THE INFLUENCE OF PRESTRESS LOSSES ON THE ANCHORAGE ZONE OF PRESTRESSED CONCRETE MEMBER

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Abstract. Article briefly discusses the factors affecting bond between concrete and reinforcement of prestressed concrete member. Behavior of prestressed concrete member during release of reinforcement is discussed. Comparative analysis of equations, which describes relation between strand draw-in and transfer length, is presented. Article analyses distribution of loss of prestress at the anchorage zone of prestressed concrete member. Estimation of rational position of strands at the anchorage zone of flexural prestressed concrete member was performed. Influence of constant and linearly variable stress distribution at the end of the strand and member cross-section height on the position of the reinforcement at the member cross-section is analysed.

Keywords: transfer length, anchorage zone, bond, loss of prestress, slip.

Introduction

Prestressed concrete structure can be pretensioned or post-tensioned depending on when the prestressing force is introduced. Special anchors are used to ensure reinforcement anchorage in the post-tensioned structure. Bond between concrete and reinforcement usually ensures anchorage of pretensioned strands. The prestressing force is transferred to the concrete by releasing the prestressed strands. Prestressed strands are shortening and slip occurs along the transfer length during release. Transfer length is influenced by bond between reinforcement and concrete, which is ensured by three factors: adhesion at the interface between concrete and steel; friction between the concrete and steel; mechanical resistance due to interlocking of the twisted strand wires and the surrounding concrete. The following characteristics are influenced by type of reinforcement and surface condition, concrete strength, concrete age, strand positioning etc. Adhesion and friction at the interface between concrete and steel don't have a major impact for the strands anchorage in the concrete. Therefore, the main bond characteristic is the mechanical interaction between the prestressing strand and the concrete. Mechanical resistance due to interlocking of the twisted strand wires and the surrounding concrete occurs after adhesion failure between concrete and steel. Mechanical bond between concrete and steel is ensured by surface roughness of the strand (helical shape of the strand, indentations). Strands can be composed of different amount of twisted wires, which takes a helical shape. Mechanical bond between strand and concrete is achieved by cement paste filling in the strands surface indentations and other roughness. Mechanical bond is increased by Hoyer or wedge effect and additional indentations on every wire in the strand.

During release of pretensioned strands there are no anchoring devices at the end of prestressed concrete member. Therefore, stresses at the end of the strand are equal to zero. Sufficient stress transfer from the pretensioned strand to the concrete depends on quality and stiffness of the concrete surrounding the strand.
1. Strand draw-in at prestress transfer

Diameter of the strand decreases during tensioning. Strands are smoothly covered by concrete during production of prestressed concrete member. Therefore a bond between concrete and strand surface occurs.

During release of the pretensioned reinforcement from the prestressing frame strands are shortening and adhesion and mechanical bond are damaged at the ends of prestressed concrete member. When the strands are cut, some small end slip is expected due to loss of stress within the transfer length. At the ends of the strands stresses become equal to zero. Then after diameter reduction during tensioning the ends of the strands regain an initial diameter. A diameter of the strand gradually decreases at transfer length and it is constant beyond the transfer length. This effect is called Hoyer or wedge effect (Hoyer, Friedrichy 1939; Arab et al. 2011). In prestressed concrete, the expansion of the strand is prevented by the hardened concrete. Therefore compressive forces perpendicular to the steel-concrete interface are induced in concrete. The pressure created as the strand tries to expand produces the normal forces needed to create a friction reaction during the strand draw-in (Fig. 1). The friction bond between concrete and strand depends on surface characteristics of the reinforcement, the coefficient of friction between the strand and concrete, and the strength of concrete. Whereas the transfer length is affected by the type (wire, strand or bar), diameter, surface (smooth, with indentations) of reinforcement, stresses in the reinforcement, strength of the concrete at the release of the reinforcement, type of release of the reinforcement (flame cutting, gradual release), consolidation of the concrete, protective layer of the concrete.

Strands slip only at transfer length during release. Compatibility of strains between prestressing strand and concrete \( (e_c = e_s) \) is in the middle of the prestressed concrete member, where there is no slip.

