THE ANALYSIS ON THE POLICY OF ACCESS TO ECONOMICALLY AFFORDABLE HOUSING IN CHINA: AN AREA CALCULATION MODEL BASED ON THE INCENTIVE MECHANISM DESIGN

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ABSTRACT. Currently, the subject of housing in China is of great interest to many researchers, since improvements in living conditions for the largest population in the world have not kept pace with the territory’s economic growth. This paper describes basic social housing situations in China, and assesses the inequality in housing reform since 1990s as a result of overall economic improvement. In order to reduce the degree of inequality in China’s social housing, this paper is focusing on the policy management of access to economically affordable housing because housing affordability in China is a pressing social and economic issue, particularly for low-income families due to an unreasonable mechanism. The related policies are analyzed in is paper by using incentive mechanism design, which reveals that supervision and announcement system, the restriction in current mechanism, and the area of economically affordable house are critical for the effectiveness of present polices. Furthermore, a housing area calculation model based on the incentive mechanism design is presented and an illustrated case of Beijing City is also conducted, which can maximize the utility of government considering incentive compatibility. Finally, some policy suggestions are explored including enhancing external environment and establishing internal mechanism.

KEYWORDS: Affordable housing; Housing conditions; Area calculation model; China

1. INTRODUCTION

Since 2006, Chinese Central Government made great efforts to start new housing reform on the housing security system due to the social problems resulting from high speed economical development in China (General Office of the State Council of the People’s Republic of China, 2006). Though lots of achievements have been made in the housing reform in China from 1990s to present, many social problems occurred with the development (Wang, 2004). Actually, many important international and national issues, like worldwide economic crisis in 2008 and very high housing price in Chinese market, force Chinese government adjust macro-control policies to protect and safeguard the benefits of low-income families (General
Office of the State Council of the People’s Republic of China, 2006; General Office of the State Council of the People’s Republic of China, 2007; Ministry of Construction of the People’s Republic of China, 2007; Stephens, 2010). An important change is that the populations to be safeguarded by economically affordable housing are transferred from lower middle-income families to low-income families (General Office of the State Council of the People’s Republic of China, 2007; Ministry of Construction of the People’s Republic of China, 2007). The responsibility of social security for Chinese government has been paid more and more attention to by Chinese government itself, which means more and more social housing demands of low-income families should be met.

New Housing Reform in China focuses on how to further improve the housing conditions of Chinese low-income families (General Office of the State Council of the People’s Republic of China, 2006). Since 1990s, Chinese housing market has been commercialized step by step. However, house market price becomes higher and higher, and the housing demands in market can not be met, where an important problem is that less than 40% Chinese people can afford the market house price (China Statistical Yearbook, 2007-2009; Huang, 2009). Though some measures had been made to help residents buy house (e.g. Provident Fund Scheme, National Comfortable Housing Project, and Jingji Shiyong Fang, etc.) (Burell, 2006), general public in China still believe that they can not afford the price and the government should adjust housing security system (Zhao and Bourassa, 2003; Wang, 2004). In May 2006, the Ministry of Construction investigated China’s housing security system together with relevant policy research institutes to overcome the shortcomings of the economically affordable housing system, and draw a final conclusion that China’s housing security system needed an adjustment (Guo and Chang, 2008). The proposed method is to further segment the groups with the meddle-to-low incomes allowing the coexistence of the low-rent housing, the economically affordable housing and the commodity housing at restricted price, and provide the housing security product at different levels for different groups. Therefore, New Housing Reform is not a new policy, but a development of prior policies to build more affordable and accessible social housing and to establish an adequate supply of economically affordable housing and low-rent housing for urban poor to improve their housing conditions (Qi et al., 2007).

As mentioned above, the background of New Housing Reform is actually that many social problems companied with China’s economic development are questioned by domestic media, researchers, and general public. These social problems include that the gap between rich and poor is becoming wider than before, the population of low-income is increasing, the prices rising too fast including house prices, and many common people can not afford the house prices (Wang, 2004; Duda et al., 2005; Mostafa et al., 2006; Mak et al., 2007; Xiao, 2010). Therefore, the government determined to strengthen the provision of economically affordable housing and low-rent housing to enlarge the population that can be safeguarded (General Office of the State Council of the People’s Republic of China, 2006; General Office of the State Council of the People’s Republic of China, 2007; Ministry of Construction of the People’s Republic of China, 2007; Stephens, 2010). However, many deceptive methods (e.g. providing false personal and family information) are used by some unqualified applicants when applying for the affordable houses. And many families do not quit the affordable housing when their incomes become higher than before and economic situation become well. Therefore, the population that should be indeed provided affordable houses can not be safeguarded, which strongly influence social fairness and justice. The authority and
credibility of government are also challenged. Furthermore the housing affordability and social housing accessibility are weakened (Rosen and Ross, 2000; Liu et al., 2008). As a result, strong measures should be made by government to improve economically affordable housing provision.

Usually, there are two types of affordable housing in China (Shaw, 1997; Mostafa et al., 2006; Guo and Chang, 2008). The first one is low-rent housing targeting the lowest income families. For low-rent housing, the residents just pay very low rents (Guo and Chang, 2008). The second one is economically affordable housing targeting low- and lower-middle-income families before 2006, and low-income families after 2006, for which the residents can buy the house and obtain property rights by lower price than market price (Qi et al., 2007; Wang, 2004). This paper aims to discuss how low-income families can obtain the approval to get the economically affordable housing. The second part intends to give an overview on China’s social housing system and tries to find the shortcomings of economically affordable housing in China by extensive literature review. The third part is the analysis on current policies of access to economically affordable housing. The fourth part presents a method using incentive mechanism design for the policy of access to China’s economically affordable housing by house area control, and case of Beijing City is also illustrated. Finally, the conclusion is drawn in the sixth part.

2. ECONOMICALLY AFFORDABLE HOUSING IN CHINA

2.1. Social housing system in China

The social housing development in China is divided by three stages by Mostafa et al. (2006) based on the different economy models, which are Planned economy (1950–1977), Socialist market economy (1978–1989), and Rapid economic growth (1990–now). In the first stage, housing sectors were partially nationalized and managed by collectively or state owned enterprises, or work units (Danwei) at the local level from 1949-1957, and over 90% of the urban housing stock was under the control of the state. Housing investment restricted due to Cultural Revolution from 1958-1977. In the second stage, initial reform measures were made. Work units were allowed to invest in housing for their employees. The sale of publicy owned housing was carried out. The State Council’s leading team for Housing System Reform was set up. In the third stage, the 1991 Housing Reform Program is a milestone is the history. During this stage, the watershed event is that premier Zhu Rongji made accelerating residential development and elevating urban homeownership as a top policy priority after taking office in 1998 (Yu, 2006).

