# PARAMETERS FOR AUTOMATED STAR IDENTIFICATION 

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#### Abstract

The determination of parameters for identifying stars sensed by charge-coupled device (CCD) is discussed. Numerical experiments are summarized which support the parameter space bound estimation practicality of the proposed star pattern recognition and identification by matching with coordinate list in star catalogue. The parameter analysis performed to apply them for proper identification algorithm which is developed and used at the Institute of Geodesy and Geoinformatics. This algorithm is applied for identification of large volume star sets.


Keywords: geodetic astronomy, CCD astrometry, star identification.

## Introduction

Highly developed algorithms are used for star trackers designed by ESA, NASA, JPL, Italian Space Agency and many others (Quan, Fang 2010; Lee, Bang 2007; Cole, Crassidis 2006; Mortari, Romoli 2002; ESA 2006; Kosik 2014). However, in this paper is not an aim to analyse and compare them. At the University of Latvia has been developed a software programme for automated star identification (Balodis 1988; Balodis et al. 1990) a long time ago. Looking forward to the aim of application of the automated star identification in the field of view of the newly developed satellite laser ranging system (Balodis 2008) the test prototype of the digital zenith camera for vertical deflection studies was applied (Abele et al. 2012) for identification and analysis of star imagery grasped on the CCD device. The properties of the optical system of digital camera were unstable during the development period of about one year. The test results were carried out continuously in order to control the development process of both the hardware and software. One of the modules for development was the software for automated star identification, i.e. by matching of star images on CCD (plate stars) in a virtual set of catalogue stars, which is a list of star coordinates in a volume of 1 billion stars (Zacharias et al. 2005). The task of star automated
identification was solved long time ago (Balodis 1988; Balodis et al. 1990), when SAO star catalogue with a coordinates of 256,000 stars was used. However, several peculiarities rose now in test operations of CCD astrometry with a many times bigger data volume.

The vertical deflection observations were carried out at various climatic conditions and in a various temperature conditions. Unfortunately, the summer/ winter temperature differences caused small changes in the position (fixation of some mechanics) of optical elements. The small inclination of the focal plane was changed as a sequence.

One more peculiarity was met by the manner of getting the digital images of the sky. At the each site the several frames were obtained with an exposure of the parts of second of time. The each next frame after the first was taken by turning the camera around the vertical by 5-10 degrees. It means that the orientation of field of view at the plate was changing in the plane of tangential coordinate system for each frame as well.

Density of the star images on a plate was very different in dependency of the sky in various dates of the year. There were listed very many catalogue stars in field of view when the Milky Way was in zenith (up to 1.5 thousand stars) and just few tens of stars on the several other dates. Of course, in city environment
the clouds and smog affected the sky visibility as well. Practically star images on the plate (CCD) haven't reached more than 60.

In conclusion, various sources affected the quality of star imagery on the plate and, consequently, the star identification conditions sometimes were very different.

In Figure 1 the obtained CCD imagery is shown with all star images together - those which are listed in the star catalogue and those which are exposed on the CCD camera. Both the CCD created noise images and the stars from the star catalogue list denoted by the small size criss-cross. The really obtained star images which are identified are denoted by bigger circle and criss-cross inside. The axis of tangential coordinate system in focal plane are denoted by two perpendicular lines.

The star identification software programme was developed many years ago for much simpler conditions. It was necessary to apply this programme for quite different data sets and their volumes now. However, in spite of the various situations on CCD imagery and for very large amount of catalogue stars the star identification algorithm appears suitable to overcome all the circumstances. The only necessity was to analyse the new conditions and to find the bounds of the various identification parameters space. This is explained in the further part of paper.

## 1. Relations of two star sets

The relations between the coordinates of the plate stars and the standard coordinates of the catalogue stars in general form is described by:

$$
\begin{equation*}
C=\varphi(P), \tag{1}
\end{equation*}
$$



Fig. 1. Selected catalogue star background and plate stars at the field of view
or with the following basic formulas of linear transformation;

$$
\left.\begin{array}{l}
\xi_{i}=M \cos \tilde{a} x_{s}+M \sin \tilde{\mathrm{a}} y_{s}+e \\
\eta_{i}=M \cos \tilde{\mathrm{a}} y_{s}-M \operatorname{simã} x_{s}+f \tag{2}
\end{array}\right\} .
$$

The case is discussed, when both the transformation parameters and the correspondence of the indexes, i.e. the numbers of the star images at the plate and matching star numbers (images) in the catalogue are unknown. Typically there are 20-50 stars on the plate which are unidentified among up to the 9001500 stars of catalogue at the field of view of camera.

