
6) binary options – options with binary and discontinuous payoff patterns;
7) lookback options – options whose payoff depends not only possibly on the price
at expiration of the underlying asset but also on the minimum or maximum price experienced by the underlying asset during at least some portion of the life of the option;

8) “Asian” options – options whose payoff depends not only possibly on the price at expiration of the underlying asset but also on the average price experienced by the underlying asset during at least some portion of the life of the option;

9) exchange options – options to exchange one asset for another which can be exercised early;

10) currency translated options - options whose underlying asset or strike price is denominated in a foreign currency at a random or prespecified exchange rate;

11) two-colour “rainbow” options – options on two risky assets which cannot be interpreted as if they were options on a single underlying asset.

Other authors (Hull 2000; Cuthbertson, Nitzsche 2003; Masson 2011; Jian et al. 2011; Zhang 1995) besides the mentioned types of exotic options, emphasize non-standard American options, such as Bermudan options and corporate warrants, and shout options.

According to the analyzed literature (Hull 2000; Bellalah 2010; Zhang 1995; Masson 2011; Rubinstein, Reiner 1992) Bermuda options, Asian options, banner options, compound options and digital options are mostly analyzed because of their complex structure and pricing characteristics. The many scientific articles devoted to analysis of pricing problems of those options (Predota 2005; Bin, Fei 2010; Ye 2009; Schoutens, Symens 2003; Nielsen, Sandmann 2003; Andreeva et al. 2010; Buchen 2004). But chooser options are most interesting ones to discuss, because they are characterized as exotic options for their difficult pricing and complex structure. They have been traded since July of 1990 with the initial contracts traded by Bankers Trust (Bampou 2008).

A chooser option is part of the compound option family. It is an option on options and is one of path-dependent options. A standard chooser option is purchased in the present, but gives its holder the right to decide at some point in time, but before maturity, whether the option will finally be a put or a call (Fig. 1). This option is sometimes named “you-choose” or “as-you-like” option (Rubinstein, Reiner 1992; Hull 2000). The strike prices of either the put or the call can be the same but need not necessarily be the same (De Weert 2008). If the strikes are the same, the chooser option is referred to as a simple chooser. When the strikes or even expiry dates are not the same, the chooser is referred to as a complex chooser (De Weert 2008). The chooser option can be European or American.

A chooser option is suitable for investors who expect strong volatility of the underlying asset but who are not uncertain about direction of the change. When the underlying asset rises over the period of time, the holder of the option will choose the call option because it will have a higher value than the put option. In the case of falling value of the underlying asset, the choice will be the put option (Simons 2000; Deacon et al. 2004).
The chooser option is held to be similar to holding a straddle strategy which consists of a put and call options sharing the same strike price. They are similar in that the option not betting on the stock will be bullish or bearish (Whaley 2006). But the chooser holder does not have to pay for both options entirely. He is flexible to decide which option to buy later. A straddle holder has to pay for both options immediately. For this reason the chooser options are less expensive than straddles (Rubinstein, Reiner 1992; Avellaneda, Laurence 2000).

3. Main principles of option pricing

Because of the complex valuation of option contracts, the main scientific studies are devoted to analysis of separate methods of options pricing (Hull 2000; Jarrow, Rudd 1983; Martin 2001; Kancerevyčius 2003; Franke, Hardle 2008).

The most popular valuation model for options is the Black-Scholes model. The model is based on the theory that markets are arbitrage free and assumes that the price of the underlying asset is characterized by a Geometric Brownian motion. This method is commonly used for pricing European options as there is an analytic solution for their price (Bampou 2008).

Another technique for pricing options is the binomial lattice model. In essence, it is a simplification of the Black-Scholes method as it considers the fluctuation of the price of the underlying asset in discrete time. This model is typically used to determine the price of European and American options (Bampou 2008).

