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INVESTMENT DECISION MAKING ALONG THE B&R USING CRITIC APPROACH IN PROBABILISTIC HESITANT FUZZY ENVIRONMENT

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Abstract. The Belt and Road (B&R) Initiative receives enthusiastic response, the aim of which is to develop cooperative partnerships with countries along the routes and build a community of common destiny. So far, Chinese companies have invested in many different countries along the B&R. Generally, the investment decision making problems are characterized by high risk and uncertainty. Then how to make an appropriate investment decision will be a thorny issue. In this paper, probabilistic hesitant fuzzy set (PHFS) is used for handling uncertainty in multiple attribute decision making (MADM), and the criteria importance through intercriteria correlation (CRITIC) approach is extended to obtain attribute weights, no matter whether the weight information is incompletely known or not. Considering that the existing probabilistic hesitant fuzzy distance measures fail to meet the condition of distance measure, a new distance between PHFSs is proposed and applied to investment decision making for countries along the B&R. In the last, comparative analyses are performed to illustrate the advantages of the presented approach.

Keywords: investment decision making, CRITIC, attribute weights, distance measure, the Belt and Road, probabilistic hesitant fuzzy sets.

JEL Classification: C49, C61, D81.

Introduction

With the deep integration of the multilateral economy, it is increasingly difficult for the World Bank and the International Monetary Fund to perform a regulatory part in world economy (Di & You, 2018). The effect of the global financial crisis in 2008 has not been fully removed. The world economy is recovering slowly, and the development problems facing all countries are still severe. Therefore, it is necessary to adjust the current international investment and trade pattern. In 2013, China proposed the cooperation initiative for co-

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constructing the B&R, which is short for the Silk Road Economic Belt and the 21st Century Maritime Silk Road. The B&R initiative aims at providing the countries along the routes with investment and financing support for resource development, infrastructure construction and so on. It breaks the limitations of the original international economic assistance framework and broadens the international collaboration and division of labor mechanism, and thus achieves win-win goals. In the future, China will further promote the healthy, standardized and sustainable development of outbound investment, expand the breadth and depth of co-operation, and improve the marginal return on investment (Yang et al., 2019).

The investment decision making can be thought as a MADM problem. It is a very important financial decision that can directly affect the enterprise's development. As mentioned before, the nature of investment decision making problems along the B&R is often complicated and uncertain, and MADM methods will be effective for dealing with such problems (Yuan et al., 2019; Duan et al., 2018). It is noted that the investment environment often remains uncertain, and uncertainty is an important factor in MADM process (Bolturk, 2018; Liu et al., 2019). To handle uncertainty, fuzzy set theory is proposed (Zadeh, 1965). Then, to better reflect the objective world and model human thinking, some enhanced versions for fuzzy set are presented, such as intuitionistic fuzzy set (IFS)(Atanassov, 1986), HFS (Torra, 2010), Pythagorean fuzzy set (PFS) (Yager, 2014), PHFS (Zhu & Xu, 2018). They are widely applied in MADM. Among them, PHFS has attracted an increasing attention from researchers due to its strong ability to simulate the reality. It can be considered as a random variable in probabilistic hesitant fuzzy MADM (PHFMADM), where the evaluation value consists of two parts, i.e. membership degree values and their corresponding probabilities, and thus can reserve much more information in MADM (Zhang et al., 2017). Up to now, PHFSs have been used in emergency response (Gao et al., 2017; Wu et al., 2019), cluster analysis (Song et al., 2019) and consensus-based decision making (Li & Wang, 2018; Wu et al., 2018; Xu & Zhou, 2017). Li and Wang (2017) adopted the QUALIFLEX approach based on PHFSs to evaluate the green suppliers. Su et al. (2019) proposed several entropy measures for PHFSs. It is noted that the distance measures between PHFSs play a key role in solving these problems mentioned above. However, the current approaches cannot ensure that the distance measure between any two PHFSs is equal to zero if and only if the two PHFSs are the same. Therefore, in practical application, unreasonable decision results may be derived. To make up the defect, a novel approach for calculating the distance measure between PHFSs will be developed in this paper.

Moreover, the attribute weights, which are considered to be an important factor affecting the decision results, are not easy to determine. Particularly, people may be confronted with the situation that the weight information is incompletely known. In addition, there is usually interaction among attributes. However, the existing methods for deriving attribute weights under probabilistic hesitant fuzzy environment neglect the mutual relations between attributes (Ding et al., 2017; Su et al., 2019). Then, to overcome these defects, two approaches based on CRITIC method (Diakoulaki et al., 1995) for determining attribute weights will be presented. The presented approach can not only be utilized for handling the situation that weight information is incompletely known, but also take the correlation among attributes into account. To demonstrate the superiority of the presented approach, an application on investment decision making for the countries along B&R will be offered. The main contributions of this paper are as below:

1) An approach for obtaining attribute weights in the setting of PHFSs is presented. In the MADM, the attribute weights have become an important factor for affecting decision results. Aiming at the situation where weight information is incomplete or unknown, two mathematical models based on CRITIC method are constructed. The proposed model can well capture the correlation structure among attributes, which is common in MADM.
2) Novel probabilistic hesitant fuzzy distances are proposed and used in investment decision making. Given the shortcomings of the available distances between PHFSs, novel distances for PHFSs are proposed. Besides, a method using the proposed distance measures is put forward for assessing the countries along B&R.

The remainder of this article is arranged as below. A literature review on distance measure and PHFMADM is offered in Section 1. In Section 2, some preliminaries related to HFS and PHFS are given. Section 3 provides novel distance measures between PHFSs and a novel approach to PHFMADM. Section 4 presents an application on investment decision making. Conclusions are given in last Section.

1. Literature review

Distance measure is considered to be an effective tool in distinguishing the difference between two objects (Li et al., 2015). It has been used in MADM (Xu & Xia, 2011a), pattern recognition (Hatzimichailidis et al., 2012), clustering analysis (Zhang & Xu, 2015). Among them, the Hamming, Euclidean and Hausdorff distances are most widely used distance measures, based on which a series of distances for IFSs have been developed (Szmidt & Kacprzyk, 2000; Grzegorzewski, 2004). Singh (2014) proposed several distances for type-2 fuzzy sets. Zhang and Xu (2014) developed a distance measure between PFSs. Afterwards, to reflect the properties of PFSs, Li and Zeng (2018) and Zhou and Chen (2019) put forward novel distance measures for PFSs respectively. In addition, some distances between HFSs have been developed and applied to MADM (Xu & Xia, 2011a; Li et al., 2015; Liu et al., 2017).

As an extension of HFS, PHFS can reserve more information than HFS (Xu & Zhou, 2017; Zhang et al., 2017). Since its appearance, PHFS has acted as a useful tool in decision analysis. Zhou and Xu (2018) presented the fuzzy preference relations in the setting of PHFSs. Then a new consensus reaching process is presented and applied to group decision making (Wu et al., 2018). Moreover, the axiomatic definition of probabilistic hesitant fuzzy distance measure is provided (Ding et al., 2017). Gao et al. (2017) put forward the Hamming and Euclidean distances for PHFSs, and a dynamic decision making approach for emergency response is presented. The QUALIFLEX method based on the Hausdorff distance between PHFSs is utilized for the selection of green suppliers (Li & Wang, 2017). Su et al. (2019) put forward several distance and entropy formulas for PHFSs. Besides, they offered an entropy-based approach for investment decision making. With the aid of hesitant degree of PHFE, Wu et al. (2019) presented a new distance for PHFSs. Obviously, distance measures have been proven to be useful in MADM (Liu et al., 2018). However, the available distances for PHFSs have failed to meet the condition of distance measure, which implies that they are not appropriate distance measures for PHFSs. Then, it is essential to exploit novel distance measures for PHFSs, which is also one of the motivations of this paper.