Generally, a star i from the catalogue have a prototype from the plate belonging to the intersection of sets of pairs of stars with the mentioned properties:
$\varphi^{-1}(i) \in \cap\left(\{s, t\}: \exists j\left(\frac{d(i, j)}{d(s, t)}=M \& \alpha(i, j)-\alpha(s, t)=\tilde{\mathbf{a}}\right)\right)=$ $\left\{s: \exists t \exists j\left(\frac{d(i, j)}{d(s, t)}=M \& \alpha(i, j)-\alpha(s, t)=\tilde{\mathrm{a}}\right)\right\}$,
where the coefficient M equals the quotient of the modules:

$$
\begin{equation*}
\frac{d(\varphi(s), \varphi(t))}{d(s, t)}=M \tag{4}
\end{equation*}
$$

but the difference of the directions in one system of coordinates and the other one correspondingly gives the angle:

$$
\begin{equation*}
\alpha(\varphi(s), \varphi(t))-\alpha(s, t)=\tilde{\mathbf{a}} . \tag{5}
\end{equation*}
$$

In such a case every plate star has an image in the catalogue:

$$
\forall s \quad \exists i \quad i=\varphi(s) .
$$

Unfortunately, in focal plane due to the properties of central projection there are certain discrepancies for both parameters $M$ and a for the various couples of stars and for the various frames as well. For proper camera it is possible to predetermine the approximate discrepancies $\varepsilon_{1}$ and $\varepsilon_{2}$ for transformation parameters M and ã correspondingly. The additional parameters $r_{1}$ and $r_{2}$ are used as a possible variation of $\varepsilon_{1}$ and $\varepsilon_{2}$ correspondingly. Therefore the star automated identification algorithm appears as a multi-argument function with a very high noise level in input data. For example, to identify $20-50$ coordinated star and CCD noise image dots on the plate which are unidentified among the 900-1500 coordinated star dots of
catalogue. For understanding the relations between plate and catalogue data sets some analysis has been performed in order to have the representation on the values of parameters $\varepsilon_{1}, r_{1}, \varepsilon_{2}, r_{2}$.

## 2. Disposition of two star sets

The set P of coordinates of star images obtained on the CCD will be named the plate stars and the set C of catalogue star spherical coordinates converted into the tangential (or standard) coordinates will be named catalogue stars in the further discussion. The algorithm of star automated identification has been described in previous papers as it is mentioned above. The further description is performed on the basis of successfully identified star sets in order to determine the identification parameters. The analysis is curried out after the star identification already has been completed.

In Table 1 represented data of 14 frames (col. 2). Number of identified plate star images in col. 3, which is in \% of all plate images in col. 4. Not all plate stars are successfully identified. Partly it is caused by the noisy behaviour of CCD device, partly due to the limited star selection from catalogue within the evidently predicted interval of star magnitudes. For example, the images of the stars of magnitude 15 and weaker
brightness will not be obtained on CCD in sub-second exposure and therefore these weak star coordinates are not useful to extract from catalogue.

The control of the star identification is followed by solving the equations (2), which simultaneously confirm the success of identification process. Mostly MSE of the tests of zenith camera observation solutions is less then 1 arc second (col. 5 and 6). In columns $7-10$ represented average Cartesian plane coordinate differences of identified star coordinates in CCD and their catalogue tangential coordinates (col. 7 and 9) and standard deviations (in col. 8 and 10) correspondingly. This explains how accurate were the predicted site coordinates for the zenith which serves for the catalogue star selection. The origins in both coordinate systems practically close to zero when the pointing coordinates in sky of the optical system is correct. For the automated star identification non precise pointing coordinates makes the identification task more complicated. In analysis all the coordinates are expressed in CCD pixels. Approximately 1 pixel corresponds to the 1 arc second in the scale in focal plane. For some frames the discrepancies are large ( $\mathrm{t} \_\mathrm{t}, \mathrm{t} 2, \mathrm{f} 5, \mathrm{f} 19 \mathrm{a}, \mathrm{f} 24 \mathrm{a}$, t_A). Enlarged standard deviation is an evidence of significant value of rotation angle.