Monte Carlo simulation is a numerical method for pricing options. It assumes that in order to value an option, we need to find the expected value of the price of the underlying asset on the expiration date. Since the price is a random variable, one possible way of finding its expected value is by simulation. This model can be adapted to price almost any type of option (Bampou 2008).
The main options pricing models contain five factors that are used to determine a theoretical value for an option and which have to be taken into account when pricing option contracts (Hull 2000):

1) market price of the underlying asset;
2) strike price;
3) time to expiration;
4) volatility of the underlying asset;
5) interest rates;
6) dividends expected during the life of the option.

*Market price and strike price.* The payoff from a call option will be the amount by which the stock price in the market exceeds the strike price dealt with the option. Call options therefore become more valuable as the stock price increases and less valuable as the strike price increases. For a put option, the payoff on exercise is the amount by which the strike price exceeds the stock price (Laurence, Avellaneda 2000). So the put option becomes less valuable as the stock price increases and more valuable as the strike price increases.

*Time to expiration.* Both put and call American options become more valuable as the time to expiration increases. European put and call options do not necessarily become more valuable as the time to expiration increases. This is because it is not true that the owner of a long-life European option has all the exercise opportunities open to the owner of a short-life European option.

*Volatility.* The volatility of a stock price is a measure of how uncertain we are about future stock price movements. As volatility increases, the chance that the stock price will change in both directions increases. The value of both calls and puts therefore increases as volatility increases (Hull 2000; Martin 2001).

*Risk-free interest rate.* The risk-free interest rate affects the price of an option in a less clear-cut way. Without additional assumptions it is difficult to gauge the effect of increasing interest rates. Since increasing interest rates decrease the present value of the exercise price, there is a tendency for call values to increase and put values to decrease. It should be emphasized that these results assume that all variables remain fixed. In practice, when interest rates fall (rise), stock prices tend to rise (fall). The net effect of an interest rate change and the accompanying stock price change therefore may be different from that just given (Hull 2000; Jarrow, Rudd 1983).

*Dividends.* Dividends have the effect of reducing the stock price on the ex-dividend date. The values of call options are negatively related to the size of any anticipated dividend, and the value of a put option is positively related to the size of any anticipated dividend.

As vanilla options are traded in exchange markets, more possibilities to find historical information about real market prices. In the case of exotic options there are not many of such possibilities because these contracts in many cases are over-the-counter contracts. Choosing adequate for market conditions pricing model is crucially important.
4. Empirical research

It is not easy to determine the right price of option contract in practice. A big number of pricing models and programs were generated to solve this problem. The Black-Scholes model and the Cox, Ross and Rubinstein binomial model are the primary pricing models. The Black-Scholes model is used to calculate a theoretical price of the option (ignoring dividends paid during the life of the option) using the five key determinants of an option’s price mentioned before: stock price, strike price, volatility, time to expiration and risk-free (short-term) interest rate (Brenner, Subrahmanyam 1994). Some assumptions were made to derive Black-Scholes model: there are no transaction costs, there are no dividends during the life of the option, there are no riskless arbitrage opportunities, security trading is continuous, the stock price follows geometric Brownian process with mean and standard deviation constant and the risk-free rate of interest is constant and the same for all maturities (Martin 2001).

Although Black-Scholes model is derived for valuing European call and put options on a non-dividend-paying stock, this model can be extended to deal with European call and put options on dividend-paying stocks, American options or options with different underlying assets (Hull 2000; Martin 2001; Jarrow, Rudd 1983; Laurence, Avellaneda 2000; Whaley 2006):

\[
c = S e^{-q(T-t)} N(d_1) - X e^{-r(T-t)} N(d_2),
\]
\[
p = X e^{-r(T-t)} N(-d_2) - S e^{-q(T-t)} N(-d_1),
\]
\[
d_1 = \frac{\ln(S/X) + (r-q+\sigma^2/2)(T-t)}{\sigma \sqrt{T-t}},
\]
\[
d_2 = d_1 - \sigma \sqrt{T-t}.
\]

where \( c \) – premium of European call option; 
\( p \) – premium of European put option; 
\( S \) – stock price; 
\( X \) – exercise price; 
\( T-t \) – time to maturity; 
\( r \) – risk free interest rate; 
\( q \) – dividends; 
\( \sigma \) – volatility of stock price; 
\( N_1, N_2 \) – the cumulative normal distribution function.