Many prior researches have focused on the China’s social housing system, most of which focused on the second and third stages as mentioned above, and have offered a general picture of the housing market behavior at different stages of modern China (Zhang, 2006). These researches can be divided to two stages. In 1990s, many researches provided overview of the Chinese housing reform (Barlow and Renaud, 1989; Wu, 1992; Tong and Hays, 1996; Shaw, 1997; Wang and Murie, 1996). Since 2000, the studies were more diversification, whose emphasizes are not much diverted to housing conditions, tenure, problems of affordability, housing prices, housing investment, housing policies and their interactions (Rosen and Ross, 2000; Hui, 2001; Huang, 2002; Liu et al., 2002; Zhao and Bourassa, 2003; Sun et al., 2004; Ye and Wu, 2008). Zhang (2006) presented that housing reforms have paved the way to changing Chinese housing conditions, accelerate urbanization, and facilitate economic reform in China, which deserve further investigation into improvement of present housing provision mechanism. Actually, there are few studies on New Housing Reform in recent
three years, which can not provide a closer view on the latest change in China’s housing policies. Thus this paper aims to fill this research gap.

So far, the housing reform in China has made great achievements (Stephens, 2010). As presented by Mak et al. (2007), the housing conditions in China have been improved largely since housing reform. In 1985, the first household survey on housing conditions was carried out in urban China, which revealed that the majority of urban residents experienced very poor living conditions at that time. Over 27% of the urban population shared their dwellings with others, and 7.4% had an average living space below 4m². Compounding the situation was the poor housing quality. About 76% of urban residents did not have their own toilets, 37% had to share their kitchens with others, and 27% did not have running water (Ho and Kwong, 2002). By the late 2000s, it was due to the economic and housing reforms that per-capita living space in China approached almost 10m², a more than 100% increase (Mak et al., 2007).

2.2. The inequality in housing reform

However, the affordability of social housing in China has been questioned greatly in recent years. The complete abandonment of the socialist housing allocation system in the late 1990s has led to profound changes in housing provision and consumption in urban China (Yu, 2006). Housing reform seems to be successful in increasing distributional inequality as a way to introduce market-based incentives and improve productivity. Through housing reform, those who were in power, who formerly are rich, who originally have ownership appear to have maintained and extended their advantages in the new system (Yu, 2006; Zhang, 2006; Mak et al., 2007; Wong et al., 2006). Consequently, the inequality of housing distribution system is resulted from housing marketisation, housing privatization, and urbanization.

In fact, housing marketisation is the primary method used by housing reform. The role of the government or the state-centered system has been greatly reduced since 1990s, and the role of non-state institutions such as work units, households, developers, cooperatives and financial institutions in housing has been greatly increased (Wong et al., 2006). Therefore, the demands of vulnerable groups/the disadvantaged (e.g. rural-urban migrants, college graduates, low-income groups, and urban poor, etc.) have been relatively ignored in recent years. Since 2000, the housing price in China has become higher and higher, which strongly influence housing conditions of vulnerable groups/the disadvantaged (Huang, 2004). Evidently, housing privatization is the result of housing reform, which means more residents in China can live in their own houses. Homeownership rate in urban China shot up sharply in recent ten years, from about 30% in 1995 to more than 82% in 2006 (Ma et al., 2010). However, those vulnerable groups/the disadvantaged can not share the achievements of housing reform. On the contrast, they are the victims. It is urbanization in China put them into cities or makes them earn very low wages. Thus urbanization is the catalyst for the inequality of housing distribution system (Deng et al., 2010). This kind of inequality in housing reform also was revealed by (Sato, 2006; Stephens, 2010), which confirms to the findings that the biggest winners in China’s housing reform are those who were favored in the previous system, based on such factors as residence status, education, and occupation (Logan et al., 2010). Like income inequality, housing inequality in urban China is strongly affected by state policies that give preferential treatment to insiders (Logan et al., 2009). In this case, the responsibility of government is to help vulnerable groups/the disadvantaged improve their housing conditions, of which affordable housing provision is a very important method.
2.3. Research problem in the provision of economically affordable housing in China

Currently, China’s economically affordable houses are supplied to the urban low-income families that have housing difficulty, and the target users of such houses shall be linked up with the target of the low-rent housing security; the building area shall be controlled within about 60 square meters; the buyers shall only own the limited property right, and shall not directly enter the market for sale within five years after the purchase; when it is indeed necessary to transfer the house, the house shall be bought back by the government in line with the original price, the depreciation, and the general price level; and in the case of the transfer five years after the purchase, the buyer shall pay a specified proportion of the difference between the price of the common commodity housing in the same area and the price of the economically affordable housing to the government as the profit from land (General Office of the State Council of the People’s Republic of China, 2006, General Office of the State Council of the People’s Republic of China, 2007; Ministry of Construction of the People’s Republic of China, 2007; Stephens, 2010).

However, the problem is that the policy for access to economically affordable housing is not clear (Yu, 2006). The applicator for economically affordable house would hide private information and provide false information in order to obtain economically affordable house because the economically affordable house price is much lower than market price (Li et al., 2009). Especially in China, the information is not perfect and symmetrical. Nobody can completely grasp other’s private information. Hence the applicator can carry on strategic selection act, and can not be revealed by others. The management of government is therefore very important in economically affordable housing provision.

During the management process of economically affordable housing in China, the public sector is the manager of economically affordable houses and the client of general public, which lacks the related information of applicators for economically affordable houses, e.g. their economic status and family structure. As the result of tending to pursue the economic benefits, many unqualified applicators also apply for the economically affordable houses, some of which would be approved. Although these unqualified applicators do not directly take the resource of qualified applicators, they invade and occupy the benefits of those qualified applicators, which will create the inequality and unfairness of society and waste of resource because the resources provided by Chinese government are very limited (Wang et al., 2005; Sato, 2006; Stephens, 2010). The public sector is very subjective in the process of making policy, during which the asymmetry of information can not be carefully considered. The result is that many policies have been made one by one, while the effect is unsatisfactory (Dong et al., 2010; Zeng and Lu, 2005). In term of access to economically affordable housing, a perfect mechanism is very necessary and should be designed carefully to keep the equality and fairness of economically affordable housing provision (Zhao and Bourassa, 2003).

2.4. Research methodology to analyze and resolve the problem: incentive mechanism design

The theory of incentive mechanism design was originally introduced in 1970s (Demsetz, 1969; Hurwicz, 1973), which have been applied in many fields like market design, distributed scheduling, and combinatorial resource allocation (Baliga and Sjöström, 2007). The theory of mechanism design helps us understand institutions ranging from simple trading rules to political constitutions. We can understand institutions as the solution to a well-defined
planner’s problem of achieving some objective or maximizing some utility function subject to incentive constraints (Bergemann and Morris, 2005).

The main idea of incentive mechanism design is to design a mechanism by planner. Firstly, the mechanism can be deconstructed into a frame model for analyzing the problems and can be refined to unified variables (Hurwicz, 1973). The value of variables will be different because of the difference amongst different mechanisms. The purpose of incentive mechanism design is to induce agent to select favorable activities for planner by the way of adjusting the value of variables. Actually, the planners intend to maximize their expected utility by using incentive mechanism design. The agents also satisfy the hypothesis of finite rationality economic man, which means they will also maximize their expected utility in a given mechanism. The agents can determine whether they will participate in the mechanism and which kind of actions they will select (Demsetz, 1969). As a result, the maximization of clients’ expected utility can be converted to constraints that should be considered by the planners when they design incentive mechanism, which are participation constraint and incentive compatibility constraint.