Table 1. Disposition of the plate and catalogue star sets

| Frames |  | Stars |  | MSE |  | (Plate-Cat) coord.diferences |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \# | $\stackrel{\text { E. }}{\substack{\tilde{y} \\ \hline}}$ |  |  | W | io |  | 分 |  | 会 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | t_t | 13 | 81 | 0.15 | 0.52 | -967.8 | 594.1 | -684.3 | 506.1 |
| 2 | t2 | 20 | 57 | 0.08 | 0.06 | -330 | 242.3 | 57.8 | 392.9 |
| 3 | t3 | 31 | 88 | 0.14 | 0.13 | -2 | 0.7 | 1.1 | 0.7 |
| 4 | t5 | 12 | 36 | 0.17 | 0.09 | -2 | 0.8 | 0.9 | 0.4 |
| 5 | 5A | 19 | 57 | 0.18 | 0.15 | -2.5 | 0.7 | 1 | 0.6 |
| 6 | f5 | 19 | 100 | 0.22 | 0.11 | 128.5 | 646.4 | -510.5 | 488.2 |
| 7 | f16 | 14 | 51 | 0.15 | 0.08 | -2.7 | 0.5 | 1 | 0.3 |
| 8 | f19 | 36 | 92 | 0.08 | 0.06 | -2.8 | 0.5 | 0.8 | 0.4 |
| 9 | f19a | 17 | 43 | 0.14 | 0.09 | -1065 | 54.5 | 1014.8 | 52.4 |
| 10 | f24 | 46 | 86 | 0.13 | 0.07 | -0.9 | 0.9 | 1.3 | 0.5 |
| 11 | f24a | 22 | 41 | 0.18 | 0.13 | -152.1 | 53.9 | -44.4 | 92.4 |
| 12 | 111 | 12 | 12 | 0.15 | 0.07 | 4.4 | 0.5 | 4 | 0.3 |
| 13 | b | 10 | 52 | 0.15 | 0.19 | -2.5 | 0.8 | 0.1 | 0.6 |
| 14 | t_A | 31 | 88 | 0.11 | 0.15 | -371.1 | 232.6 | 38.2 | 380.8 |

## 3．Shortly on the algorithm of star identification

The applied algorithm of star identification is presen－ ted in（Balodis 2008）．In general，the main idea is defi－ ned in formula（3）．The beginning of identification al－ gorithm can be described as a search of two congruent sets of diverged vector＇s beams．However，due to the discrepancies two similar beams of vectors are being searched，not congruent ones．

At first the modules from set $P$ compared with random modules from set $C$ ．When the two sets of modules with a length coincidence accuracy $d=M-1 \leq \varepsilon_{1}$ are found，then two sets of modules and corresponding star numbers are formed．The in－ tegers $E\left(d / r_{1}\right)$ are calculated and those star numbers are picked out which forms the modules with max ti－ mes repetition rate of some integer value．This helps to find a pole in one diverging set of vectors and cor－ respondingly in another one．The following vector direction tests are performed now．The vectors with similar modules are eliminated if the difference of di－ rections doesn＇t exceed the threshold $\varepsilon_{2}$ ．The remin－ der of difference divided by $r_{2}$ and converted into in－ teger helps to complete the finding of a two similar sets of vectors．Again，directions with integer＇s max ti－ mes repetition rate of some integer value is distinguis－ hed．Solution of linear equation（3）confirms or rejects the result of search of the similar diverging beams of vectors in set $P$ and their prototype in set $C$ ．

In order to perform the intersection of various needed data sets the greatest common divisor is used to convert the quotient of real numbers of the modu－ les and direction values correspondingly into integers． In practice this accelerates the intersection of data sets for the search of similar beams of vectors．For this purpose the parameters $r_{1}$ and $r_{2}$ are applied．First one is used as common divisor for vector modules in order to pick out the pretenders for similarity，the second one－for directions．