As it was mentioned before, a chooser option allows the investor to choose at a specific point in time \( t (t<T) \) whether the option is to be call or put. Once this choice was made at \( t \), the option stays as either a call or a put to maturity \( (T) \) (Cuthbertson, Nitzsche 2003).

At this moment the chooser has the same payoff as the straddle strategy, but it will be cheaper. The reason for the lower premium is that the straddle always has a payoff and the chooser can end in out-of-the-money.
An analytical solution for pricing the chooser option is possible because if the options underlying the chooser are both European and have the same strike price, put-call parity can be used to provide a valuation formula. This was proven by Rubinstein in 1991.

At time $t$ the investor will choose the higher valued of the two options, that is he will choose the call if (Cuthbertson, Nitzsche 2003):

$$C(S^*_t, K, T-t) > P(S^*_t, K, T-t).$$

According to the put-call parity $(P + C = C + Ke^{-r(T-t)})$, then the equation reduces to:

$$C > C + Ke^{-r(T-t)} - S^*_t.$$  \hspace{1cm} (6)

The investor will choose the call at $t$ when the current stock price exceeds the present value of the strike price $S^*_t > Ke^{-r(T-t)} = K^*$. The payoff from the chooser at time $T$ is presented in Table 1 (Cuthbertson, Nitzsche 2003; Deacon et al. 2004; Sandman, Wittke 2010):

**Table 1. Payoff from the chooser options at time $T$**

<table>
<thead>
<tr>
<th>Choice at time $t$</th>
<th>Payoff at $T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_T \leq K$</td>
<td></td>
</tr>
<tr>
<td>Call</td>
<td>$0$</td>
</tr>
<tr>
<td>Put</td>
<td>$K - S_T$</td>
</tr>
</tbody>
</table>

At the time when the choice is made the value of the chooser option is (Cuthbertson, Nitzsche 2003; Rubinstein, Reiner 1992)

$$Chooser_{simple} = \max\{C(S^*_t, K, T-t), P(S^*_t, K, T-t); t\}.$$ \hspace{1cm} (7)

Where $C$ is the value of the call underlying the option and $P$ is the value of the put underlying the option.

In the case of a simple chooser (Cuthbertson, Nitzsche 2003; Rubinstein, Reiner 1992):

$$P(S^*_t, K, T-t) = C(S^*_t, K, T-t) + Ke^{-r(T-t)} - S^*_t e^{-\eta(T-t)}.$$ \hspace{1cm} (8)

It follows that the payoff of a simple chooser is (Cuthbertson, Nitzsche 2003; Rubinstein, Reiner 1992):

$$C(S^*_t, K, T-t) + \max\{0, Ke^{-r(T-t)} - S^*_t e^{-\eta(T-t)}\}.$$ \hspace{1cm} (9)

And the current value of the option is

$$Chooser = e^{-r(T-t)}E[C(S^*_t, K, T-t)] + e^{-r(T-t)}E[\max\{0, Ke^{-r(T-t)} - S^*_t e^{-\eta(T-t)}\}].$$ \hspace{1cm} (10)

The expectations are taken at time $t$.

The first term evaluates the current value of a call option with underlying asset price $S$, strike price $K$ and time to maturity $T-t$, the second term is the value of a
put with underlying asset price $S e^{-q(T-t)}$, strike price $K e^{-r(T-t)}$ and time to expiration $T-t$ (Cuthbertson, Nitzsche 2003; Rubinstein, Reiner 1992). Thus:

$$Chooser = C(S, K, T-t) + P(S, e^{-q(T-t)}, K e^{-r(T-t)}, T-t).$$ (11)

Using Black-Scholes formulas (1–4) presented above the value of a simple chooser option can be calculated (Cuthbertson, Nitzsche 2003).

A complex chooser option is similar to a standard chooser except that either the call/put striking prices, call/put time to expiration or both are not identical. The payoff from a complex chooser is written as (Hull 2000):

$$Chooser \text{ max } [C(S, K_c, T_c), P(S, K_p, T_p)]$$ (12)

Complex choosers can be valued using the concept of an option on an option, or a compound option. An option on an option is an option for which the underlying asset is an option (Cuthbertson, Nitzsche 2003).