In addition, the weight of attribute reflects its relative importance, and has played a central role in MADM. Many different approaches have been suggested for determining attribute weights and could be grouped into three categories in the following: objective (Hwang & Yoon, 1981; Wang, 1998; Deng et al., 2000), subjective (Horsky & Rao, 1984; Hwang & Yoon, 1981) and integrated (Ma et al., 1999; Wang & Parkan, 2006). The objective method mainly uses objective decision information for ascertaining attribute weights. The subjective method adopts subjective preference information for obtaining attribute weights. And the integrated approach combines two sources of information above for assessing attribute weights. However, there is little research on the method for determining attribute weights in the setting of PHFSs. To handle the situation where weight information is incompletely known, Ding et al. (2017) put forward a TOPSIS-based approach for PHFMADM. Moreover, Su et al. (2019) used the entropy weight-based approach for evaluating attribute weighs. The above researches provide a design foundation for obtaining attribute weights in the setting of PHFSs. However, they have overlooked the fact that there is usually strong correlation among attributes in MADM. To overcome the defect, two mathematical models for determining attribute weights will be constructed. The CRITIC method has been proven to be effective in obtaining objective weights (Diakoulaki et al., 1995; Wang & Zhao, 2016). It can capture the correlation structure well among the attributes (Zhao et al., 2011). In this research, the CRITIC method will be extended for solving the PHFMADM problem.

2. Preliminaries

2.1. HFS and PHFS

To model the hesitancy of people in offering his or her preferences over objects, Torra (2010) defined the HFS. After that, a concise representation method for HFS is provided as below.

Definition 1 (Xia & Xu, 2011). Assume X is a universe of discourse. The HFS on X takes the following form:

$$A = \left\{ \left\langle x, h_A(x) \right\rangle \middle| x \in X \right\}, \tag{1}$$

here, $h_A(x)$ represents the membership values of x to A and is called a HFE. It contains several distinct values in $\left[0,1\right]$.

Obviously, the probability information for HFE is ignored. To fill this gap, Zhu and Xu (2018) proposed PHFS.

Definition 2 (Zhu & Xu, 2018). Assume X is a universe of discourse. The PHFS on X is as below:

$$H = \left\{ \left\langle x, h_x(\gamma_l | p_l) \right\rangle \middle| x \in X \right\}, \tag{2}$$

here, $h_x(\gamma_l | p_l)$ represents the membership values of x to H and is called a PHFE. It includes the membership degrees $\gamma_l \left(l = 1,2,\cdots,\left|h_x\right|\right)$ and their probabilities $p_l \left(l = 1,2,\cdots,\left|h_x\right|\right)$ such that $p_l \in \left[0,1\right]$ and $\sum_{l=1}^{|h_x|} p_l = 1$. Here, $\left|h_x\right|$ denotes the number of the possible membership degrees in $h_x\left(\gamma_l | p_l\right)$. In what follows, h_x denotes the PHFE and is short for $h_x\left(\gamma_l | p_l\right)$.

Let h_1 and h_2 be two PHFEs. Generally, $\left|h_1\right| \neq \left|h_2\right|$. To operate correctly, the shorter PHFE is extended so that the compared PHFEs are equal in length (Gao et al., 2017; Zhang et al., 2017). According to risk preferences of decision makers, the appropriate values can be chosen and added to the shorter PHFE. Without loss of generality, the shorter PHFE can be extended through adding the minimum membership degree in it with corresponding probability 0. Li et al. (2015) suggested the distance should be calculated in a unified space. Otherwise, misleading results are obtained. Then, the PHFEs are extended uniformly in this paper.

Remark 1. Assume $h_1\left(\gamma_i \middle| p_i\right)$ and $h_2\left(\gamma_j \middle| p_j\right)$ are two PHFEs. Then

$$h_1\left(\gamma_i \middle| p_i\right)=h_2\left(\gamma_j \middle| p_j\right) \Leftrightarrow h_1^{\sigma(l)}\left(\gamma_i p_i\right)=h_2^{\sigma(l)}\left(\gamma_j p_j\right);$$

$$h_1^{\sigma(l)}\left(\gamma_i\right)=h_2^{\sigma(l)}\left(\gamma_j\right),\quad h_1^{\sigma(l)}\left(p_i\right)=h_2^{\sigma(l)}\left(p_j\right)\ \left(l=1,2,\cdots,|h|\right),$$

here, $|h|=\max\left\{|h_1|,|h_2|\right\}$, $h_1^{\sigma(l)}\left(\gamma_i p_i\right)$ and $h_2^{\sigma(l)}\left(\gamma_j p_j\right)$ represent the lth largest value in $h_1\left(\gamma_i \middle| p_i\right)$ and $h_2\left(\gamma_j \middle| p_j\right)$, respectively. $h_1^{\sigma(l)}\left(\gamma_i\right)$ and $h_2^{\sigma(l)}\left(\gamma_j\right)$ are the corresponding membership values with probabilities $h_1^{\sigma(l)}\left(p_i\right)$ and $h_2^{\sigma(l)}\left(p_j\right)$, respectively.

Definition 3 (Takahashi, 2000). Assume X is a nonempty set. The distance measure d on X satisfies the following properties:

1) $d(x,y)\ge 0$; 2) $d(x,y)=0 \Leftrightarrow x=y$; 3) $d(x,y)=d(y,x)$; 4) $d(x,y)\le d(x,z)+d(z,y)$.

2.2. Existing distances for PHFEs

Gao et al. (2017) and Su et al. (2019) proposed a series of distance measures for PHFEs.

Definition 4 (Gao et al., 2017). Assume $h_1\left(\gamma_i \middle| p_i\right)$ and $h_2\left(\gamma_j \middle| p_j\right)$ are two PHFEs. The distance measure $d(h_1,h_2)$ between $h_1\left(\gamma_i \middle| p_i\right)$ and $h_2\left(\gamma_j \middle| p_j\right)$ satisfies the following properties:

1) $0\le d(h_1,h_2)\le 1$; 2) $d(h_1,h_2)=0 \Leftrightarrow h_1=h_2$; 3) $d(h_1,h_2)=d(h_2,h_1)$.