## 4．The tests of similarity of two beams

In Table 2 the results of the test of similarity of two diverging vector＇s sets（vector beams）is shown．In each of the frames among the identified sets of stars one beam of vectors is selected．The closest identified star to the origin of plate coordinate system has been chosen as a pole in set $P$ and its image in set $C$ corres－ pondingly．Both the distances and directions to other stars have been calculated in set $P$ and from the pole image in set $C$ ．In col． 9 the parameter（M－1．）and its standard deviation in col． 10 is shown．Rotation angle ã and its standard deviation shown in col． 11 and 12. In the star identification procedure the task is to find a beam of vectors with such an unknown parameters beforehand．In col． 3 and 4 are the volumes of the sets of plate stars and catalogue stars in each frame．

Table 2．Test of similarity of two converging vector＇s beams

| Frames |  | STARS |  |  |  | MSE |  | Scale－1． |  | Rotation angle（ ${ }^{\circ}$ ） |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \＃ | 茫 |  |  |  |  | ＊ | io | $\begin{aligned} & -i \\ & \dot{\Sigma} \end{aligned}$ | 会 | 20 | 令 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | t＿t | 16 | 544 | 13 | 81 | 0.15 | 0.52 | 0.001 | 0.001 | 141.7 | 0.1 |
| 2 | t2 | 35 | 32 | 20 | 57 | 0.08 | 0.06 | 0.003 | 0.003 | 304.8 | 0.1 |
| 3 | t3 | 35 | 32 | 31 | 88 | 0.14 | 0.13 | 0.003 | 0.003 | 0.0 | 0.1 |
| 4 | t5 | 33 | 256 | 12 | 36 | 0.17 | 0.09 | 0.003 | 0.002 | －0．1 | 0.1 |
| 5 | 5A | 33 | 128 | 19 | 57 | 0.18 | 0.15 | 0.002 | 0.002 | 0.0 | 0.1 |
| 6 | f5 | 19 | 19 | 19 | 100 | 0.22 | 0.11 | 0.005 | 0.007 | 181.0 | 0.1 |
| 7 | f16 | 27 | 1194 | 14 | 51 | 0.15 | 0.08 | 0.002 | 0.001 | 0.0 | 0.1 |
| 8 | f19 | 39 | 37 | 36 | 92 | 0.08 | 0.06 | 0.002 | 0.002 | 0.1 | 0.1 |
| 9 | f19a | 39 | 568 | 17 | 43 | 0.14 | 0.09 | 0.004 | 0.003 | 9.9 | 0.1 |
| 10 | f24 | 53 | 48 | 46 | 86 | 0.13 | 0.07 | 0.002 | 0.003 | 0.0 | 0.1 |
| 11 | f24a | 53 | 506 | 22 | 41 | 0.18 | 0.13 | 0.002 | 0.002 | 14.0 | 0.1 |
| 12 | 111 | 100 | 110 | 12 | 12 | 0.15 | 0.07 | 0.002 | 0.002 | 0.0 | 0.1 |
| 13 | b | 19 | 1162 | 10 | 52 | 0.15 | 0.19 | 0.002 | 0.001 | 0.0 | 0.0 |
| 14 | t＿A | 35 | 928 | 31 | 88 | 0.11 | 0.15 | 0.003 | 0.003 | －55．3 | 0.1 |

More information is given in Table 3 where catalogue (col. 8) and plate (col. 9) area in pixels is given, the centre of star set coordinates in col. 3, 4 and 5, 6 correspondingly and the distance of coordinate system origins in col. 7. These data demonstrates the variety of circumstances for the star identification.