Mathematically, the model of incentive mechanism design can be defined as

$$\max_{\{q_k, \theta_k\}_{k=1, \ldots, k}} \sum_{k=1}^{k} u_i(\pi(q_k) - t(q_k)) \times p_k$$

s.t. (IR) $$u(t(q_k)) - C(q_k, \theta_k) \geq \bar{u}, k \in (1, k)$$ (2)

(IC) $$u(t(q_k)) - C(q_k, \theta_k) \geq u(t(q_i)) - C(q_i, \theta_i), \ (i, k) \in (1, k), i \neq k$$ (3)

where: $$\theta_{k(k \neq 0)}$$ is the type of agent; $$q_k$$ is the decision vector provided by planner to agent; $$p_k$$ is the possibility that agent belongs to $$\theta_k$$; $$\pi(q_k)$$ is the outcome of planner; $$t(q_k)$$ is the payoff to agent from planner based on $$q_k$$, also can be called as incentive fee; $$u_0(\pi(q_k) - t(q_k))$$ is the V-N-M utility function of planner; $$u_{i(i \neq 0)}$$ is the outcome of agent to select $$q_k$$; $$\bar{u}$$ is the reservation utility of agent, which is the expected utility of agent to reject the contract; $$C(\theta_k, q_k)$$ is the costs of agent to select $$q_k$$.

3. THE ANALYSIS ON THE ACCESS POLICY OF ECONOMICALLY AFFORDABLE HOUSING IN CHINA

Based on the Management Instrument of Economically affordable House (Ministry of Construction of the People’s Republic of China, 2007) jointly released by seven ministries of the State Council of the People’s Republic of China in December 2007, family income and family structure are the most important information, which will directly and strongly influence the economical benefits of applicators as well as social outcome. Therefore, these two kinds of information should be carefully considered in the analysis on current mechanism of access to economically affordable housing.

3.1. Present mechanism of access to economically affordable house in China

Players and preferences

The planner of allocation mechanism for economically affordable housing is government, and the applicator of economically affordable housing is the subordinate and the owner of private information. Thus government and applicator can be respectively called planner and agent of mechanism design (Hurwicz, 1973).

The preferences of mechanism analysis are shown in Appendix 1.

Actions

According to the Management Instrument of Economically affordable House (Ministry of Construction of the People’s Republic of China, 2007), three optional actions of applicators can be described as followed.
(1) Do not apply for the economically affordable house;
(2) Apply for the economically affordable house and purchase, the approved purchase area is $S^*$;
(3) Apply for the economically affordable house and purchase, the approved purchase area is $S^*$. Additionally, the applicator will buy excess area. Hence total purchase area is $S^* + S'$.

Based on the current regulation, government will approve the application upon the applicator can pass the qualification. Thus the action space of government just has only one value, which is to approve.

Payoff
1) The analysis on applicators
   (1) Income
   The regulator can obtain the land to build economically affordable housing as free. The land area is $m$, and floor area ratio is $\eta$. The land price is $P$. The potential applicators select to accept or reject. The area of one economically affordable house is $S^*$. If applicators decide to accept, the degree of support from government to each applicator (also is each house) is
   \[
   t(S) = \frac{S^*}{m \times \eta} \times m \times P = \frac{S^* \times P}{\eta}
   \]
   (4)

   Although, the floor area ratio $\eta$ is controlled by government to some extent, $\eta$ has strong proportional relation with land market price $P$.

   Therefore, the degree of support from government to each applicator can be changed to
   \[
   t(S^*) = S^* \times P'
   \]
   (5)

   where: $P'$ is the land-price of floor, which is to some extent controlled by market. Neither government nor applicators can not change $P'$. It can be concluded that government can influence its degree of support to each applicators by regulating the area of economically affordable house. Thus the income of potential applicator is
   \[
   u(t(S^*)) = S^* \times P' \times t\%
   \]
   (6)

   The applicators can not obtain all land income transferred from government because the construction of economically affordable house shall be conducted by developers. Some of income will be obtained by the developers. $t\%$ is the percent of land income transferred from government for applicators.

   (2) The costs of payoffs
   As shown in Figure 1, there are two types of applicators that are $\theta_{\text{high}}$ and $\theta_{\text{low}}$. In Figure 1, $V(\theta_k, S)$ is marginal utility of house area for different types of applicators (the detailed descriptions for the costs of payoffs can be found in Appendix 2). Therefore, the cost for applicator to adopt the strategy to apply for the economically affordable house with area $S^*$ is defined based on Eq. (13) (see in Appendix 2) as followed.

![Figure 1. Function figure of consumer costs](image)
\[
C(\theta_k, S^*) = \begin{cases} 
S = S_A & \int_{S=0}^{S} V(\theta_{low}, S) \times S, \theta_k = \theta_{low} \\
S = S_B & \int_{S=0}^{S} V(\theta_{high}, S) \times S, \theta_k = \theta_{high} 
\end{cases}
\]

where: \( S^* > S_A \) according to the real status, otherwise the government need not to implement the policy of economically affordable housing. On the other hand, the relationship between \( S^* \) and \( S_B \) is uncertainly.

The retained earnings for applicator to adopt the strategy not to apply for the economically affordable house is described as followed.

\[
\overline{u} = \begin{cases} 
S = S_A & \int_{S=0}^{S} V(\theta_{low}, S) \times S - c \times S_A, \theta_k = \theta_{low} \\
S = S_B & \int_{S=0}^{S} V(\theta_{high}, S) - c \times S_B, \theta_k = \theta_{high} 
\end{cases}
\]

(3) Net income

The net income for applicator to adopt the strategy to apply for the economically affordable house with area \( S^* \) is described as followed.

\[
u(t(S^*)) - C(\theta_k, S^*) = \begin{cases} 
S^* \times P' \times t\% - (c \times S^* - S = S^*) & \int_{S=0}^{S} V(\theta_{low}, S) \times S, \theta_k = \theta_{low} \\
S^* \times P' \times t\% - (c \times S^* - S = S^*) & \int_{S=0}^{S} V(\theta_{high}, S) \times S, \theta_k = \theta_{high} 
\end{cases}
\]

Eq. (9) can be converted to

\[
u(t(S^*)) - C(\theta_k, S^*) = \begin{cases} 
S^* \times P' \times t\% - (c \times S^* - S = S^*) & \int_{S=0}^{S} V(\theta_{low}, S) \times S - (c - P' \times t\%) \times S^*, \theta_k = \theta_{low} \\
S^* \times P' \times t\% - (c \times S^* - S = S^*) & \int_{S=0}^{S} V(\theta_{high}, S) \times S - (c - P' \times t\%) \times S^*, \theta_k = \theta_{high} 
\end{cases}
\]

Accordingly, the net income of applicator can be demonstrated by Figure 2. Thus the retained utility of applicator not to apply for the economically affordable house respectively is the area surrounded by line \( AK' \), line \( K'V_1 \) and curve \( AV_1 \) (\( AK'V_1 \)) and the area surrounded

![Marginal revenue and Marginal cost measured by currency, V](image_url)

**Figure 2.** The net income of applicator
by line $BK_2$, line $K_2V_2$, and curve $BV_2 (BK_2V_2)$ when the type of applicator is $\theta_{\text{low}}$ and $\theta_{\text{high}}$. On the other hand, the retained utility of applicator to apply for the economically affordable house respectively is the area of $OS'T_4V_1$ and the area of $OS'T_5K_1$ when the type of applicator is $\theta_{\text{low}}$ and $\theta_{\text{high}}$.