The Hamming, Euclidean and Hausdorff distances are widely used for MADM (Xu & Xia, 2011a, 2011b), based on which some probabilistic hesitant fuzzy distances are obtained (Gao et al., 2017; Su et al., 2019):

Normalized Hamming distance between PHFEs h_1 and h_2:

$$d_1\left(h_1,h_2\right)=\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}\left(\gamma_i p_i\right)-h_2^{\sigma(j)}\left(\gamma_i p_i\right)\right|. \tag{3}$$

Normalized Euclidean distance for PHFEs h_1 and h_2:

$$d_2\left(h_1,h_2\right)=\sqrt{\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}\left(\gamma_i p_i\right)-h_2^{\sigma(j)}\left(\gamma_i p_i\right)\right|^2}. \tag{4}$$

Normalized Hamming-Hausdorff distance for PHFEs h_1 and h_2:

$$d_3\left(h_1,h_2\right)=\max_j\left\{\left|h_1^{\sigma(j)}\left(\gamma_i p_i\right)-h_2^{\sigma(j)}\left(\gamma_i p_i\right)\right|\right\}. \tag{5}$$

The generalized probabilistic hesitant fuzzy normalized distance between PHFEs h_1 and h_2:

$$d_4\left(h_1,h_2\right)=\left[\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}\left(\gamma_i p_i\right)-h_2^{\sigma(j)}\left(\gamma_i p_i\right)\right|^{\lambda}\right]^{\frac{1}{\lambda}},\lambda>0, \tag{6}$$

here, $|h|=\max\left\{\left|h_1\right|,\left|h_2\right|\right\}$. $h_1^{\sigma(j)}\left(\gamma_i p_i\right)$ and $h_2^{\sigma(j)}\left(\gamma_i p_i\right)$ represents the jth largest value in h_1 and h_2, respectively. Based on hesitant degree, Wu et al. (2019) presented a novel distance between PHFEs h_1 and h_2:

$$d_5\left(h_1,h_2\right)=\frac{1}{|h|}\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}\left(\gamma_i p_i\right)-h_2^{\sigma(j)}\left(\gamma_i p_i\right)\right|+\left|\widetilde{h_1}-\widetilde{h_2}\right|, \tag{7}$$

where $\widetilde{h_k}\left(k=1,2\right)$ denotes the hesitant degree of PHFE $h_k\left(k=1,2\right)$ and is calculated as

$$\widetilde{h_k}=\frac{1}{2}\left\{\frac{1}{\left|h_k\right|}\sum_{i=1}^{\left|h_k\right|}\left[h_k\left(\gamma_i p_i\right)-\left(\frac{1}{\left|h_k\right|}\sum_{i=1}^{\left|h_k\right|}h_k\left(\gamma_i p_i\right)\right)\right]^2+\left(1-\frac{1}{\left|h_k\right|}\right)\right\},\ k=1,2. \tag{8}$$

Li and Wang (2017) presented a probabilistic hesitant fuzzy Hausdorff distance between PHFEs h_1 and h_2, which does not require adding any values in the shorter PHFE.

$$d_6\left(h_1,h_2\right)=\frac{1}{2}\left\{\frac{1}{\left|h_1\right|}\sum_{i=1}^{\left|h_1\right|}\min_{\gamma_j\left(p_j\right)\in h_2}\left|h_1\left(\gamma_i p_i\right)-h_2\left(\gamma_j p_j\right)\right|+\frac{1}{\left|h_2\right|}\sum_{j=1}^{\left|h_2\right|}\min_{\gamma_i\left(p_i\right)\in h_1}\left|h_1\left(\gamma_i p_i\right)-h_2\left(\gamma_j p_j\right)\right|\right\}. \tag{9}$$

Moreover, Li and Wang (2018) put forward a novel Hausdorff distance for PHFEs.

$$d_7\left(h_1,h_2\right)=\max\left\{\max_{\gamma_i\left(p_i\right)\in h_1}\min_{\gamma_j\left(p_j\right)\in h_2}\left|\gamma_i-\gamma_j\right|p_i p_j,\ \max_{\gamma_j\left(p_j\right)\in h_2}\min_{\gamma_i\left(p_i\right)\in h_1}\left|\gamma_j-\gamma_i\right|p_i p_j\right\}. \tag{10}$$

Based on the expected values of PHFEs, Su et al. (2019) presented the Hamming like-distance measure:

$$d_8\left(h_1,h_2\right)=\left|\sum_{i=1}^{\left|h_1\right|}h_1^{\sigma(j)}\left(\gamma_i p_i\right)-\sum_{i=1}^{\left|h_2\right|}h_2^{\sigma(j)}\left(\gamma_i p_i\right)\right|. \tag{11}$$

However, these distances do not satisfy the property 2) in Definition 4.

Example 1. Assume $h_1=\left\{0.8\middle|0.6,0.2\middle|0.4\right\}$ and $h_2=\left\{0.6\middle|0.8,0.4\middle|0.2\right\}$ are two PHFEs. Then,

$$d_\eta\left(h_1,h_2\right)=d_8\left(h_1,h_2\right)=0,\ \eta=1,2,\cdots,6.$$

However, it must be admitted that $h_1\neq h_2$, whether from the possible membership degrees or the corresponding probabilities.

Example 2. Assume $h_3=\left\{0.5\middle|0.5,0.4\middle|0.3,0.2\middle|0.2\right\}$ and $h_4=\left\{0.5\middle|0.45,0.4\middle|0.375,0.2\middle|0.175\right\}$ are two PHFEs. Then, $d_7\left(h_3,h_4\right)=0$. However, $h_3\neq h_4$. Even though the possible membership degrees in h_1 are the same to those in h_2, their corresponding probabilities are not the same and it does not follow that $h_3=h_4$. Since PHFE includes the membership degrees and their probabilities, they should be differentiated from these two different aspects.

Therefore, it can be found that the distance measures above do not satisfy the property 2) in Definition 4. The main reason lies in that these distance measures integrate the possible membership degrees into the probability distribution when calculating the distance between PHFEs, which would lead to information loss. In fact, PHFE consists of two parts. Then, they should be considered separately when calculating the distance between PHFEs. And an effective way for measuring not only the deviation between the membership values but also the deviation between their corresponding probabilities should be provided. In what follows, several probabilistic hesitant fuzzy distances that satisfy the axiomatic definition of distance measure will be proposed.

3. Novel distance measure and CRITIC method for MADM

3.1. Problem description

For the PHFMADM problem, assume that there are alternatives A_i (i = 1, 2, ..., m) that are assessed under attributes C_j (j = 1, 2, ..., n), and the matrix $D=\left(h_{ij}\left(\gamma_l \middle| p_l\right)\right)_{m\times n}$ can be derived, where $h_{ij}\left(\gamma_l \middle| p_l\right)$ is a PHFE representing the attribute value of A_i under C_j. Generally, attributes are divided into two categories: cost and benefit. To obtain a reasonable decision result, decision matrix should be normalized in advance, and the normalized matrix $M=\left(\tilde{h}_{ij}\left(\tilde{\gamma}_l \middle| \tilde{p}_l\right)\right)_{m\times n}$ is obtained. Here,

$$\tilde{h}_{ij}\left(\tilde{\gamma}_l \middle| \tilde{p}_l\right)=\begin{cases} h_{ij}\left(\gamma_l \middle| p_l\right) & \text{for benefit attribute } C_j \\ h_{ij}\left(\left(1-\gamma_l\right) \middle| p_l\right) & \text{for cost attribute } C_j \end{cases} \quad \left(i=1,2,\cdots,m,\ j=1,2,\cdots,n\right). \tag{12}$$

Let $w_j \in \left[0,1\right]$ be the weight of attribute C_j, such that $\sum_{j=1}^{n} w_j = 1$. Sometimes, weight information cannot be determined completely. Assume Ω denotes the set for given weight information and it has the characteristics as below (Park & Kim, 1997; Kim et al., 1999):

1) $w_i \geq w_j$; 2) $w_i - w_j \geq \alpha_i,\ \alpha_i > 0$; 3) $w_i - w_j \geq w_m - w_n$, for $j \neq m \neq n$;

4) $\alpha_i \leq w_i \leq \alpha_i + \varepsilon_i,\ 0 \leq \alpha_i \leq \alpha_i + \varepsilon_i \leq 1$; 5) $w_i \geq \beta_i w_j,\ \beta_i \in \left[0,1\right]$.

3.2. Novel distances between PHFEs

Based on Definition 3 and 4, the definition of distance for PHFEs is modified as below.