## 5. Star magnitudes

Before the automated star identification some parameters are fixed by person who performs the computation. The star set magnitudes upper and lower bond are freely selected by this operator. Not always these decisions of operator were right. In col. 3 and 4 of Table 2 the volumes of star sets is described. In col. 5 is a number of identified stars in each frame and in col. 6 the percentage of identified plate stars. It is useful to compare these data with a col. 10 and 11 of Table 3. Just $12 \%$ of stars identified in Frame 111 where stars from catalogue selected of the magnitude segment $8.8-13.0$. In col. 11 of Table 3 presented the distribution of the catalogue star magnitudes. First is the number of stars of magnitudes up to 5.9 , then segment 6.0-6.9, then 7.0-7.9, etc. Last number in this row shows the number of stars of magnitude 14.0 and upwards. For the Frame 111 no stars of magnitude up to 7.9 selected for identification. Similar situation is with a Frame 5A where just $19 \%$ of stars identified. For
the Frame t_5 just $36 \%$ of stars are identified. Seems, that in this case a reason for such a low percentage is the lack of stars of magnitude 11.0 and upward. $100 \%$ of stars are identified in Frame f5 where the catalogue stars selected from magnitude segment 5.6-11.8. From last case and others this experiment proves that stars from catalogue should be selected up to magnitude 13 and brighter.

For one observation test (f5a) with 128 stars from catalogue and 33 obtained prototypes, 19 suitable 'star-prototype' pairs were identified using algorithms with varying parameters. Star and prototype magnitudes have been under consideration. All stars from catalogue have magnitudes from 5th to 10th. In the case of 33 prototypes magnitude distribution is from 7th to 15 th magnitude. As a result, $42 \%$ of the prototypes have been treated as noise. Concerning identified 'star-prototype' pairs magnitude distributions are from 6th to 10th for catalogue stars and from 7th to 13th for prototypes (see Figure 2). There is still a discrepancy between two distributions. It can be explained by optical properties of used CCD matrix and approach for the determination of prototype magnitudes, which has higher impact in comparison with CCD camera parameters. Differences between star magnitudes and prototype magnitudes have been derived after pair fitting. Estimated star magnitudes on plate are close to catalogue star magnitudes near the

Table 3. Star set placements and the chosen catalogue star magnitudes

| \# | Fr | $\xi$ | $\eta$ | $x$ | $y$ | Dist of | Cat | PL | Magn | Distribution of cat,star magn, |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | cntr | cntr | cntr | cntr | origins | area | area | segment | $5,6,7,8,9,10,11,12,13,14$ magn, |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 1 | t_t | 1156 | 1168 | 558 | 471 | 917 | 1643 | 731 | 0.0-15.0 | 10102121840126344 |
| 2 | t_2 | 661 | 622 | 645 | 442 | 180 | 908 | 782 | 6.8-12.6 | 0114898100 |
| 3 | t_3 | 661 | 622 | 645 | 442 | 180 | 908 | 782 | 6.8-12.6 | 0114898100 |
| 4 | t_5 | 1331 | 1367 | 689 | 457 | 1113 | 1908 | 827 | 5.4-11.0 | 461430521500000 |
| 5 | 5A | 1331 | 1367 | 689 | 457 | 1113 | 1908 | 827 | 5.4-11.0 | 2371526750000 |
| 6 | f5 | 580 | 463 | 579 | 462 | 1 | 743 | 741 | 5.6-11.8 | 1101493000 |
| 7 | f16 | 1591 | 1614 | 637 | 467 | 1491 | 2267 | 790 | 4.8-12.5 | 4812269621048635200 |
| 8 | f19 | 661 | 463 | 661 | 462 | 0 | 807 | 807 | 5.2-12.1 | 101091510100 |
| 9 | f19a | 1441 | 1440 | 661 | 462 | 1251 | 2038 | 807 | 4.8-11.5 | 46122896218204000 |
| 10 | f24 | 671 | 476 | 672 | 476 | 1 | 823 | 824 | 4.8-12.6 | 1114101314400 |
| 11 | f24a | 1460 | 1440 | 672 | 476 | 1244 | 2051 | 824 | 4.8-11.5 | 4683090196172000 |
| 12 | b | 1476 | 1471 | 579 | 462 | 1350 | 2084 | 741 | 5.6-12.5 | 2612228220647635600 |
| 13 | 111 | 938 | 1126 | 675 | 478 | 700 | 1466 | 827 | 8.8-13.0 | 00021012226220 |
| 14 | t_A | 1253 | 1249 | 645 | 442 | 1010 | 1769 | 782 | 4.0-13.0 | 254234214225544690 |



Fig. 2. Comparison of catalogue star magnitudes and assumed star image magnitudes in plate