The measurement of the strand draw-in is an indirect method to determine the transfer length in prestressed concrete members. There are many proposed formulas describing relation between pre-tensioned strand draw-in \((\delta)\) during release and transfer length \((l_{pt})\).

Two expressions were proposed by G. Balazs to determine the transfer length considering the strand diameter and concrete compressive strength (Balazs 1992, 1993):

\[
l_{pt} = 105 \cdot \delta \cdot \frac{\delta^{3/2}}{f_{ck}}; \quad (1)
\]

\[
l_{pt} = \frac{111 \cdot \delta^{0.625}}{f_{ck}^{0.15} \cdot \left(\frac{f_{pk}}{E_p}\right)^{0.4}}, \quad (2)
\]

where \(l_{pt} - \) transfer length; \(\delta - \) strand draw-in at the free end of a prestressed concrete member; \(f_{ck} - \) characteristic compressive strength of concrete at the time of prestress transfer; \(f_{pk} - \) characteristic strand stress immediately before release; \(E_p - \) modulus of elasticity of the prestressing strand.

W. T. Marshall and D. Krishnamurthy proposed a formula, which describes relation between transfer length and strand draw-in (Marshall, Krishnamurthy 1969):

\[
l_{pt} = \frac{\delta}{\sqrt{K}}, \quad (3)
\]

where \( \delta \) - strand draw-in at the free end of a prestressed concrete member; \( f_{ck} - \) characteristic compressive strength of concrete at the time of prestress transfer; \( f_{pk} - \) characteristic strand stress immediately before release; \( E_p - \) modulus of elasticity of the prestressing strand.

D. R. Rose and B. W. Russell proposed a formula to calculate the transfer length (Rose, Russell 1997)

\[
l_{pt} = 2 \cdot \delta \cdot \frac{E_p}{f_{pk}} + 137.16, \quad (4)
\]

where \( f_{pk} - \) characteristic strand stress immediately before release.

Maximum limit of pretensioned strands draw-in can be calculated according to C. V. Herrero hypotheses (Herrero et al. 2013)

\[
\delta \leq \frac{2 \cdot f_{pk} - f_{pe}}{2 \cdot E_p} \Rightarrow l_{pt, \text{min}} = \frac{2 \cdot E_p \cdot \delta}{2 \cdot f_{pk} - f_{pe}}, \quad (5)
\]

where \( f_{pk} - \) characteristic strand stress immediately before release; \( f_{pe} - \) effective stress transferred to the pretensioned member (which considers the prestress losses due to elastic shortening, conditioned by the concrete’s modulus of elasticity).
Relationship between transfer length \((l_{pt})\) and strand draw-in \((\delta)\) can be expressed (Guyon 1960):

\[
l_{pt} = \alpha \cdot \frac{E_p}{f_{pk}} \cdot \delta, \tag{6}
\]

where \(\alpha\) – coefficient representing the shape factor of the bond stress distribution along the transfer zone: \(\alpha = 2\) for uniform bond stress distribution (linear variation in strand stress); \(\alpha = 3\) for linear descending bond stress distribution (parabolic variation in strand stress); \(f_{pk}\) – characteristic strand stress immediately before release.

Other authors (Oh, Kim 2000; Logan 1997; Russell, Burns 1996) performed experimental research and calculated transfer length \((l_{pt})\) according to formula (6), and theoretical and experimental results showed sufficient coincidence. Therefore transfer length \((l_{pt})\) may be calculated, with enough accuracy, according to measurements of pretensioned strands draw-in \((\delta)\).

Strand transfer length directly depends on the shape of bond stress distribution between strand and concrete. Many authors suggest that bond stress distribution should be evaluated with (Guyon) coefficient that represents the shape factor of the bond stress distribution along the transfer zone. Several experimental and theoretical studies subsequent to Guyon’s theoretical analysis have reported \(\alpha\) values ranging from 1.5 to 4.

