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POLYNOMIAL ERROR APPROXIMATION OF A PRECISION ANGLE MEASURING SYSTEM

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Abstract. The paper deals with the calibration of a flat angle measuring system. The article describes a comparative method for using a multi-angular prism (polygon) with autocollimator along with an angle measuring system. The components of angle measuring system is analysed in the paper. The article also presents polynomial error approximation and the obtained results of calibrating the precision angle measuring system as well as discusses preliminary assumptions for further research.

Keywords: accuracy, angle encoders, angle measurement, calibration, measuring systems.

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Introduction

Precision angle measuring systems are widely used for manufacturing various types of machinery embedding them into devices and instruments such as total stations, metal cutting machines, etc. An angle encoder is the main part of an angle measuring system. They may vary in sizes, types and accuracy. Depending on the required accuracy, the whole angle measuring system must be accordingly calibrated. Calibration of such systems is essential to determine errors and increase measurement accuracy.

Calibration process is frequently a very complicated procedure requiring specific equipment, including standards for achieving the best results. Precision angle measurement is compulsory for engineering at various levels of precision, and therefore many artefacts and instruments such as angle gauge blocks, optical polygons and rotary tables are used for angle metrology. To ensure high reliability, they should be calibrated using standard instruments such as an indexing table along with an autocollimator (Just *et al.* 2003; Kim *et al.* 2011).

1. Equipment and method used for flat angle calibration

To determine the accuracy of an angle measuring system, a comparative method was chosen. This method is based on a comparison of the measurand and standard angle determining a deviation between those angles. Equipment selected for realisation of this method consists of a precision angle comparator, 36 mirror-sided polygon and an autocollimator. The angle standard as a mirror polygon is an important measurement object used in applied for an angle calibration world wide. When measuring the angle accuracy of a product, many application fields of using the polygon could be used (Watanabe *et al.* 2003). The angle comparator consists of the basis that includes a precision mechatronicrotary device, a device for detecting circular scale graduation and its relative rotation according to the circular scale measuring system. A section view of the angle comparator is presented in Fig. 1. Another critical element is a calculation system for both the control of the calibration process and data processing. The basic mechatronic system is designed for direct limb and angular encoder calibration. The basis of such system consists of a massive small-grain grey granite brick with aerostatic rotational mechanism mounted on it through a suspension ring. It is rotated using a worm gear. To ensure proper rotation of this gear, combined aerostatic and rolling bearings are installed. The worm gear is mounted on aerostatic bearings and supported radially by rolling bearings. The rotation axis approximately matches the axis of the aerostatic rotational device. For this reason, suspension and radial bearings are mounted on position-correction

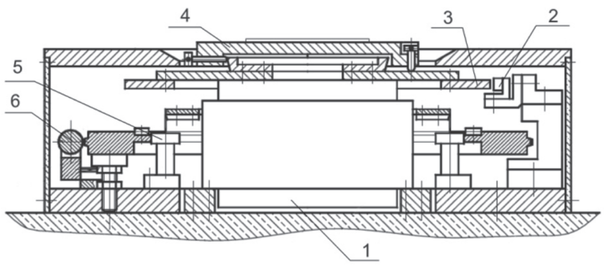

Fig. 1. Section view of an angle comparator

devices. The whole system is covered with a special cover and a precisely finished top surface protecting the system of damage coming from environmental pollutants. Additional measurement equipment can also be placed on this surface (Kasparaitis, Šukys 2008).

There are two diverse requirements for the gear of the comparator: it has to ensure constant angular rotation and the possibility of precise positioning while providing a stable angular position within given time.

The gear has not to create any other forces except for torque or tangential rotational force. It is also important that the gear does not generate intensive thermal activity or vibration. To perform accurate measurements, the laboratory should be isolated from any external vibrations under stable temperature.

Important parts of an angle measuring system are shown in the section view in Fig. 1: 1 – precision aerostatic shaft, 2 – reading head, 3 – reference circular scale, 4 – measured table, 5 – rolling bearing, 6 – worm gear.

One of the variants of the gear that meets the requirements mentioned before is a worm gear offering autonomous mechanism for the rotation of the gear combined with a precise connection with a rotation device that is rigid along the tangent direction of the spindle and slender in other directions.

The examined comparator has an elastic worm gear connection with the spindle generating pure torque without any radial or axial forces and ensures the stability of the spindle. The worm gear is rotated by an electric motor through a mechanical reducer and can be disconnected to rotate the spindle manually.