To illustrate chooser option pricing in practice the empirical example was analyzed. Consider the European chooser option on stock ABC with a maturity of one year. The underlying asset price $S$ is 50EUR, the risk free rate is 10%, the dividend yield on the asset is 5% and the volatility of the asset is 20%. The chooser option provides the choice at date $t$ between a call and put of the same strike $K=50$EUR and maturity date $T=1$ year.

As the choice date $t$ varies, the values of the simple chooser option change too and are as presented in Table 2.

<table>
<thead>
<tr>
<th>T</th>
<th>0.00</th>
<th>0.09</th>
<th>0.16</th>
<th>0.25</th>
<th>0.33</th>
<th>0.41</th>
<th>0.50</th>
<th>0.58</th>
<th>0.67</th>
<th>0.75</th>
<th>0.83</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chooser</td>
<td>4.96</td>
<td>5.26</td>
<td>5.57</td>
<td>5.87</td>
<td>6.13</td>
<td>6.36</td>
<td>6.57</td>
<td>6.77</td>
<td>6.96</td>
<td>7.14</td>
<td>7.31</td>
<td>7.61</td>
</tr>
</tbody>
</table>

The relationship between analyzed two factors is almost linear (see Fig. 2).
As it was mentioned in the article if the choice time is equal to 0, the value of a chooser option is equal to the value of the simple call option under analyzed circumstances. If the choice time is equal to 1, the chooser value is equal to the value of simultaneous purchase of the call and put that is for the straddle strategy results. In order to check the statement, Black-Scholes option pricing model was used and call and straddle values were calculated:

**Table 3.** The value of simple call option and straddle strategy

<table>
<thead>
<tr>
<th>t = 0</th>
<th>t = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call = 4.97</td>
<td>Call = 4.97; put = 2.65, straddle = 7.62</td>
</tr>
</tbody>
</table>

The results received from the calculations confirmed that in time moments 0 one chooser is equal to the value of simple call and straddle strategy values (Tables 2 and 3).

Another important factor having influence on the value of chooser option is strike price agreed with the option. The results showing the relationship between the simple chooser value and its strike price are shown in Table 4 and Fig. 3.

**Table 4.** Relationship between the simple chooser value and its strike price

<table>
<thead>
<tr>
<th>K</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chooser</td>
<td>29.47</td>
<td>20.42</td>
<td>11.75</td>
<td>6.59</td>
<td>8.96</td>
<td>16.22</td>
<td>24.92</td>
<td>33.90</td>
</tr>
</tbody>
</table>

Fig. 3. Graphical presentation of relationship between strike price and chooser price

In order to find out the relationship and influence of two analyzed factors, the covariance coefficient was evaluated between the value of chooser option and its strike price and choice time. The results showed that correlation between value and strike price is not so strong and equals to 0.21. The correlation between option value and time
of choice is very strong and is equal to 0.99. Another factor that should be taken into consideration is volatility measurement of the underlying asset because time of choice highly depends on how volatile underlying asset is.

5. Conclusions

The study examined the variety of existing exotic options and main aspects of option pricing. For deeper analysis chooser options were taken. The following conclusions can be made:

1. Using financial engineering exotic options can be applied almost in every market situation. According to the analyzed literature the biggest attention is paid to Asian, Bermuda and barrier options. Chooser options are worth analyzing because of their dual nature. Using these options investors are not sure about market direction.

2. The main factors in option pricing are market price of the underlying asset, strike price, volatility of the asset, time to maturity of the contract, interest rates and dividends. The same factors influence prices of exotic options too.

3. The pricing formulas for chooser options can be obtained in an environment of Black-Scholes.

4. Calculations made in the article confirmed that if the choice time of contract is the current date the value of a chooser option is equal to the value of the simple call option. If the choice time is equal to one, the chooser value equals the value of a straddle strategy.

5. Correlation between the chooser value and strike price is not strong but the influence of time of choice must be taken into consideration because of its influence on chooser option price.

References


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