The following equations for calculating \(\alpha\) coefficient where developed by C.V. Herrero (Herrero et al. 2013):

\[
\alpha = \frac{1}{1 + \ln \left( \frac{f_{pk}}{f_{pk} - B \cdot f_{pe}} \right) \cdot \frac{f_{pk}}{B \cdot f_{pe}}}; \tag{7}
\]

\[
B = 1 + \frac{V_c \cdot C_2}{V_p}; \tag{8}
\]

\[
C_2 = \frac{A_p \cdot E_p}{A_c \cdot E_c}; \tag{9}
\]

where \(V_c\) – Poisson’s modulus of concrete; \(V_p\) – Poisson’s modulus of the prestressing strand or wire; \(A_p\) – cross-sectional area of the prestressing strand; \(A_c\) – concrete area of the prestressing strand’s zone of influence; \(E_c\) – concrete modulus of elasticity at the time of transfer; \(f_{pk}\) – characteristic initial stress of the pretensioned strand; \(f_{pe}\) – effective stress transferred to the pretensioned member (which considers the prestress losses due to elastic shortening, conditioned by the concrete’s modulus of elasticity).

According to equations (7), (8) and (9) coefficient \(\alpha\) depends on the initial prestress, the effective prestress transferred to the pretensioned member and Poisson’s effect of the concrete and pretensioned strand, which causes radial tensile stresses.

Most of that factors aren’t estimated in calculation of transfer length according to Guyon’s formula (6). At the time of release measurement accuracy of strands draw-in depends on bond between concrete and strand. Formulas for the transfer length calculation (1)...(6) are applicable when a bond between concrete and strand is sufficient. Otherwise the strand draw-in will be too deep and the wrong transfer length will be calculated.

![Graphical dependence of pretensioned strands draw-in and transfer length](image)

Fig. 2. Graphical dependence of pretensioned strands draw-in and transfer length: 1 – (1); 2 – (2); 3 – (3); 4 – (4); 5 – (5); 6 – (6), \(\alpha = 2\); 7 – (6), \(\alpha = 3\)

Graphical dependence of pretensioned strands draw-in and transfer length is presented in Fig. 2. Results according to formulas (1)...(6) show the same tendency, i.e., more strand slips into the concrete during release, longer transfer length will be needed to ensure strand stress transfer to the concrete. Most of the curves in the graph ((4), (5), (6), (7)) are linear and some are parabolic ((1), (2), (3)). Expression (5) presented by C. V. Herrero describes upper limit of the strand draw-in. Every formula is derived on the basis of experimental results of specimens with different seven wire strands diameter.

2. Stress distribution at transfer zone

Sudden shortening of the strand occurs and stress induced in the strand changes after strand release from abutments. Because of the strand draw-in stresses in the strand at the ends of the prestressed concrete element are equal to zero. Strand anchors in the con-
crete at the prestressed concrete member end due to Hoyer’s effect. Strand stress of the prestressed concrete element increases from zero at the element ends to the maximum value at the end of the transfer length. Strand stress increases linearly in this zone. Maximum stresses are reached at the end of transfer length and beyond the transfer length stresses are constant (see Fig. 3) (Oh, Kim 2000; Russell, Burns 1996; Rose, Russell 1997).

Longitudinal strain increases in the concrete along the prestressed concrete member over time. Strain increase in the concrete is conditioned by strains due to shrinkage and creep. Strain due to shrinkage is uniform along the member and strain due to creep is variable. Variation of strain due to creep depends on the stress induced by pretensioning force in the separate element cross-sections (Herrero et al. 2013).

Concrete strain increases and strand stress decreases due to time-dependent factors, however transfer length doesn’t change significantly (Oh, Kim 2000). Nevertheless, concrete strain and stress, and strand prestress variation at the transfer zone should be assessed during design of the prestressed concrete elements.

Strand slips at the end of the prestressed concrete member during release of reinforcement and consequently bond between concrete and reinforcement is damaged. Stresses in reinforcement become equal to zero at the damaged zone of the prestressed concrete member (see Fig. 4.).