Definition 5. Assume $h_1\left(\gamma_i \middle| p_i\right)$, $h_2\left(\gamma_j \middle| p_j\right)$ and $h_3\left(\gamma_k \middle| p_k\right)$ are PHFEs. The probabilistic hesitant fuzzy distance measure d possesses the properties as below:

1) $0 \leq d\left(h_1,h_2\right) \leq 1$; 2) $d\left(h_1,h_2\right)=0 \Leftrightarrow h_1 = h_2$;

3) $d\left(h_1,h_2\right)=d\left(h_2,h_1\right)$; 4) $d\left(h_1,h_3\right) \leq d\left(h_1,h_2\right)+d\left(h_2,h_3\right)$.

Holding these properties in mind, a novel distance measure between PHFEs is provided as below.

Definition 6. Assume $h_1\left(\gamma_i \middle| p_i\right)$ and $h_2\left(\gamma_j \middle| p_j\right)$ are two PHFEs. Then a hybrid probabilistic hesitant fuzzy Hamming distance (HPHFHD) for PHFEs $h_1\left(\gamma_i \middle| p_i\right)$ and $h_2\left(\gamma_j \middle| p_j\right)$ can be obtained:

$$d_9\left(h_1,h_2\right)=\alpha d\left(h_1\left(\gamma_i p_i\right),h_2\left(\gamma_i p_i\right)\right)+\beta d\left(h_1\left(\gamma_i\right),h_2\left(\gamma_i\right)\right)+\left(1-\alpha-\beta\right)d\left(h_1\left(p_i\right),h_2\left(p_i\right)\right), \quad (13)$$

here, $\alpha,\beta\in\left[0,1\right]$ are used for ascertaining the linear combination. And

$$d\left(h_1\left(\gamma_i p_i\right),h_2\left(\gamma_i p_i\right)\right)=\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}\left(\gamma_i p_i\right)-h_2^{\sigma(j)}\left(\gamma_i p_i\right)\right|; \quad (14)$$

$$d\left(h_1\left(\gamma_i\right),h_2\left(\gamma_i\right)\right)=\frac{1}{|h|}\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}\left(\gamma_i\right)-h_2^{\sigma(j)}\left(\gamma_i\right)\right|; \quad (15)$$

$$d\left(h_1\left(p_i\right),h_2\left(p_i\right)\right)=\frac{1}{|h|}\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}\left(p_i\right)-h_2^{\sigma(j)}\left(p_i\right)\right|. \quad (16)$$

1) If $\alpha=1,\ \beta=0,$ the HPHFHD is reduced to probabilistic hesitant fuzzy normalized Hamming distance (PHFNHD) (Gao et al., 2017; Su et al., 2019).
2) If $\alpha=0,\ \beta=1,$ the HPHFHD is reduced to hesitant fuzzy normalized Hamming distance (HFNHD) (Xu & Xia, 2011b).
3) If $\alpha=0,\ \beta=0,$ then the HPHFHD reduces to the normalized Hamming distance between the probability distributions of PHFEs h_1 and h_2.

Example 3. Let $h_1=\left\{0.3\middle|0.8,0.5\middle|0.2\right\}$ and $h_2=\left\{0.4\middle|0.6,0.5\middle|0.3,0.2\middle|0.1\right\}$ be two PHFEs. h_1 is transformed to $\tilde{h}_1=\left\{0.3\middle|0.8,0.5\middle|0.2,0.3\middle|0\right\}$. Then, $d_9\left(h_1,h_2\right)$=0.09 $\left(\alpha=\beta=1/3\right)$.

Example 4. In Example 1, it is found that $d_9\left(h_1,h_2\right)$=0.1333 $\left(\alpha=\beta=1/3\right)$. If the data in Example 2 is used, then $d_9\left(h_3,h_4\right)$=0.0367. Obviously, $h_1\neq h_2$ and $h_3\neq h_4$, which are consistent with reality.

Definition 7. Assume $h_1\left(\gamma_i \middle| p_i\right)$ and $h_2\left(\gamma_j \middle| p_j\right)$ are two PHFEs. Then a generalized hybrid probabilistic hesitant fuzzy distance GHPHFD between PHFEs h_1 and h_2 can be derived:

$$d_{10}^{\lambda}\left(h_1,h_2\right)=\alpha d^{\lambda}\left(h_1\left(\gamma_i p_i\right),h_2\left(\gamma_i p_i\right)\right)+\beta d^{\lambda}\left(h_1\left(\gamma_i\right),h_2\left(\gamma_i\right)\right)+\left(1-\alpha-\beta\right)d^{\lambda}\left(h_1\left(p_i\right),h_2\left(p_i\right)\right), \quad (17)$$

here, $\lambda\geq 1$, and

$$d^{\lambda}\left(h_1\left(\gamma_i p_i\right),h_2\left(\gamma_i p_i\right)\right)=\left[\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}\left(\gamma_i p_i\right)-h_2^{\sigma(j)}\left(\gamma_i p_i\right)\right|^{\lambda}\right]^{\frac{1}{\lambda}}; \quad (18)$$

$$d^{\lambda}\left(h_1\left(\gamma_i\right),h_2\left(\gamma_i\right)\right)=\left[\frac{1}{|h|}\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}\left(\gamma_i\right)-h_2^{\sigma(j)}\left(\gamma_i\right)\right|^{\lambda}\right]^{\frac{1}{\lambda}}; \quad (19)$$

$$d^{\lambda}\left(h_1\left(p_i\right),h_2\left(p_i\right)\right)=\left[\frac{1}{|h|}\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}\left(p_i\right)-h_2^{\sigma(j)}\left(p_i\right)\right|^{\lambda}\right]^{\frac{1}{\lambda}}. \quad (20)$$

1) If $\lambda=1$, the GHPHFD reduces to the HPHFHD.

2) If $\lambda = 2$, the hybrid probabilistic hesitant fuzzy Euclidean distance (HPHFED) can be obtained:

$$d_{10}^{2}\left(h_1,h_2\right)=\alpha\left[\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}\left(\gamma_i p_i\right)-h_2^{\sigma(j)}\left(\gamma_i p_i\right)\right|^2\right]^{\frac{1}{2}}+\beta\left[\frac{1}{|h|}\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}\left(\gamma_i\right)-h_2^{\sigma(j)}\left(\gamma_i\right)\right|^2\right]^{\frac{1}{2}}+$$

$$\left(1-\alpha-\beta\right)\left[\frac{1}{|h|}\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}\left(p_i\right)-h_2^{\sigma(j)}\left(p_i\right)\right|^2\right]^{\frac{1}{2}}. \tag{21}$$

3) Let $\lambda \to +\infty$, the hybrid probabilistic hesitant fuzzy Hausdorff distance (HPHFHD) can be obtained as below:

$$d_{10}^{\infty}\left(h_1,h_2\right)=\alpha\max_j\left\{\left|h_1^{\sigma(j)}\left(\gamma_i p_i\right)-h_2^{\sigma(j)}\left(\gamma_i p_i\right)\right|\right\}+\beta\max_j\left\{\left|h_1^{\sigma(j)}\left(\gamma_i\right)-h_2^{\sigma(j)}\left(\gamma_i\right)\right|\right\}+$$

$$\left(1-\alpha-\beta\right)\max_j\left\{\left|h_1^{\sigma(j)}\left(p_i\right)-h_2^{\sigma(j)}\left(p_i\right)\right|\right\}. \tag{22}$$