Table 4. Identification results (f24) by choosing stepwise values of the parameters $\varepsilon_{2}, r_{2}$

| $\stackrel{\sim}{\sim}$ |  | $\begin{aligned} & 8 . \\ & 8 . \end{aligned}$ | $\stackrel{\rightharpoonup}{8}$ | Oio | $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { H } \\ & \hline 0 \end{aligned}$ | $\begin{aligned} & n \\ & 8 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \circ \\ & \hline 8 . \\ & 0 \end{aligned}$ | N |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\varepsilon_{2}$ | 0.060 | 47 | 47 | 47 | 47 | 47 | 47 | 47 | 47 |
| $\varepsilon_{2}$ | 0.220 | 47 | 47 | 47 | 47 | 47 | 47 | 47 | 47 |
| $\varepsilon_{2}$ | 0.380 | 47 | 47 | 47 | 47 | 47 | 47 | 47 | 47 |
| $\varepsilon_{2}$ | 0.540 | 47 | 47 | 47 | 47 | 47 | 47 | 47 | 47 |
| $\varepsilon_{2}$ | 0.700 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 0.860 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 1.020 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 1.180 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 1.340 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 1.500 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 1.660 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 1.820 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 1.980 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 2.140 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 2.300 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 2.460 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 2.620 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 2.780 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 2.940 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 3.100 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 3.260 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 3.420 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 3.580 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 3.740 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 3.900 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 4.060 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 4.220 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 4.380 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 4.540 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 4.700 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 4.860 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 5.020 | 15 | 47 | 47 | 19 | 17 | 13 | 13 | 23 |
| $\varepsilon_{2}$ | 5.180 | 14 | 47 | 11 | 47 | 47 | 47 | 47 | 47 |
| $\varepsilon_{2}$ | 5.340 | 14 | 47 | 11 | 47 | 47 | 47 | 47 | 47 |
| $\varepsilon_{2}$ | 5.500 | 14 | 47 | 11 | 47 | 47 | 47 | 47 | 47 |
| $\varepsilon_{2}$ | 5.660 | 12 | 47 | 47 | 14 | 47 | 47 | 47 | 47 |
| $\varepsilon_{2}$ | 5.820 | 14 | 47 | 47 | 47 | 12 | 47 | 47 | 47 |
| $\varepsilon_{2}$ | 5.980 | 14 | 47 | 47 | 47 | 11 | 47 | 47 | 9 |
| $\varepsilon_{2}$ | 6.140 | 14 | 47 | 47 | 47 | 11 | 47 | 47 | 9 |
| $\varepsilon_{2}$ | 6.300 | 14 | 47 | 47 | 47 | 11 | 47 | 47 | 9 |

centre of plate. However, at the edge of plate star magnitude discrepancies exceeds 4 . But strict disposition can't be detected.

## 6. Variable identification parameters $\varepsilon_{1}, r_{1}, \varepsilon_{2}, r_{2}$

According to the col. 11 of Table 2 the rotation angle ã is very variable within the set of frames. Similarly, taking into account the properties of central projection a variable is also the parameter M. Consequently, it is necessary to use variable identification parameters. The following computation has been carried out in order to detect the parameter space.

The identification programme was applied in (Balodis 1988) by systematically choosing the input values of various identification parameters for maximization of the number of identified stars in output evaluating the objective function on a grid in the entire 4-dimensional parameter space. The aim is to find the maximum of the objective function in the presence of many local minima and maxima. The objective function is the entire identification algorithm. The maximum has to be found iteratively by evaluating the value of the objective function, and performing small steps in parameter space. As a result the 4 -dimensional cryptosystem's lattice of identification parameters is found for the proper optical system.

In Figure 3 the output results of objective function depicted for the frame f16 when computation was performed by stepwise choosing the input of parameter $\varepsilon_{1}$ in segment $\left[0.005,0.050\right.$ ] and parameter $\varepsilon_{2}$ in segment [ $0,6.12$ ]. In Table 4 the output results of objective function depicted for the frame f 24 when computation was performed by stepwise choosing the input of parameter $r_{2}$ in segment [0, 0.007] and parameter $\varepsilon_{2}$ in segment $[0.06,6.3]$.