As shown in Figure 3, the value of $S^*$ will increase when $S^*$ moving from $S_A$ to the right. During this process, the utility for applicator with type $\theta_{\text{low}}$ and $\theta_{\text{high}}$ will also change with the change of the value of $S^*$. The rule of change is indicated by Figure 3.

When $S^*$ changes on the interval $(S_A, S_T)$, the applicator with type $\theta_{\text{low}}$ will apply for economically affordable house, and the applicator with type $\theta_{\text{high}}$ will not apply for economically affordable house;

When $S^*$ changes on the interval $(S_P, S_A')$, the applicator with type $\theta_{\text{low}}$ and $\theta_{\text{high}}$ will all apply for economically affordable house;

When $S^*$ changes on the interval $(S_A', S_C)$, the applicator with type $\theta_{\text{low}}$ and $\theta_{\text{high}}$ will all apply for economically affordable house;

When $S^*$ changes on the interval $(S_C, S_C')$, the applicator with type $\theta_{\text{low}}$ will not all apply for economically affordable house, and the applicator with type $\theta_{\text{high}}$ will apply for economically affordable house;

When $S^* > S_T$, the applicator with type $\theta_{\text{low}}$ and $\theta_{\text{high}}$ will all not apply for economically affordable house;

Evidently, $S^*$ should be on the interval $(S_A, S_T)$, so that really qualified applicator would apply for economically affordable house, and unqualified applicator would not apply for economically affordable house.

The Phenomenon “Adverse Selection” will occur when $S^*$ changes on the interval $(S_A', S_C)$, which means really qualified applicator would not apply for economically affordable house, and unqualified applicator would apply for economically affordable house. The qualified will squeezed by the unqualified under this kind of situation. Even if “Adverse Selection” is not completely, the unqualified would go into the applicators as long as $S^* > S_T$.

Currently, the applicators in China can be agreed to purchase extra area apart from allowed area, and the favorable policies will not be applied in excess area. In this case, the unqualified can not be excluded from applicators. Although $S^*$ changes on the interval $(S_A', S_T)$, consumer surplus obtained by the qualified can make net income exceed retained income when

![Figure 3](image-url)
the unqualified increase purchase area (e.g. \( S_B \)) regardless of any value of \( S^* \). The reason is that

\[
S^* \times P \times t\% - (c \times S_B) = \int_{S=0}^{S_B} V(0_{\text{high}}, S) \times S
\]

is always greater than

\[
\int_{S=0}^{S_B} V(\theta_{\text{high}}, S) \times S - c \times S_B
\]

Consequently, the unqualified will give false information of income to apply for economically affordable house regardless of any value of \( S^* \).

2) The analysis on the action of government

At the present stage in China, the government can not completely know the economical information of applicators. If the real economical information of applicators is divided into two types, corresponding payoff matrixes of two types are respectively proposed as shown in Table 1 and 2.

**Table 1.** Applying for economically affordable house: qualified applicator (Type 1)

<table>
<thead>
<tr>
<th>Player</th>
<th>Government</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action</td>
<td>Approve</td>
</tr>
<tr>
<td>Applicator</td>
<td>Apply</td>
</tr>
<tr>
<td>Do not apply</td>
<td>–</td>
</tr>
</tbody>
</table>

**Table 2.** Applying for economically affordable house: unqualified applicator (Type 2)

<table>
<thead>
<tr>
<th>Player</th>
<th>Government</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action</td>
<td>Approve</td>
</tr>
<tr>
<td>Applicator</td>
<td>Apply</td>
</tr>
<tr>
<td>Do not apply</td>
<td>–</td>
</tr>
</tbody>
</table>

In this game, the government do not know real type of applicator, but know that the possibility that applicator is qualified is \( p \), and the possibility that applicator is unqualified is \( 1 - p \). Therefore, the expected income when government selecting to approve the application are \( 150p - 150(1 - p) \), and the expected income when government selecting not to approve the application are \( -50p + 150(1 - p) \). In this case, the expected income to approve will be more than the expected income not to approve when \( p > 0.6 \). Thus to approve the application is optimal.

Although the values in the above-mentioned payoff matrixes are not static data, the characteristics of them can indicate the optimal strategic of government. As the representation of the benefits of qualified applicator in the proposed model, the utility that is assumed as 100 obtained by qualified applicator when the economically affordable house being approved will be viewed as one part of government income. Meanwhile, government can obtain good social reputation that is assumed as 50. Accordingly, government will obtain 150 utility because the application of qualified applicator is approved.

The zero-sum relationship between qualified applicator and unqualified applicator indeed exists for the reason that the quantity of economically affordable housing is fixed, which means qualified applicator will lose an opportunity when unqualified applicator obtaining the economically affordable house. Hence government will lose 100 utility, moreover bad social reputation will be obtained by government. Then the utility of government is –150 when the application of unqualified applicator is approved.

When government does not know real type of applicator, the optimal strategic depends on the influence from the actions of applicator and the identified possibility of type of applicator by government. Therefore, strict qualification is very necessary, so that the possibility that unqualified applicators can obtain economically affordable houses will be reduced greatly, although small part of unqualified applicators will also obtain houses. As a conclusion, the only action for government is to approve.
Based on the above-mentioned analysis, the supervision on the agent is very significant, which can provide more information of applicator and increase the accuracy to identify the type of applicator.

3.2. The discussion on current mechanism

(1) Strengthening supervision and announcement system is reasonable and necessary;

(2) The restriction in current mechanism of economically affordable housing in China is excessively loose. For example, sometimes the applicator could be agreed to purchase extra area, which means unqualified applicator has incentive to deceive the government. This situation does not meet the requirements of incentive compatibility. Although certain measures have been taken to verify the qualification of applicators, some unqualified applicators can apply for economically affordable houses.

(3) The area of economically affordable house is able to greatly affect whether the policy is incentive compatibility. However, there are no explicit strategies in present policy about the house area. Based on the analysis, to determine the house area does not have the relationship with economical income and current house area of applicator as well as regional situation, but also have strong relationship with the type of applicators.

Therefore, the house area is a very complex issue, in which the influence of economical and living status of applicator should be considered. Meanwhile, the most important point is that the preferences of applicators can be considered in the rule of incentive compatibility to protect the benefits of qualified applicators based on their real status as well as to strengthen the reputation of government. Thus following mechanism design will focus on how to determine the area of economically affordable house.

4. INCENTIVE MECHANISM DESIGN FOR THE POLICY OF ACCESS TO ECONOMICALLY AFFORDABLE HOUSING IN CHINA

4.1. The objectives and principles of mechanism design

Objectives

The objective of proposed incentive mechanism is designed to legally avoid the situation that unqualified applicators pretend to be the qualified to apply for and purchase economically affordable house.

Principles

1) The principle of participation: to ensure that qualified residents have drive to purchase economically affordable houses;

2) The principle of incentive compatibility: to ensure unqualified applicators’ incomes obtained from pretending to be the qualified to apply for and purchase economically affordable house will be less than their incomes obtained from not pretending.