Proof. Suppose $a_j=\left|h_1^{\sigma(j)}\left(\gamma_i p_i\right)-h_2^{\sigma(j)}\left(\gamma_i p_i\right)\right|$ and $a=\max_j\left\{a_j\right\}$. Then

$$\lim_{\lambda\to+\infty} d^{\lambda}\left(h_1\left(\gamma_i p_i\right),h_2\left(\gamma_i p_i\right)\right)=\lim_{\lambda\to+\infty}\left(\sum_{j=1}^{|h|}a_j^{\lambda}\right)^{\frac{1}{\lambda}}=e^{\lim\limits_{\lambda\to+\infty}\frac{\ln\left(a_1^{\lambda}+a_2^{\lambda}+\cdots a_{|h|}^{\lambda}\right)}{\lambda}}=$$

$$e^{\lim\limits_{\lambda\to+\infty}\frac{a_1^{\lambda}\ln a_1+a_2^{\lambda}\ln a_2+\cdots+a_{|h|}^{\lambda}\ln a_{|h|}}{a_1^{\lambda}+a_2^{\lambda}+\cdots a_{|h|}^{\lambda}}}=e^{\ln a}=a.$$

Similarly,

$$\lim_{\lambda\to+\infty} d^{\lambda}\left(h_1\left(\gamma_i\right),h_2\left(\gamma_i\right)\right)=\max_j\left\{\left|h_1^{\sigma(j)}\left(\gamma_i\right)-h_2^{\sigma(j)}\left(\gamma_i\right)\right|\right\};$$

$$\lim_{\lambda\to+\infty} d^{\lambda}\left(h_1\left(p_i\right),h_2\left(p_i\right)\right)=\max_j\left\{\left|h_1^{\sigma(j)}\left(p_i\right)-h_2^{\sigma(j)}\left(p_i\right)\right|\right\}.$$

Therefore, the proof is complete.

As a metric, the GHPHFD satisfies the properties 1)–4) in Definition 5.

Proof.

1) It is straightforward.

2) "$\Leftarrow$" It is straightforward.

"$\Rightarrow$" If $d_{10}^{\lambda}\left(h_1,h_2\right)=0$, then $d^{\lambda}\left(h_1\left(\gamma_i p_i\right),h_2\left(\gamma_i p_i\right)\right)=d^{\lambda}\left(h_1\left(\gamma_i\right),h_2\left(\gamma_i\right)\right)=d^{\lambda}\left(h_1\left(p_i\right),h_2\left(p_i\right)\right)=0$. Therefore, $h_1^{\sigma(j)}\left(\gamma_i p_i\right)-h_2^{\sigma(j)}\left(\gamma_i p_i\right)=0$, $h_1^{\sigma(j)}\left(\gamma_i\right)-h_2^{\sigma(j)}\left(\gamma_i\right)=0$ and $h_1^{\sigma(j)}\left(p_i\right)-h_2^{\sigma(j)}\left(p_i\right)=0$, $j=1,2,\cdots,|h|$, which imply that $h_1=h_2$.

3) It is straightforward.

4) $d_{10}^{\lambda}(h_1,h_3)=\alpha d^{\lambda}(h_1(\gamma_i p_i),h_3(\gamma_i p_i))+\beta d^{\lambda}(h_1(\gamma_i),h_3(\gamma_i))+(1-\alpha-\beta)d^{\lambda}(h_1(p_i),h_3(p_i))$.

Then,

$$d^{\lambda}(h_1(\gamma_i p_i),h_3(\gamma_i p_i))=\left[\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}(\gamma_i p_i)-h_2^{\sigma(j)}(\gamma_i p_i)+h_2^{\sigma(j)}(\gamma_i p_i)-h_3^{\sigma(j)}(\gamma_i p_i)\right|^{\lambda}\right]^{\frac{1}{\lambda}}\le$$

$$\left[\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}(\gamma_i p_i)-h_2^{\sigma(j)}(\gamma_i p_i)\right|^{\lambda}\right]^{\frac{1}{\lambda}}+\left[\sum_{j=1}^{|h|}\left|h_2^{\sigma(j)}(\gamma_i p_i)-h_3^{\sigma(j)}(\gamma_i p_i)\right|^{\lambda}\right]^{\frac{1}{\lambda}}=$$

$$d^{\lambda}(h_1(\gamma_i p_i),h_2(\gamma_i p_i))+d^{\lambda}(h_2(\gamma_i p_i),h_3(\gamma_i p_i)).\ \text{[By Minkowski inequality]}.$$

Similarly, the following results can be derived:

$$d^{\lambda}(h_1(\gamma_i),h_3(\gamma_i))\le d^{\lambda}(h_1(\gamma_i),h_2(\gamma_i))+d^{\lambda}(h_2(\gamma_i),h_3(\gamma_i));$$

$$d^{\lambda}(h_1(p_i),h_3(p_i))\le d^{\lambda}(h_1(p_i),h_2(p_i))+d^{\lambda}(h_2(p_i),h_3(p_i)).$$

Therefore, $d_{10}^{\lambda}(h_1,h_3)\le d_{10}^{\lambda}(h_1,h_2)+d_{10}^{\lambda}(h_2,h_3)$, which completes the proof.

In the process of MADM, attribute weight plays an important role in decision results. Suppose $w_i\in[0,1]$ is the weight of $x_i\in X$, such that $\sum_{i=1}^{n}w_i=1$. Then a generalized weighted hybrid probabilistic hesitant fuzzy distance (GWHPHFD) can be derived.

Definition 8. Assume A and B are two PHFSs defined on $X=\{x_k\}_{k=1}^{n}$. The GWHPHFD for PHFSs A and B is as below:

$$d_{11}^{\lambda}(A,B)=\sum_{k=1}^{n}w_k\left\{\alpha\left[\sum_{j=1}^{|h|}\left|h_A^{\sigma(j)}(\gamma_i p_i)(x_k)-h_B^{\sigma(j)}(\gamma_i p_i)(x_k)\right|^{\lambda}\right]^{\frac{1}{\lambda}}+\right.$$

$$\beta\left[\frac{1}{|h|}\sum_{j=1}^{|h|}\left|h_A^{\sigma(j)}(\gamma_i)(x_k)-h_B^{\sigma(j)}(\gamma_i)(x_k)\right|^{\lambda}\right]^{\frac{1}{\lambda}}+$$

$$\left.(1-\alpha-\beta)\left[\frac{1}{|h|}\sum_{j=1}^{|h|}\left|h_A^{\sigma(j)}(p_i)(x_i)-h_B^{\sigma(j)}(p_i)(x_i)\right|^{\lambda}\right]^{\frac{1}{\lambda}}\right\},\ \lambda\ge 1. \tag{23}$$

1) If $\lambda=1$, the GWHPHFD reduces to the WHPHFD as below:

$$d_{11}^{1}(A,B)=\sum_{k=1}^{n}w_k\left\{\alpha\left(\sum_{j=1}^{|h|}\left|h_A^{\sigma(j)}(\gamma_i p_i)(x_k)-h_B^{\sigma(j)}(\gamma_i p_i)(x_k)\right|\right)+\right.$$

$$\beta\left(\frac{1}{|h|}\sum_{j=1}^{|h|}\left|h_A^{\sigma(j)}(\gamma_i)(x_k)-h_B^{\sigma(j)}(\gamma_i)(x_k)\right|\right)+$$

$$\left(1-\alpha-\beta\right)\left(\frac{1}{|h|}\sum_{j=1}^{|h|}\left|h_A^{\sigma(j)}\left(p_i\right)\left(x_k\right)-h_B^{\sigma(j)}\left(p_i\right)\left(x_k\right)\right|\right)\Bigg\}. \tag{24}$$