The limb of the angle measuring system has a rigid connection avoiding any intermediate sliding mechanical parts. It helps in eliminating negative effects of hysteresis and reverse errors that have a negative effect on precision and are common to mechanical coupling devices (Kasparaitis, Šukys 2008; Sydenham, Thorn 1992).

There are two precise guiding devices made of small-grain granite mounted on the basis of the comparator that have two sliding carriages with stroke-detection microscopes mounted on them. They are placed orthogonally to the axis of the spindle on aerostatic supports.

A 36 sided polygon is placed on the top of the precision angle measuring system with the embedded Renishaw angle encoder. The system is precisely centred and levelled. The autocollimator Hilger & Watts is pointed directly to the mirror face of the polygon and the system is set to the position in which the readout of the autocollimator is 0.00 arc seconds as shown in Fig. 2.

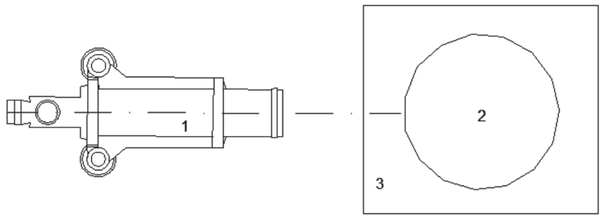

Fig. 2. Schematic view of instrument layout: 1 – autocollimator, 2 – 36 sided polygon placed on the top of the angle measuring system, 3 – angle measuring system

2. Data processing

A systematic error along with the evaluation of its possible cause is one of the key procedures in data processing (Rabinovich 2010). One of the main features influencing the precision of any rotary device is the eccentricity of mounting a rotating or measuring element (eccentricity of bearing or the measuring scale etc.). Having obtained scale calibration results, the eccentricity of the scale (or a disc mounting) can be calculated. Data on encoder systematic errors allow determining the mechanical systematic errors of the elements of the test rig such as the eccentricity of bearings or the scale itself (Kim *et al.* 2011, Stone *et al.* 2004). The encoder was calibrated using one reading head and the first harmonic was noticed in the chart. The results of calibrating the encoder are displayed in Fig. 3.