The model of mechanism design

Government actually is the representation of qualified applicators when economically affordable house being facilitated. Therefore, the government’s incomes can be represented by qualified applicators’ incomes (Bergemann and Morris, 2005). Another important issue is the value of social reputation $M$. When the application of unqualified applicator obtains approval from government, government will lose an opportunity to provide house to qualified applicator. In this case, government will lose social reputation $M$, and the utility of government is $-M$. In proposed model, all preferences are as same as the preferences of mechanism analysis.

A model of mechanism design for the mechanism of access to economically affordable house in China under information asymmetry is presented in Eq. (11) based on Eq. (14) - Eq. (20) in Appendix 3 (The detailed process can be found in Appendix 3).
4.2. Case study of access to economically affordable house in Beijing City

In China, applying for the economically affordable house is conducted by family as the unit. Household population, especially the population that has economical ability, can influence the family purchasing ability. In fact, the local governments of different cities have put income of family that could be composed of different members into consideration when setting the standard for application. In the presented case as followed, Beijing City as the capital of China will be taken as an example to illustrate the proposed model as mentioned above.

In released standard for application of Beijing City in Oct. 5, 2007, the annual family income of a family of one, a family of two, a family of three, a family of four, and a family of five respectively is 22,700 RMB, 36,300 RMB, 45,300 RMB, 52,900 RMB, and 60,000 RMB. and standard household net assets respectively are less than 240,000 RMB, 270,000 RMB, 360,000 RMB, 450,000 RMB, and 480,000 RMB (local Government of Beijing City, 2007).

Before this standard being released, the standard annual family income for applying for economically affordable house in Beijing City is 60,000 RMB, which is a unified standard for all kinds of family structure and does not distinguish based on the number of family member. Therefore, new released standard in 2007 intended to focus on the different family structure, on the basis of which the standard economically affordable house area for a family of three in Beijing City will be discussed because it is primary family structure in China right now.

Based on the data presented before, annual income of qualified applicator is set as 45,000 RMB and income of unqualified applicator is set as 60,000 RMB. Thus $\theta_{\text{low}} = 45,000$, $\theta_{\text{high}} = 60,000$.

In 2007, average market price of house transaction is 14,411 RMB/m² according to government transaction data (Research Group on Beijing Housing Price, 2007). For the conveniences of calculation, market price is set as 14,400 RMB/m² in illustrated case that is $c = 14,400$. Meanwhile land price per floor area in 2007 is 3,799 RMB/m², which is set as 3,800 RMB/m² in illustrated case that is $P' = 3,800$. According to the suggestions of experts from Ministry of Construction, $t\%$ is set as 85%.

Thus the reference value of parameter in presented case can be shown in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\theta_{\text{low}}$</th>
<th>$\theta_{\text{high}}$</th>
<th>$c$</th>
<th>$P'$</th>
<th>$t%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference value</td>
<td>45000</td>
<td>60000</td>
<td>14400</td>
<td>3800</td>
<td>85%</td>
</tr>
</tbody>
</table>

The utility of government is then can be calculated by Eq. (12).
Accordingly, the utility of government can achieve maximum when \( S^* = 29.46 \approx 30 \), which can make qualified applicants apply for the economically affordable houses and unqualified applicants not apply for the economically affordable houses. The maximum utility of government is 81700 + \( M \).

4.3. Discussion

The illustrated case shows how to calculate the economically affordable house area for a family of three to maximize the utility of government considering incentive compatibility. Slightly unsatisfactory is that the average area for each person is only 10 m² that is relatively small, but this result is also rescannable. The average housing area per person in China right now is about 25 m², which includes urban housing and a great amount of rural housing. For a family of three in urban, a house with 30 m² can meet basic requirements for live.

1) The range of numerical value for \( S \) is on the interval (11.89, 36.93), which is small. This range is based on the survey on the house consumption tendency of residents, and is also related to the incomes of unqualified applicants \( \theta_{\text{high}} \) and the characteristics. The former focuses on the influence on the area that unqualified applicant is willing to purchase. The latter focuses on the position of range on the axis. Actually, the incomes of residents will strongly influence their house consumption activities.

2) The rule of incentive compatibility was well utilized in proposed model. The ratio of housing price to income based on the calculation result is 30m² × 3800/45000 = 2.533, which is much lower than the ratio 25 according to the latest reports from Beijing (Lu et al., 2010). If the ratio achieves the UK level between 3 and 6\(^1\), the annual family income will be between 38,000 RMB and 19,000 RMB. This is the level that should apply for low-rent housing in China. On the other hand, the ratio will be 1.9 if applicants \( \theta_{\text{high}} \) get the houses, which will be very low and unreasonable. Thus the proposed model can well protect the benefits of qualified applicants and exclude unqualified applicants. Furthermore, a relatively small result (30m²) can further control the opportunistic practices of applicants \( \theta_{\text{high}} \).

3) The utility function of government in Eq. (1) is difficult to model and parameterization. The Table 1 and Table 2 are used to qualitatively describe the current action of government. The utilities of government and applicants in the game are assumed based on the real status in China, which are not used in proposed model. Actually, the government’s incomes are described by qualified applicants’ incomes as shown in Eq. (11) and Appendix 3. Meanwhile, the value \( M \) of utility for government to obtain social reputation is a constant and could be influenced by macro environment. \( M \) would be related to the area of economically affordable house. When \( S \) is changing larger, \( M \) will be larger.

\[^{1}\text{Data source: http://www.mortgageguideuk.co.uk/blog/house-prices/house-prices-income/}\]
5. POLICY SUGGESTIONS

Therefore, some suggestions can be made to help Chinese decision-maker improve system of economically affordable housing according to the analysis on present mechanism and area calculation as mentioned above, which focus on how to enhance external environment and establish internal mechanism.

5.1. Enhancement of external environment

Credit evaluation and income monitor

The credit evaluation system of residents should be established, which has very close relationship with social security system. In China, social security system completely relies on the information of population to be safeguarded that can be provided by the populations themselves, the companies or work unit (Danwei) they work in, and the communities they live in (Duda et al., 2005; Ye and Wu, 2008; Guo and Chang, 2008). Though there are some channels to obtain the private information of applicators, the information are still imperfect for government. Under this situation, the credit evaluation can help government identify who are the most appropriate people should be safeguarded, which can give an objective measurement on the applicators relating to their income level and the possibility that they deceive the government. Thus the incomes of applicators should be monitored, which can provide scientific and accurate definition for the applicator type and application standard benefiting the external environment for the system of economically affordable housing provision.

Long-term investigate on house consumption psychology

Whether the consumers will apply for economically affordable houses and how much area they will purchase depend on their measurement of incomes by the purchasing activities, which has weak relationship with whether the activities are permitted, especially in the case that the punishment is not strict (Wang and Li, 2006; Li, 2009). In order to make appropriate policies, the house consumption psychology and preferences of different type of consumers should be deeply and accurately analyzed from a long-term perspective.

5.2. Establishment of internal mechanism

Further specification and verification of population to be safeguarded

Based on the classification of China Statistical Yearbook (2007), the residents are classified to 7 groups, which are the lowest, low, lower middle, middle, higher middle, high, and the highest. However, this kind of classification is not accurate enough for making policies. Present security objectives for economically affordable housing are low-income families in China, which means at least 50-60% families can benefit from this policy based on a wide definition of Population to be safeguarded\(^1\) (China Statistical Yearbook, 2007-2009). No matter which model and means adopted by the policy of public housing, the governmental resource should be ensured in an affordable range, which is the basis of long-term sustainable development of economically affordable housing system (Liu et al., 2008). Due to present situation in China, prior consideration should be put into the most needed population. Thus real safeguarded objectives should be further specified and verified.