2) If $\lambda = 2$, the GWHPHFD reduces to the WHPHFED as follows:

$$d_{11}^2\left(h_1,h_2\right)=\sum_{k=1}^{n} w_k\left\{\alpha\left[\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}\left(\gamma_i p_i\right)\left(x_k\right)-h_2^{\sigma(j)}\left(\gamma_i p_i\right)\left(x_k\right)\right|^2\right]^{\frac{1}{2}}+\right.$$

$$\beta\left[\frac{1}{|h|}\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}\left(\gamma_i\right)\left(x_k\right)-h_2^{\sigma(j)}\left(\gamma_i\right)\left(x_k\right)\right|^2\right]^{\frac{1}{2}}+$$

$$\left.\left(1-\alpha-\beta\right)\left[\frac{1}{|h|}\sum_{j=1}^{|h|}\left|h_1^{\sigma(j)}\left(p_i\right)\left(x_k\right)-h_2^{\sigma(j)}\left(p_i\right)\left(x_k\right)\right|^2\right]^{\frac{1}{2}}\right\}. \tag{25}$$

3) Let $\lambda \to +\infty$, the WHPHFHD can be obtained as below:

$$d_{11}^{\infty}\left(h_1,h_2\right)=\sum_{k=1}^{n} w_k\left\{\alpha\max_j\left\{\left|h_1^{\sigma(j)}\left(\gamma_i p_i\right)\left(x_k\right)-h_2^{\sigma(j)}\left(\gamma_i p_i\right)\left(x_k\right)\right|\right\}+\right.$$

$$\beta\max_j\left\{\left|h_1^{\sigma(j)}\left(\gamma_i\right)\left(x_k\right)-h_2^{\sigma(j)}\left(\gamma_i\right)\left(x_k\right)\right|\right\}+$$

$$\left.\left(1-\alpha-\beta\right)\max_j\left\{\left|h_1^{\sigma(j)}\left(p_i\right)\left(x_k\right)-h_2^{\sigma(j)}\left(p_i\right)\left(x_k\right)\right|\right\}\right\}. \tag{26}$$

Definition 9. Assume A, B and C are three PHFSs defined on $X=\left\{x_1,x_2,\cdots,x_n\right\}$. Then,

1) $0 \le d_{11}^{\lambda}\left(A,B\right) \le 1$; 2) $d_{11}^{\lambda}\left(A,B\right)=0 \Leftrightarrow A=B$;

3) $d_{11}^{\lambda}\left(A,B\right)=d_{11}^{\lambda}\left(B,A\right)$; 4) $d_{11}^{\lambda}\left(A,C\right) \le d_{11}^{\lambda}\left(A,B\right)+d_{11}^{\lambda}\left(B,C\right)$.

3.3. An extended CRITIC method

The CRITIC approach is utilized for determining attribute weights (Diakoulaki et al., 1995), which takes the mutual relationship between attributes into account. Up to now, little research has been conducted on the determination of attribute weight in probabilistic hesitant fuzzy setting. In this part, the CRITIC method is extended to the setting of PHFSs for determining attribute weights. According to Diakoulaki et al. (1995), if the correlation coefficient between attribute C_j and other attributes C_k (k = 1, 2, ..., n) is high, removing the attribute C_j will have little influence on decision results. Thus, the attribute C_j is given a smaller weight. In addition, C_j with big standard deviation among alternatives is given a larger weight. Based on these facts, a mathematical model is established as below:

$$\begin{cases} \max f(w) = \sum_{j=1}^{n}\sum_{k=1}^{n}\delta_j\left(1-\rho_{jk}\right)w_j \\ s.t.\ \sum_{j=1}^{n} w_j^2 = 1, w_j \geq 0,\ j=1,2,\cdots,n \end{cases}, \tag{27}$$

here, the constrained condition for attribute weights is set as $\sum_{j=1}^{n} w_j^2 = 1$, which is motivated by Wang (1998). Generally, the sum of weights is 1, and they can be normalized by the transformation formula $w_j^* = w_j \Big/ \sum_{j=1}^{n} w_j$. In addition, δ_j denotes the standard deviation for attribute C_j:

$$\delta_j = \sqrt{\frac{1}{m}\sum_{i=1}^{m}\left(\sum_{t=1}^{|h|} h_{ij}^{\sigma(t)}\left(\gamma_l p_l\right) - \frac{1}{m}\sum_{i=1}^{m}\sum_{t=1}^{|h|} h_{ij}^{\sigma(t)}\left(\gamma_l p_l\right)\right)^2},\ j=1,2,\cdots,n \tag{28}$$

ρ_{jk} represents the correlation coefficient between the attribute C_j and C_k, and can be calculated as (Song et al., 2019)

$$\rho_{jk} = \frac{\sum_{i=1}^{m}\left(\sum_{t=1}^{|h|} h_{ij}^{\sigma(t)}\left(\gamma_l p_l\right) - \frac{1}{m}\sum_{i=1}^{m}\sum_{t=1}^{|h|} h_{ij}^{\sigma(t)}\left(\gamma_l p_l\right)\right)\left(\sum_{t=1}^{|h|} h_{ik}^{\sigma(t)}\left(\gamma_l p_l\right) - \frac{1}{m}\sum_{i=1}^{m}\sum_{t=1}^{|h|} h_{ik}^{\sigma(t)}\left(\gamma_l p_l\right)\right)}{\sqrt{\sum_{i=1}^{m}\left(\sum_{t=1}^{|h|} h_{ij}^{\sigma(t)}\left(\gamma_l p_l\right) - \frac{1}{m}\sum_{i=1}^{m}\sum_{t=1}^{|h|} h_{ij}^{\sigma(t)}\left(\gamma_l p_l\right)\right)^2 \cdot \sum_{i=1}^{m}\left(\sum_{t=1}^{|h|} h_{ik}^{\sigma(t)}\left(\gamma_l p_l\right) - \frac{1}{m}\sum_{i=1}^{m}\sum_{t=1}^{|h|} h_{ik}^{\sigma(t)}\left(\gamma_l p_l\right)\right)^2}}. \tag{29}$$

To derive weights of attributes, a Lagrange function is constructed as below:

$$L(w,\xi) = \sum_{j=1}^{n}\sum_{k=1}^{n}\delta_j\left(1-\rho_{jk}\right)w_j + \frac{\xi}{2}\left(\sum_{j=1}^{n} w_j^2 - 1\right). \tag{30}$$

And let

$$\begin{cases} \frac{\partial L(w,\xi)}{\partial w_j} = \sum_{k=1}^{n}\delta_j\left(1-\rho_{jk}\right) + \xi w_j = 0 \\ \frac{\partial L(w,\xi)}{\partial \xi} = \frac{1}{2}\left(\sum_{j=1}^{n} w_j^2 - 1\right) = 0 \end{cases}. \tag{31}$$

Then,

$$w_j = \frac{\sum_{k=1}^{n}\delta_j\left(1-\rho_{jk}\right)}{\sqrt{\sum_{j=1}^{n}\left(\sum_{k=1}^{n}\delta_j\left(1-\rho_{jk}\right)\right)^2}},\ j=1,2,\cdots,n. \tag{32}$$

By normalizing the weight w_j, the optimal weight can be derived in the following:

$$w_j^* = \frac{\sum_{k=1}^{n} \delta_j \left(1-\rho_{jk}\right)}{\sum_{j=1}^{n}\sum_{k=1}^{n} \delta_j \left(1-\rho_{jk}\right)},\ j=1,2,\cdots,n. \tag{33}$$