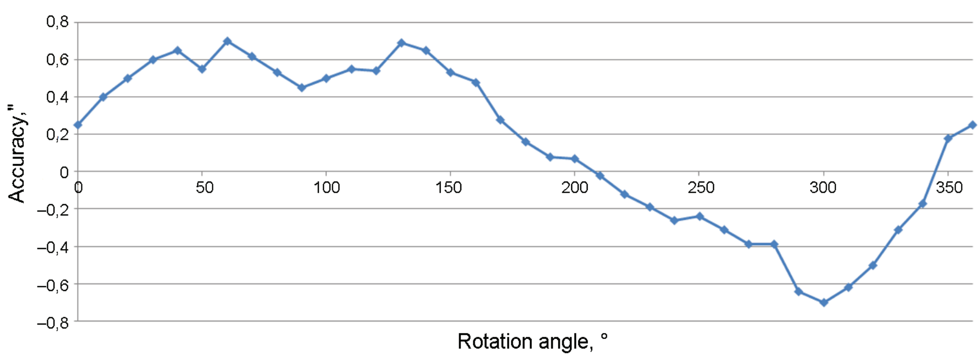

Fig. 3. Errors of the encoder

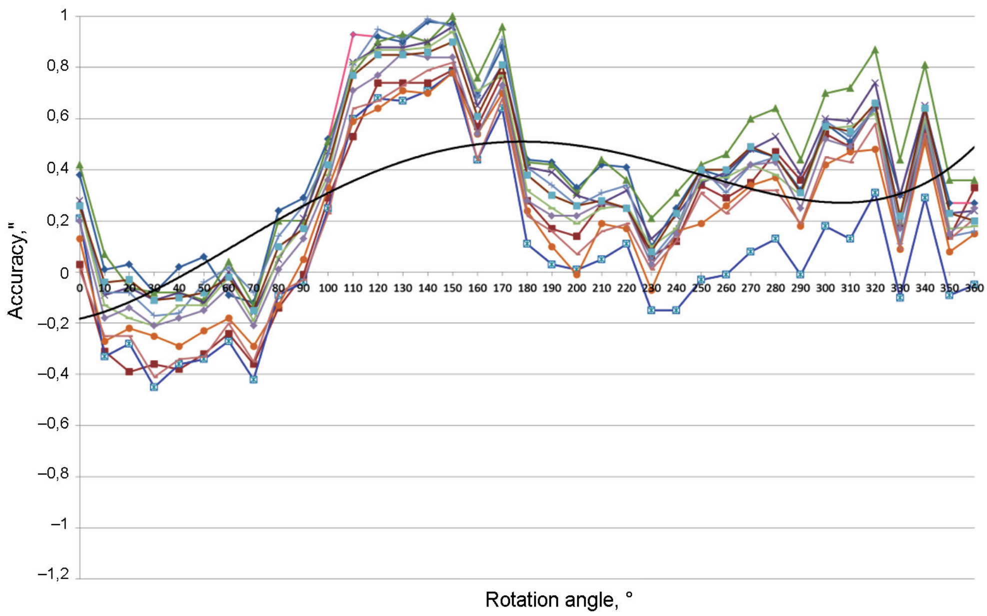

Fig. 4. The results of calibrating the angle measuring system

The measurement results are displayed in Fig. 4. The measurement errors were calculated following the evaluation of the results of polygon calibration and the accuracy of the autocollimator. Two reading sensors were embedded in the measuring system to eliminate eccentricity errors. Fig. 4 displays the results of twelve circles rotating the angle measuring system forwards and backwards.

Having the smoothened data and using Fourier analysis, the harmonics of measurements could be calculated (Bručas, Giniotis 2008; Bručas *et al.* 2006). Finite Fourier series can be expressed applying the following formula:

$$\delta\tilde{\varphi}(\varphi) = A_0 + 2\sum_{m=1}^{n-1}\left(A_m \cos 2\pi m f_1\varphi + B_m \sin 2\pi m f_1\varphi\right) + A_n \cos 2\pi m f_1\varphi . \quad (1)$$

The coefficients of the equation for each type of harmonics can be calculated as

$$A_m = \frac{1}{N}\sum_{r=-n}^{n-1}\delta\varphi_r \cos\frac{2\pi m r}{N}; \quad (2)$$

$$B_m = \frac{1}{N}\sum_{r=-n}^{n-1}\delta\varphi_r \sin\frac{2\pi m r}{N}, \quad (3)$$

where: m – the number of harmonics, N – the total number of measurements, r – the measurement number.

The amplitude of each type of harmonics is

$$R_m = \sqrt{A_m^2 + B_m^2} . \quad (4)$$

The phase shift of each type of harmonics (regarding the zero point) is

$$\varphi_m = arctg\left(-\frac{B_m}{A_m}\right). \quad (5)$$

The received results indicate that the polynomial curve of the fourth order reflects the view of the present measurement data. The equation for the fourth order polynomial curve is

$$y = (7\cdot10^{-4})x^4 - (4\cdot10^{-7})x^3 + (6\cdot10^{-5})x^2 + 0,0022x - 0,1824. \quad (6)$$

The calculation of a standard deviation is shown in Fig. 5.

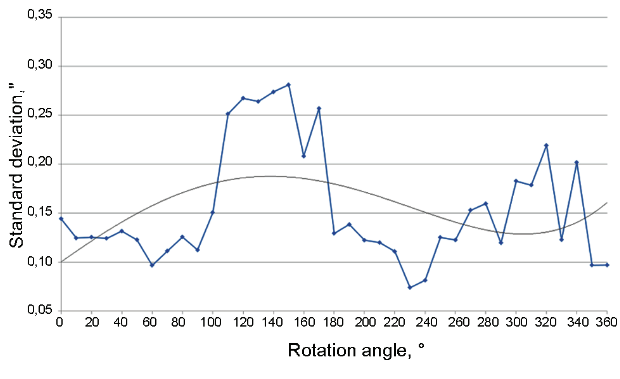

Fig. 5. Standard deviation

The equation for the polynomial curve of the standard deviation can be expressed as

$$y = (6\cdot10^{-11})x^4 - (2\cdot10^{-8})x^3 - 10^{-6}x^2 + 0,0011x + 0,1003. \quad (7)$$

Conclusions

Systematic errors of the angle measuring system have been determined. Calculating the errors of the polygon and the accuracy of the evaluated autocollimator and encoder has displayed the results of systematic errors that tend to have the highest values in the range of the 36 sided polygon of 100°–170°. This leads to the assumption that obvious systematic errors may be caused by other components of the angle measuring system (such as bearings) or the accuracy of installing the measurement system.

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