Usually during the design stage of prestressed concrete member prestress losses of pretensioned reinforcement are calculated according to constant stress distribution in the entire length of member. However, as was discussed earlier, stresses of concrete and reinforcement vary at the transfer length \( l_{pt} \) of prestressed concrete member and only in the middle part of the member stresses are constant. Consequently when calculating prestress losses of prestressed concrete member it is appropriate to assess linearly variable stress distribution along the anchorage zone. The calculated prestress losses are presented in Fig. 5. According to the analysis results prestress losses are constant along the transfer length before strand release \( \Delta \sigma_{pd}, \Delta \sigma_{pr1} \) respectively prestress losses due to strand draw-in and steel relaxation at transfer, but prestress losses after strand release are variable \( \Delta \sigma_{el}, \Delta \sigma_{cs}, \Delta \sigma_{cr}, \Delta \sigma_{pr2} \) respectively prestress losses due to elastic deformation, shrinkage and creep of concrete, and steel relaxation at service). Distribution of prestress losses due to elastic deformation and creep of concrete are linear, but prestress losses due to relaxation of reinforcement are distributed not linearly. Such distribution of relaxation losses is determined by the assessment of variation of prestress losses due to elastic deformation and creep of the concrete. Deformation due to concrete shrinkage is constant in the entire length of the element, and prestress losses are also constant.
3. Influence of stress distribution at anchorage zone of prestressed concrete member

Prestress losses of reinforcement must be assessed in order to determine rational position (eccentricity) and prestress force of the reinforcement. During calculation of prestress loss of pretensioned reinforcement, assumption could be made that stresses along the reinforcement are constant or their distribution are linearly variable at the anchorage zone, but are constant beyond the anchorage zone. Assessment of linearly variable distribution of reinforcement stresses most accurately reflects the actual behavior of prestressed concrete element, consequently calculated results would be more accurate.

During design of prestressed concrete member the reinforcement position should be estimated so that the tensile stresses would be zero at the cross-section of the element. It means that whole cross-section would be in compression and appearance of cracks would be limited.

By changing eccentricity of precompression force the similar stress state can be achieved in all cross-sections of prestressed concrete member (Krishnamurthy 1983; Boczkar 1984):

At transfer

\[ e \leq \sigma'_{\text{min}} \cdot \frac{W_v}{P_0} + \frac{M_{\text{min}}}{P_0} \cdot \frac{W_c}{A_c}; \quad (10) \]

\[ e \leq -\sigma'_{\text{max}} \cdot \frac{W_v}{P_0} + \frac{M_{\text{min}}}{P_0} \cdot \frac{W_a}{A_c}; \quad (11) \]

At service

\[ e \geq \sigma_{\text{max}} \cdot \frac{W_v}{P_0} + \frac{M_{\text{max}}}{P_0} \cdot \frac{W_c}{A_c}; \quad (12) \]

\[ e \geq -\sigma_{\text{min}} \cdot \frac{W_a}{P_0} + \frac{M_{\text{max}}}{P_0} \cdot \frac{W_a}{A_c}, \quad (13) \]

where \( \sigma'_{\text{min}}, \sigma'_{\text{max}}, \sigma_{\text{min}}, \sigma_{\text{max}} \) – respectively permissible minimal and maximal stresses at transfer and service, \( M_{\text{min}}, M_{\text{max}} \) – respectively moment at transfer due to self weight and moment at service due to self weight, additional dead loads, superimposed dead load, live loads, etc., \( W_v, W_a \) – respectively section modulus for the top (compressive layer) and for the bottom (tensile layer) of the section, \( e \) – eccentricity of the prestressed strand.

Rational position of reinforcement can be determined according to (10) and (13) inequalities. Rational position of reinforcement is determined according to tensile stress appearing at the bottom of the member.

In some cases cracks may be allowed or not allowed in prestressed concrete flexural members. Cracks are checked at the top face (compressive layers) of the member at transfer stage and at the bottom face (tensile layers) at service stage. Four cases of cracking during design have to be analysed: when cracks are allowed at transfer and at service stages, when cracks aren't allowed at transfer and at service stages, when cracks aren't allowed at transfer, but are allowed at service stage, when cracks are allowed at transfer, but aren't allowed at service stage.

Research shows that position of reinforcement at the anchorage zone is mostly influenced by prestressing force and bending moment induced by external effects. Further from the ends of the prestressed concrete member, according to linearly variable distribution of stresses, eccentricity of reinforcement decreases and approaches the values estimated in the evaluation of constant stress distribution. In both cases of stress distribution eccentricities of reinforcement become equal outside the anchorage zone and stay equal in the middle zone of the member. Pretensioning force is constant and position of the eccentricity is mainly determined by bending moments induced by the external forces.