Nevertheless, people sometimes run across the situation where weight information is partially known. Then another mathematical model is established as below:

$$\begin{cases} \max f(w) = \sum_{j=1}^{n}\sum_{k=1}^{n} \delta_j \left(1-\rho_{jk}\right) w_j \\ s.t.\ w \in \Omega,\ \sum_{j=1}^{n} w_j = 1,\ \ w_j \geq 0,\ j=1,2,\cdots,n \end{cases}, \tag{34}$$

where Ω denotes the set of known weight information. After obtaining the optimal weights $w_j^*\left(j=1,2,\cdots,n\right)$, the GWHPHFD $d_{11}^{\lambda}\left(A_i, A^+\right)$ between the alternative A_i and A^+ can be calculated:

$$d_{11}^{\lambda}\left(A_i, A^+\right) = \sum_{k=1}^{n} w_k \left\{ \alpha \left[\sum_{j=1}^{|h|} \left| h_{A_i}^{\sigma(j)}\left(\gamma_l p_l\right)\left(x_k\right) - h_{A^+}^{\sigma(j)}\left(\gamma_l p_l\right)\left(x_k\right) \right|^{\lambda} \right]^{\frac{1}{\lambda}} + \right.$$
$$\beta \left[\frac{1}{|h|} \sum_{j=1}^{|h|} \left| h_{A_i}^{\sigma(j)}\left(\gamma_l\right)\left(x_k\right) - h_{A^+}^{\sigma(j)}\left(\gamma_l\right)\left(x_k\right) \right|^{\lambda} \right]^{\frac{1}{\lambda}} +$$
$$\left. \left(1-\alpha-\beta\right) \left[\frac{1}{|h|} \sum_{j=1}^{|h|} \left| h_{A_i}^{\sigma(j)}\left(p_l\right)\left(x_k\right) - h_{A^+}^{\sigma(j)}\left(p_l\right)\left(x_k\right) \right|^{\lambda} \right]^{\frac{1}{\lambda}} \right\}, \tag{35}$$

here, $A^+ = \left\{\left\langle x_1, (1|1)\right\rangle, \left\langle x_2, (1|1)\right\rangle, \cdots \left\langle x_n, (1|1)\right\rangle\right\}$ denotes the ideal alternative. Then the alternatives can be ranked according to $d_{11}^{\lambda}\left(A_i, A^+\right)$. The smaller the $d_{11}^{\lambda}\left(A_i, A^+\right)$, the better the alternative A_i.

3.4. Decision procedure

In this part, an approach to MADM in the setting of PHFSs is presented. A detailed decision making procedure is offered as below, and Figure 1 shows the flowchart.

Step 1. A decision matrix $D = \left(h_{ij}\left(\gamma_l | p_l\right)\right)_{m\times n}$ is constructed, where $h_{ij}\left(\gamma_l | p_l\right)$ is a PHFE and represents the evaluation value of alternative A_i (i = 1, 2, ..., m) under attribute C_j (j = 1, 2, ..., n).

Step 2. The normalized matrix $M = \left(\tilde{h}_{ij}\left(\tilde{\gamma}_l | \tilde{p}_l\right)\right)_{m\times n}$ is derived by Eq. (12).

Step 3. Determine attribute weights. When weight information is unknown, Eq. (33) will be utilized for determining attribute weights. If weight information is partially known, Eq. (34) is adopted.

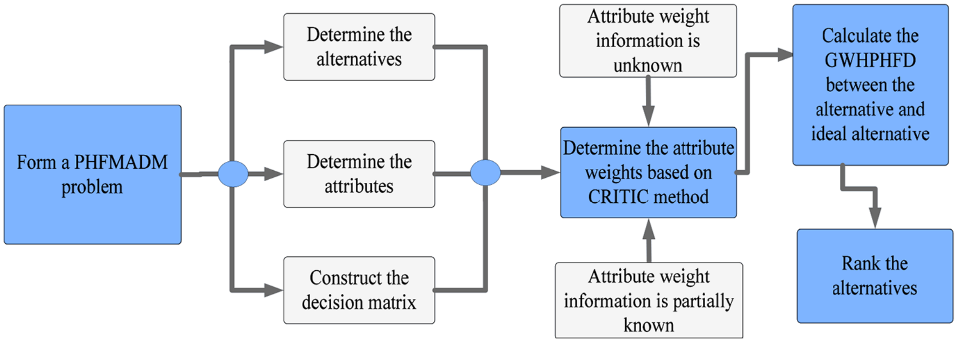

Figure 1. The flowchart for the proposed approach

Step 4. Calculate GWHPHFD between alternative A_i and A^+ by Eq. (35), and then rank the alternatives A_i, $i = 1, 2, ..., m$.

Step 5. End.

4. The application in investment decision making

4.1. A case study

In October 2013, China put forward the major initiative for jointly building the B&R, the aim of which is to realize common development by strengthening connectivity of counties along the routes. The B&R has attracted worldwide attention since its appearance. The strategic partners along the B&R are not only from Asia but also from Africa and Europe. To promote the construction, China will implement a more active opening-up strategy and strengthen cooperation among the countries along B&R, which have benefited lots of participating countries. To promote further progress in mutually beneficial cooperation, a Chinese company plans to strengthen investment and considers investing in an auto parts factory. By screening, there are four countries left for further investigation, such as Vietnam (A_1), Iran (A_2), Poland (A_3) and Russia (A_4) from Southeast Asia, West Asia and Europe respectively (Su et al., 2019).

Assume that three experts from business, science and government are invited to assess these alternatives. Nevertheless, risks go along with investment, and it is affected by a lot of factors. In this paper, five different risk factors are considered:

C_1 : Political stability. Social conflicts are under control. Specifically speaking, there is no serious social and political unrest all over the country. The citizens participate in politics by legal means, and the government does not take the means of violence to suppress civil political behavior.

C_2 : Credit risk, which is also known as default risk and refers to the risk that the counterparty fails to perform the due debts. It has the following characteristics: Asymmetric market information, dynamic accumulation, non-systemic risk, subjectivity and so on.

C_3 : Legal and regulatory, which refers to rule by law. Policies should be developed for meeting the requirements of legal and regulatory. A strong legal and regulatory framework, which is fair to poor communities, is required, and the investor has the secure property rights.

C_4 : Financial risk, which refers to the risks associated with finance. It includes financial market risk, financial products risk, and financial institutions risk. Once the systemic risks happen and the financial system seizes up, it will inevitably lead to economic chaos, and even trigger the political crisis.

C_5 : Infrastructure risk, which refers to the risks caused by lack of complete technical infrastructure. Infrastructure refers to the public service facilities that support production and life of people. It plays an indispensible part in promoting social development. In particular, improper planning of infrastructure will lead to the failure of investment.

Here, C_1 and C_3 are benefit attributes. C_2, C_4 and C_5 are cost attributes.

4.2. The decision strategy

To select the optimal investment plan, the decision strategy based on the proposed distance and CRITIC method is presented, which includes two cases as below:

Case 1. The weight information is unknown.

Step 1. The countries $A_i\left(i=1,2,3,4\right)$ are assessed on attributes $C_j\left(j=1,2,3,4,5\right)$, and the decision matrix $D=\left(h_{ij}\right)_{4\times5}$ is derived, which is shown in Table 1.

Step 2. The normalized matrix $M=\left(\tilde{h}_{ij}\left(\tilde{\gamma}_l\middle|\tilde{p}_l\right)\right)_{4\times5}$ can be obtained by Eq. (12), which is shown in Table 2.