Scientific and transparent policy for application standards

Setting standards of house area for qualified applicators should not only meet the requirement that maximize the benefits of applicators,

\(^{2}\) Based on the China Statistical Yearbook (2006-2009), the percent of lowest-income families in China is about 10%, the percent of low-income families is about 30%, and the percent of lower middle-income families is about 20%
but also prevent unqualified applicators from strategic selective actions. In present policies, family population and local average house area are main indicators when approving the area of qualified applicators (Guo and Chang, 2008; Stephens, General Office of the State Council of the People’s Republic of China, 2007). The indicators that have incentive function are ignored in the decision-making process, which will make unqualified applicators apply for economically affordable houses seeking economical benefits. Hence some measures should be taken to limit the utilities of unqualified applicators when they obtain the houses, all of which the most effective and directive method is to control house area.

Controlling the actions to develop economically affordable houses and limiting their unit area

In China, many economically affordable houses are developed by private sectors based on the requirements of local governments (Wong et al., 2006). Thus some developers would facilitate sales of economically affordable houses by increasing the area of unit, which could make approved applicators obtain additional area. Therefore, it is more difficult to control those applicators. Meanwhile social resources are still wasted.

Increasing the degree of punishment

In present laws and regulations relating to economically affordable housing, the punishment on the fraudulent activities to purchase economically affordable houses is just to make up a deficiency as same as market price. The detailed punishment is relatively vague and lack of strength, which would not an alert for the one intend to deceive. The appropriate method is to increase the degree of punishment and reduce the expected incomes of unqualified applicators based on the above-mentioned calculation, which can effectively reduce the occurrence of fraudulent purchasing and the waste of social resources.

6. CONCLUSIONS

Given the increased rates of housing marketisation, housing privatization, and urbanization in China, tremendous challenges have been imposed on governments as well as the low-income families to improve their housing conditions to cope with the increasing dynamics and uncertainties derived from economical development. To address these challenges effectively, Chinese government has adjusted social housing policies to protect and safeguard the benefits of low-income families since 2006. Despite the government’s efforts to improve low-income families’ living standards, the inequality in present housing distribution system still exists due to an unreasonable mechanism in social housing system. Among the problems, the policies on access to economically affordable housing are widely questioned, which need to be carefully analyzed and re-designed to exclude those unqualified applicators and include those qualified applicators to ensure the social fairness and not to waste the social resources.

The present policies on access to economically affordable housing was analyzed in the paper by mechanism design, which indicate that strengthening supervision and announcement system is reasonable and necessary for government, and the restriction in current mechanism of economically affordable housing in China should be strict. The most important finding was that the area of economically affordable house could greatly affect whether the policy was incentive compatibility. Therefore, a housing area calculation model based on the incentive mechanism design was presented and an illustrated case of Beijing City was also conducted. The results showed that an economically affordable house that area was 30 m² for a family of three can maximize the utility of government considering incentive compatibility, based on which some policy suggestions are made including enhancing external environment (Credit evaluation and income monitor,
Long-term investigate on house consumption psychology) and establishing internal mechanism (Further specification and verification of population to be safeguarded, Scientific and transparent policy for application standards, Controlling the actions to develop economically affordable houses and limiting their unit area, and Increasing the degree of punishment). The discussion in this paper is expected to generate interest in a more thorough understanding of the current New Housing Reform and hence a more rounded research agenda of the application of policy management in social housing system in the Chinese context.

On the other hand, the limitation is that the paper mainly focuses on academic findings based on many qualitative assumptions and aims to absorb the interests on affordable housing in China. Further researches should focus on how to make model more applicable to be applied in different region and contribute to the real public policy decision making.

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REFERENCES


SANTRAUKA

GALIMYBIŲ GAUTI ĮPERKAMĄ BŪSTĄ KINISOJE POLITIKOS ANALIZĖ: PLOTO SKAIČIAVIMO MODELIS, PAGRĮSTAS SKATINAMOJO MECHANIZMO PROJEKTAVIMU

Jiankun ZHANG, Jingfeng YUAN, Miroslaw J. SKIBNIEWSKI

Būsto tema Kinijoje šiuo metu itin domina daug mokslininkų, nes šalyje, kuri turi daugiausia gyventojų pasaulyje, gyvenimo sąlygų gerėjimas atsidūrė nuo šalies ekonominio augimo. Šiame darbe nusakoma baziinoa socialinio būsto padėtis Kinijoje ir vertinamas XX a. 10-ajame dešimtmečyje dėl bendros ūkio padėties gerėjimo prasidejusios būsto reformos netolygumas. Siekiant mažinti Kinijos socialinio būsto skirtumų lygį, šiame darbe daugiausia dėmesio skiriama politinėms priemonėms, kurias pasitelkus valdoma prieiga prie įperkamo būsto, nes dėl neracionalaus mechanizmo būsto įperkamumas Kinijoje yra neįprastai didelė problema, o ypač mažos pajamos gaminiami šeimoms. Atitinkamai politikos kryptys šiame darbe nagrinėjamos pasitelkus skatinamojo mechanizmo projektavimą, kuris rodo, kad priežiūros ir pranešimų sistema, dabartinio mechanizmo suvaržymas ir įperkamo būsto ploto turi lemiamą reikšmę dabartinų politikos kryptų veiksmingumui. Be to, pateikiami skatinamojo mechanizmo projektavimą pagrįstas būsto ploto skaičiavimo modelis ir atliekamas Pekino atvejo tyrimas, kuris, atsižvelgiant į skatinamųjų priemonių suderinamumą, gali maksimaliai padidinti vyriausybės naudingumą. Pabaigoje nagrinėjami keli politiniai siūlymai, įskaitant išorinės aplinkos gerinimą ir vidinio mechanizmo sukūrimą.
APPENDIX 1

\( \Theta \): the applicants’ type space constituted by all possible type of applicators;

\( \theta_k \): a specific type of applicants, \( \theta_k; \Theta, k = 1, \ldots, n, \theta_k \) can not only be one-dimensional but also be multidimensional. Meanwhile, \( \theta_k \) can not only be continuous but also be discrete. In model proposed in this paper, the applicants can be divided into two types based on that whether the actual economical situation of applicant consistent with the economical situation regulated by Management Instrument of Economically affordable Housing (Ministry of Construction of the People’s Republic of China, 2007). Therefore, the type of applicators would be qualified and unqualified, which are respectively defined as \( \theta_{high}, \theta_{low} \);

\( p_k \): the possibility that applicator belongs to \( \theta_k \), which can be defied as

\[
\sum_{k=1}^{n} p_k = p_{low} + p_{high} = 1;
\]

\( S_k \): the decision vector determined by planner based on the agent’s type provided by agent itself. In model proposed in this paper, \( S_k \) is the area of economically affordable housing;

\( \pi(S_k) \): the outcome/benefits of planner;

\( t(S_k) \): the payoff of agent from planner based on \( S_k \), also can be called as incentive fee;

\( \pi(S_k) - t(S_k) \): the V-N-M utility function of planner

\( u \): the reservation utility of agent, which is the expected utility when agent not applying for the economically affordable house;

\( C(\theta_k, S_k) \): the costs of agent to select \( S_k \).