At the anchorage zone rational eccentricity value of pretensioned reinforcement at transfer and service stages is with opposite sign. During service stage reinforcement should be lifted towards the top face of the cross-section of the member in order to limit or reduce cracking at the top face of the prestressed concrete member (see Fig. 6 (2)).

Analysis of rational position of pretensioned reinforcement at the anchorage zone was performed by investigating 600x400 mm cross-section beam. During research of prestressed concrete member anchorage zone, the cases of linear variation and constant reinforcement stresses were analysed. Results of this analysis are presented in Figure 6. Results showed that in any case of cracking limiting of prestressed concrete member, by assessing linearly variable distribution of the stresses at the anchorage zone, strands eccentricity should be increased more than in case of constant stress distribution (see Fig. 6). It is estimated that by assessing linearly variable or constant distribution of prestress at anchorage zone eccentricity of the reinforcement at transfer and at service stages increases respectively 2.7 and 3 times.
When cracks are allowed in prestressed concrete element there is a big difference between rational position of reinforcement assessing linearly variable and constant distribution of prestress in reinforcement (see Fig. 6b). According to linearly variable distribution of prestress, rational position of reinforcement at transfer and at service stages are out of the range of member cross-section. In this case pretensioning force or cross-section dimensions should be reduced or concrete cracking should be limited.
Linearly variable distribution of strand stresses, variation of rational position of pretensioned strands depends on the height of cross-section of prestressed concrete member (see Fig. 7). In all cases of cracking limitation, range of rational position of reinforcement decreases with decreasing height of prestressed member cross-section.

When cracks aren’t allowed in prestressed concrete member, range of rational position of reinforcement intersects at the anchorage zone (see Fig. 7a), but the range decreases with decreasing height of member cross-section. When the cross-section height of prestressed concrete member decreases from 600 mm to 500 mm and from 500 mm to 400 mm rational eccentricity of pretensioning force at transfer stage decreases respectively 62% and 53% and at service stage – respectively 64% and 65%.

When cracks are allowed in prestressed concrete member (see Fig. 7b) and cross-section height decreases from 600 mm to 500 mm and from 500 mm to 400 mm, range of eccentricity rational position of pretensioning force at transfer length varies. Closer to the middle of the member position of the reinforcement is not depending on the element height. When cracks are allowed in prestressed concrete element and cross-section height decreases from 600 mm to 500 mm and from 500 mm to 400 mm eccentricity rational position of pretensioning force at transfer stage decreases respectively 68% and 62% and at service stage – respectively 75% and 77%.

Conclusions

1. Reinforcement anchorage zone of the prestressed concrete member is important for its bearing capacity and rigidity. In order to ensure a good anchorage zone, a good bond between concrete and reinforcement and sufficient reinforcement anchorage length must be ensured.

2. Most expressions of the relation between the transfer length and strand draw-in are confirmed by sufficient coincidence with the experimental results. However provided expressions are derived for different strands diameters and some of the coefficients can only be determined experimentally.

3. Concrete stresses are variable at the anchorage zone of the prestressed concrete member. Concrete stresses mostly depend on time dependant factors such as concrete shrinkage and creep. Prestressed concrete member transfer length doesn’t change a lot over time.

4. Various distribution of prestress losses is conditioned by linearly variable distribution of stresses in the reinforcement at the anchorage zone of prestressed concrete member.

5. The difference between reinforcement eccentricities is observed when linearly variable and constant distribution of stresses are assessed at the anchorage zone. Increase of eccentricity at the anchorage zone, assessing linearly variable distribution of stresses, mostly is influenced by pretensioning force. Pretensioning force depends on prestress losses and bending moment induced by external forces.

6. Rational position of the pretensioned reinforcement is significantly influenced by cross-section height of the prestressed concrete member. Limits of eccentricity rational value of pretensioning load decreases with decreasing cross-section height. Due to greater eccentricity of pretensioning load, greater decrease of the eccentricity is in a case of allowed cracks in the prestressed concrete element.

References


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**Reikšminiai žodžiai:** įtempių perdavimo ilgis, inkaravimo zona, sukibimas, įtempių nuostoliai, praslydimas.

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