Fig. 3. Identification results (f16) by choosing stepwise values in parameter $\varepsilon_{1}$ (ESCALE) and $\varepsilon_{2}$ (EANG) space

Concluding the experimental tests for parameter search for the proper optical system the 4-dimensional parameter space has been found with a parameters that lie between given upper and lower bounds: $0 \leq \varepsilon_{1} \leq$ $0.05,0.08 \leq r_{1} \leq 0.14,0 \leq \varepsilon_{2} \leq 6.28,0.001 \leq r_{2} \leq 0.009$.

## 7. The multiple star identification tests within entire parameter space

In order to check the number of local maxima by testing the entire parameters space by stepwise changing the parameter values in all the combinations within the bounds mentioned above the multiple star identification were performed. Total 1800 tests have been carried out for each frame. For example, results depicted in Table 5 for the frame f24a. The catalogue coordinate predicted values have been calculated by using equations (2) in each case when the vector beams has been found with 4 points at least. It was assumed that a certain number of stars are identified (col. 1). Calculated mean square errors $\sigma \mathrm{X}, \sigma \mathrm{V}$ (col. 4, 5) and corresponding catalogue star magnitude segments (col. 3) depicted in Table 5. The number in col. 2 indicates, how many times in 4-dimensional parameter space the identification tests repeated the same result.

For example, 0 stars were identified in 250 tests, 5 stars in 670 tests, 6 stars in 270 tests, ...., 17 stars in 10 tests and finally 22 stars in 121 tests, where catalogue star magnitudes in segment $[4.8,11.4]$ and solution of eq. (2) gives precision $\sigma \mathrm{X}=0.5 \mathrm{arc}$ seconds and $\sigma \mathrm{V}=$ 0.7 arc second. One can see that according the Table 2 in frame f24a the number of plate stars is 56, catalogue stars 506 and just $41 \%$ of plate stars identified in catalogue. In Table 3 is shown that no stars with magnitude 12.0 and weaker were not beforehand selected in the set of catalogue stars. Probably, that was a reason for low persentage of identified stars. From other side, probably the CCD noise was high.

## Conclusion

The automated star identification software programme for large volume star sets has been developed and applied for digital zenith camera data reduction. The identification parameter space is experimentally determined for proper optical system. The analysis of the CCD data justifies the capability of this software to be used in on-line regime on the satellite laser ranging systems for pointing control. The identification process is very quick - it takes parts of second. However, for the different optical system the identification parameters should be specified.

Table 5. Multiple star identification success in frame f24a applying various parameters

| Number of id.stars | Repeat- <br> ed result | Cat.star magn. (From-to) |  | $\sigma X$ | $\sigma \mathrm{V}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 5 | 6 | 7 | 8 |
| 0 | 250 | - | - | - | - |
| 1 | 0 | - | - | - | - |
| 2 | 0 | - | - | - | - |
| 3 | 0 | - | - | - | - |
| 4 | 0 | - | - | - | - |
| 5 | 670 | 8.5 | 11.3 | 4.2 | 4.3 |
| 6 | 270 | 8.4 | 11.4 | 3.8 | 3.2 |
| 7 | 160 | 8.9 | 11.2 | 3.8 | 3.4 |
| 8 | 250 | 7.9 | 11.3 | 3.1 | 2.8 |
| 9 | 49 | 9.2 | 11.2 | 2.6 | 3.0 |
| 10 | 0 | - | - | - | - |
| 11 | 10 | 8.6 | 11.4 | 1.8 | 2.5 |
| 12 | 0 | - | - | - | - |
| 13 | 0 | - | - | - | - |
| 14 | 0 | - | - | - | - |
| 15 | 0 | - | - | - | - |
| 16 | 10 | 8.5 | 11.4 | 0.8 | 0.6 |
| 17 | 10 | 7.0 | 11.4 | 1.3 | 1.6 |
| 18 | 0 | - | - | - | - |
| 19 | 0 | - | - | - | - |
| 20 | 0 | - | - | - | - |
| 21 | 0 | - | - | - | - |
| 22 | 121 | 4.8 | 11.4 | 0.5 | 0.7 |

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