APPENDIX 2

As house is also a kind of commodity, it also coincides with the law of diminishing marginal utility (Bergemann and Morris, 2005). In Chinese life, house is the most important fixed assets for common people. Meanwhile, the total assets of Chinese common people in housing consumption are primarily comprised of house and bank deposit. The percent of other type of assets is very low. Consequently, currency and house area constitute two interchangeable commodities. Furthermore, they can be analyzed by indifference curve.

In Figure 4, the consumer believes that a house with 120 m\(^2\) plus cash of 6,000 RMB have the same utility with a house with 40 m\(^2\) plus cash of 14,000 RMB. According to Figure 1, a series of indifference curves corresponding to each level of utility can be mapped, in which the indifference curve that has the longest distance with the origin. Any indifference curve can not intersect with others, as shown in Figure 5. The indifference curve, which is the ranking of preferences between two different houses, is completely determined by subjective consciousness of consumers despite of cost differences and income status. The final decision depends on the budget constraints of consumers that are applicators in this paper (Laffont and Martimort, 2000). Based on the hypothesis of finite rationality economic man, equilibrium point of consumers to pursue utility maximization should be the intersection point that budget constraints line that can touch the indifference curve with the highest utility level, which is the cut point between budget constraints line and the indifference curve as shown in Figure 6.
**Figure 4.** Indifference curve of currency and house area

**Figure 5.** A series of indifference curves

**Figure 6.** Equilibrium point of consumers

**Figure 7.** The analysis on the equilibrium of consumer
It can prove that the ratio of two commodity prices equal to marginal rate of substitution and ratio of marginal utilities of two commodities when at the equilibrium point of consumers. If the ratio of commodity prices is viewed as an objective evaluation made by the market, ratio of marginal utilities of commodities can be viewed as a subjective evaluation made by consumers. Thus consumers can achieve utility maximization when objective evaluation exactly accord with subjective evaluation as shown in Figure 7.

The concept of Consumer Surplus can also be introduced by Figure 7. Consumer Surplus means consumer can obtain net income, which can explain the reason for occurrence of purchasing behavior from another perspective. The area surrounded by vertical axis, demand curve, and price line is Consumer Surplus (Bergemann and Morris, 2005). \( V(\theta_k, S) \) indicates the marginal utility of house with area \( S \): when \( \theta \) is constant, \( V(\theta_k, S) \) is demand curve of \( S \), which means the utility of economically affordable house per unit area intend to reduce for the applicators in pace with increase of area. Although the indifference curve, which is the ranking of preferences between two different houses, is completely determined by subjective consciousness of consumers despite of cost differences and income status, the attitudes of common people will be strongly influenced by long-term change of income.

The consume costs of applicator can be expressed by the differences between market price and consumer utility according to the origin of Consumer Surplus (Bergemann and Morris, 2005), which is

\[
C = c \times S^* - \int_{S=0}^{S=S^*} V(\theta_k, S) \quad (13)
\]

where: \( c \) is the unit market price for purchasing commercial house; \( V(\theta_k, S) \) is marginal utility of house area for applicator.

**APPENDIX 3**

1) \( \pi(\theta_k) \)

When the applicator purchase economically affordable house with area \( S^* \), \( \pi(\theta_k) \) is shown in Eq. (14).

\[
\pi(\theta_k) = \begin{cases} 
[S^* \times P' \times t\% - (c \times S^* - \int_{S=0}^{S=S^*} V(\theta_{low}, S) \times S)] + M, \theta_k = \theta_{low} \\
-[S^* \times P' \times t\% - (c \times S^* - \int_{S=0}^{S=S^*} V(\theta_{low}, S) \times S)] - M, \theta_k = \theta_{high}
\end{cases} \quad (14)
\]

2) \( t(S^*) \)

When the applicator purchase economically affordable house with area \( S^* \), \( t(S^*) \) is shown in Eq. (15).

\[
t(S^*) = S^* \times P' \quad (15)
\]
3) \( \pi(\theta_k) - t(S^*) \)

When the applicator purchase economically affordable house with area \( S^* \),

\[
\pi(\theta_k) - t(S^*) = \begin{cases} 
[S^* \times P' \times t\% - (c \times S^* - \int_{S=0}^{S=S^*} V(\theta_{\text{low}}, S) \times S)] + M - S^* \times P', \theta_k = \theta_{\text{low}} \\
-[S^* \times P' \times t\% - (c \times S^* - \int_{S=0}^{S=S^*} V(\theta_{\text{low}}, S) \times S)] - M - S^* \times P', \theta_k = \theta_{\text{high}} 
\end{cases}
\]  
(16)

4) Constraints

The objective of proposed mechanism for government is not about a specific action after signing the contract, but to accept or do not accept the application of applicators. Hence there is just the constraint of rule for participation, which can be developed by Eq. (17) and Eq. (18).

\[
S^* \times P' \times t\% - [c \times S^* - \int_{S=0}^{S=S^*} V(\theta_{\text{low}}, S) \times S] > \int_{S=0}^{S=S_A} V(\theta_{\text{low}}, S) \times S - c \times S_A 
\]  
(17)

\[
S^* \times P' \times t\% - [c \times S^* - \int_{S=0}^{S=S^*} V(\theta_{\text{high}}, S) \times S] < \int_{S=0}^{S=S_B} V(\theta_{\text{high}}, S) \times S - c \times S_B 
\]  
(18)

5) Objective function of government

By mechanism design, all applicators should be qualified, which means \( p_{\text{low}} = 1 \). Unqualified applicators will not apply for economically affordable house. Thus the objective function of government is shown in followed.

\[
\max \{[S^* \times P' \times t\% - (c \times S^* - \int_{S=0}^{S=S^*} V(\theta_{\text{low}}, S) \times S)] + M - S^* \times P'\} 
\]  
(19)

6) Model of mechanism design

A model of mechanism design for the mechanism of access to economically affordable house in China under information asymmetry is described in Eq. (20).

\[
\begin{align*}
\max & \{[S^* \times P' \times t\% - (c \times S^* - \int_{S=0}^{S=S^*} V(\theta_{\text{low}}, S) \times S)] + M - S^* \times P'\} \\
\text{s.t.} & \begin{cases} 
[S^* \times P' \times t\% - (c \times S^* - \int_{S=0}^{S=S^*} V(\theta_{\text{low}}, S) \times S)] > \int_{S=0}^{S=S_A} V(\theta_{\text{low}}, S) \times S - c \times S_A \\
[S^* \times P' \times t\% - (c \times S^* - \int_{S=0}^{S=S^*} V(\theta_{\text{high}}, S) \times S)] < \int_{S=0}^{S=S_B} V(\theta_{\text{high}}, S) \times S - c \times S_B 
\end{cases}
\end{align*}
\]  
(20)
The data range of $S^*$ can be obtained from constraints, within which the maximum of objective function will be obtained (Laffont and Martimort, 2000).

**APPENDIX 4**

The marginal utility of applicator can be obtained on the basis of the prior research on the different income groups’ preferences to housing consumption (Wang and Li, 2006; Li, 2009). In the illustrated case, the applicators with type $\theta_{\text{high}}$ are willing to pay 50% annual income for houses when they have no houses, after that the price that applicator is willing to pay will decline 5% every 5 $m^2$. On the other hand, the applicators with type $\theta_{\text{low}}$ are willing to pay 45% annual income for houses when they have no houses, after that the price that applicator is willing to pay will decline 5.5% every 5 $m^2$. The detailed information is shown in Table 4.

**Table 4.** The marginal utility of different type of applicators for different house area

<table>
<thead>
<tr>
<th>Type of applicator</th>
<th>House area</th>
<th>1–5 $m^2$</th>
<th>6–10 $m^2$</th>
<th>11–15 $m^2$</th>
<th>16–20 $m^2$</th>
<th>21–25 $m^2$</th>
<th>26–30 $m^2$</th>
<th>31–35 $m^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{\text{high}}$</td>
<td>30000</td>
<td>28500</td>
<td>27075</td>
<td>25721.25</td>
<td>24435.19</td>
<td>23213.43</td>
<td>22052.76</td>
<td></td>
</tr>
<tr>
<td>$\theta_{\text{low}}$</td>
<td>20250</td>
<td>19136.25</td>
<td>17988.08</td>
<td>16908.79</td>
<td>15894.26</td>
<td>14940.61</td>
<td>14044.17</td>
<td></td>
</tr>
<tr>
<td>$\theta_{\text{high}}$</td>
<td>36–40 $m^2$</td>
<td>41–45 $m^2$</td>
<td>46–50 $m^2$</td>
<td>51–55 $m^2$</td>
<td>56–60 $m^2$</td>
<td>61–65 $m^2$</td>
<td>66–70 $m^2$</td>
<td></td>
</tr>
<tr>
<td>$\theta_{\text{low}}$</td>
<td>20950.12</td>
<td>19902.61</td>
<td>18907.48</td>
<td>17962.11</td>
<td>17064</td>
<td>16210.8</td>
<td>15400.26</td>
<td></td>
</tr>
<tr>
<td>$\theta_{\text{high}}$</td>
<td>13201.52</td>
<td>12409.43</td>
<td>11664.86</td>
<td>10964.97</td>
<td>10307.07</td>
<td>9688.649</td>
<td>9107.33</td>
<td></td>
</tr>
<tr>
<td>$\theta_{\text{low}}$</td>
<td>71–75 $m^2$</td>
<td>76–80 $m^2$</td>
<td>81–85 $m^2$</td>
<td>86–90 $m^2$</td>
<td>91–95 $m^2$</td>
<td>96–100 $m^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_{\text{high}}$</td>
<td>14630.25</td>
<td>13898.74</td>
<td>13203.8</td>
<td>12543.61</td>
<td>11916.43</td>
<td>11320.61</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_{\text{low}}$</td>
<td>8560.89</td>
<td>8047.24</td>
<td>7564.40</td>
<td>7110.54</td>
<td>6683.91</td>
<td>6282.87</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For $c = 14,400$, it can be conclude that $S_A = 30 m^2$, $S_B = 75 m^2$ from Table 4.

**Figure 8.** The utility of different type of applicators
The total utility figure of different type of applicators is shown in Figure 8, which can be described by Eq. (21).

\[
\int V(\theta, S) \times S = (l_1 S^4 + l_2 S^3 + l_3 S^2 + l_4 S)(l_5 \theta + l_6)
\]  

(21)

By numerical simulation using MATLAB software, fitting curve can be obtained as shown in Eq. (22)

\[
\begin{align*}
    l_1 &= -1.09e - 9 \\
    l_2 &= 7.698e - 4 \\
    l_3 &= -0.5445 \\
    l_4 &= 135.1 \\
    l_5 &= 6.093e - 3 \\
    l_6 &= -138.7
\end{align*}
\]  

(22)

The comparison between fitting curve and discrete data is shown in Figure 9.

[Figure 9. The fitting curve when \( \theta = 45000 \) and 60000]

When \( \theta \) is changing between 45000 and 60000, the fitting curves can be obtained as shown in Figure 10.
Furthermore, there are different values of $S$ corresponding to $\int V(\theta, S)$ for different combinations of $S$ and $\theta$, which will constitute a camber in 3D total utility space. The camber for $[\theta \in (45000, 60000), S \in (0, 300)]$ is shown in Figure 11.
Therefore, the total utility of residential in the illustrated case can be described as followed.

\[
\int V(\theta, S) \times S = \left\{ (-1.09 \times 10^{-9})S^4 + (7.698 \times 10^{-4})S^3 - 0.5445S^2 + 135.1S \right\} \left\{ (6.093 \times 10^{-3})\theta - 138.7 \right\} \tag{23}
\]

Eq. (23) is equivalent to Eq. (24)

\[
\int V(\theta, S) \times S = \begin{cases} 
135.485 \left\{ (-1.09 \times 10^{-9})S^4 + (7.698 \times 10^{-4})S^3 - 0.5445S^2 + 135.1S \right\}, & \theta = \theta_{low} = 45000 \\
226.880 \left\{ (-1.09 \times 10^{-9})S^4 + (7.698 \times 10^{-4})S^3 - 0.5445S^2 + 135.1S \right\}, & \theta = \theta_{high} = 60000 
\end{cases} \tag{24}
\]

Then Eq. (24) can be put into Eq. (17) and Eq. (18), which is shown in Eq. (25) and Eq. (26).

\[
135.485 \left\{ (-1.09 \times 10^{-9})S^4 + (7.698 \times 10^{-4})S^3 - 0.5445S^2 + 135.1S \right\} - 11170S > \\
135.485 \left\{ (-1.09 \times 10^{-9}) \times 30^4 + (7.698 \times 10^{-4}) \times 30^3 - 0.5445 \times 30^2 + 135.1 \times 30 \right\} - 14400 \times 30 \tag{25}
\]

\[
226.880 \left\{ (-1.09 \times 10^{-9})S^4 + (7.698 \times 10^{-4})S^3 - 0.5445S^2 + 135.1S \right\} - 11170S < \\
226.880 \left\{ (-1.09 \times 10^{-9}) \times 75^4 + (7.698 \times 10^{-4}) \times 75^3 - 0.5445 \times 75^2 + 135.1 \times 75 \right\} - 14400 \times 75 \tag{26}
\]

Eq. (25) and Eq. (26) can be further converted to

\[
(-1.09 \times 10^{-9})S^4 + (7.698 \times 10^{-4})S^3 - 0.5445S^2 + 52.66S > 374.41 \tag{27}
\]

\[
(-1.09 \times 10^{-9})S^4 + (7.698 \times 10^{-4})S^3 - 0.5445S^2 + 85.87S < 2309.46 \tag{28}
\]

When \(S < 100\), the value of \((-1.09 \times 10^{-9})S^4\) is very small and can be negligible. Thus Eq. (27) and Eq. (28) can be simplified. Then

\(11.89 < S < 107.98\) or \(S > 593.43\) is obtained by Eq. (27), and \(S < 36.93\) or \(174.72 < S < 493.69\) is obtained by Eq. (28).