Step 3. Using Eq. (27), a mathematical model for determining attribute weights is constructed as below:

$$\begin{cases}\max f\left(w\right)=0.5860w_1+0.2907w_2+0.3112w_3+0.5007w_4+0.1582w_5\\ s.t.\ \sum_{j=1}^{5}w_j^2=1,\ \ w_j\geq0,\ \ j=1,2,\cdots,5.\end{cases}$$

Then the optimal attribute weights can be derived:

$$w_1=0.3173,\quad w_2=0.1574,\quad w_3=0.1685,\quad w_4=0.2711,\quad w_5=0.0857.$$

Step 4. The GWHPHFD between alternative A_i and A^+ is calculated by Eq. (35), and the ranking results are shown in Table 3 $(\alpha=\beta=1/3)$.

Step 5. End.

Therefore, Vietnam (A_1) is the most suitable country for investment. Indeed, Vietnam's economy has grown rapidly in the past decade. A large number of cheap labor forces are promoting foreign investment in Vietnam. With the improvement of investment environment, Vietnam has obtained achievements in attracting foreign investment.

Case 2. The weight information is partially known:

$$\Omega=\left\{0.1\leq w_1\leq0.25,\ 0.1\leq w_2,\ 0.15\leq w_3\leq0.25,\ 0.3\leq w_4\leq0.4,\ 0.2\leq w_5\leq0.3,\ \sum_{j=1}^{n}w_j=1\right\}.$$

Step 1. The decision matrix $D=\left(h_{ij}\right)_{4\times5}$ is derived as Table 1.

Step 2. The normalized matrix $M=\left(\tilde{h}_{ij}\left(\tilde{\gamma}_l\middle|\tilde{p}_l\right)\right)_{4\times5}$ is obtained as Table 2.

Table 1. The decision information for the countries

	C_1	C_2	C_3	C_4	C_5
A_1	$\{0.8\|1\}$	$\{0.6\|0.7,0.7\|0.3\}$	$\{0.7\|0.2,0.8\|0.8\}$	$\{0.8\|0.7,0.9\|0.3\}$	$\{0.6\|0.8,0.8\|0.2\}$
A_2	$\{0.6\|0.8,0.7\|0.2\}$	$\{0.5\|0.3,0.6\|0.7\}$	$\{0.6\|0.1,0.7\|0.6,0.8\|0.3\}$	$\{0.5\|0.6,0.6\|0.4\}$	$\{0.6\|0.3,0.7\|0.6,0.8\|0.1\}$
A_3	$\{0.6\|0.1,0.7\|0.2,0.8\|0.7\}$	$\{0.7\|0.5,0.8\|0.5\}$	$\{0.8\|0.8,0.9\|0.2\}$	$\{0.6\|0.2,0.7\|0.2,0.8\|0.6\}$	$\{0.6\|0.1,0.7\|0.3,0.8\|0.6\}$
A_4	$\{0.8\|0.2,0.9\|0.8\}$	$\{0.7\|0.2,0.8\|0.7,0.9\|0.1\}$	$\{0.6\|0.9,0.8\|0.1\}$	$\{0.7\|0.9,0.8\|0.1\}$	$\{0.7\|0.5,0.8\|0.5\}$

Table 2. The normalized matrix $M=\left(\tilde{h}_{ij}\left(\tilde{\gamma}_l\middle|\tilde{p}_l\right)\right)_{4\times5}$ for the countries

	C_1	C_2	C_3	C_4	C_5
A_1	$\{0.8\|1\}$	$\{0.3\|0.3,0.4\|0.7\}$	$\{0.7\|0.2,0.8\|0.8\}$	$\{0.1\|0.3,0.2\|0.7\}$	$\{0.2\|0.2,0.4\|0.8\}$
A_2	$\{0.6\|0.8,0.7\|0.2\}$	$\{0.4\|0.7,0.5\|0.3\}$	$\{0.6\|0.1,0.7\|0.6,0.8\|0.3\}$	$\{0.4\|0.4,0.5\|0.6\}$	$\{0.2\|0.1,0.3\|0.6,0.4\|0.3\}$
A_3	$\{0.6\|0.1,0.7\|0.2,0.8\|0.7\}$	$\{0.2\|0.5,0.3\|0.5\}$	$\{0.8\|0.8,0.9\|0.2\}$	$\{0.2\|0.6,0.3\|0.2,0.4\|0.2\}$	$\{0.2\|0.6,0.3\|0.3,0.4\|0.1\}$
A_4	$\{0.8\|0.2,0.9\|0.8\}$	$\{0.1\|0.1,0.2\|0.7,0.3\|0.2\}$	$\{0.6\|0.9,0.8\|0.1\}$	$\{0.2\|0.1,0.3\|0.9\}$	$\{0.2\|0.5,0.3\|0.5\}$

Table 3. The ranking results derived by the GWHPHFD

	A_1	A_2	A_3	A_4	Ranking results
$\lambda = 1$	0.3862	0.4979	0.5113	0.4392	$A_1 \succ A_4 \succ A_2 \succ A_3$
$\lambda = 2$	0.3766	0.4620	0.4762	0.4276	$A_1 \succ A_4 \succ A_2 \succ A_3$
$\lambda = 5$	0.3811	0.4692	0.4877	0.4377	$A_1 \succ A_4 \succ A_2 \succ A_3$
$\lambda = 10$	0.3852	0.4777	0.5005	0.4452	$A_1 \succ A_4 \succ A_2 \succ A_3$
$\lambda = 100$	0.3911	0.4906	0.5207	0.4568	$A_1 \succ A_4 \succ A_2 \succ A_3$
$\lambda = 1000$	0.3918	0.4921	0.5232	0.4582	$A_1 \succ A_4 \succ A_2 \succ A_3$
$\lambda \to +\infty$	0.3919	0.4923	0.5234	0.4584	$A_1 \succ A_4 \succ A_2 \succ A_3$

Step 3. Utilize Eq. (34), a mathematical model is constructed as below:

$$\begin{cases} \max f(w) = 0.5860w_1 + 0.2907w_2 + 0.3112w_3 + 0.5007w_4 + 0.1582w_5 \\ s.t.\ w \in \Omega,\ \ w_j \geq 0,\ \ j = 1,2,\cdots,5. \end{cases}$$

The optimal attribute weights can be derived:

$$w_1 = 0.25, \quad w_2 = 0.1, \quad w_3 = 0.15, \quad w_4 = 0.3, \quad w_5 = 0.2.$$

Step 4. The GWHPHFD between alternative A_i and A^+ is calculated by Eq. (35), and the final decision results can be obtained, which are shown in Table 4 $(\alpha = \beta = 1/3)$.

Table 4. The final decision results derived by the GWHPHFD

	A_1	A_2	A_3	A_4	Ranking results
$\lambda = 1$	0.4250	0.5213	0.5358	0.4713	$A_1 \succ A_4 \succ A_2 \succ A_3$
$\lambda = 2$	0.4183	0.4859	0.5004	0.4610	$A_1 \succ A_4 \succ A_2 \succ A_3$
$\lambda = 5$	0.4245	0.4943	0.5126	0.4708	$A_1 \succ A_4 \succ A_2 \succ A_3$
$\lambda = 10$	0.4291	0.5037	0.5260	0.4776	$A_1 \succ A_4 \succ A_2 \succ A_3$
$\lambda = 100$	0.4360	0.5190	0.5483	0.4882	$A_1 \succ A_4 \succ A_2 \succ A_3$
$\lambda = 1000$	0.4369	0.5208	0.5510	0.4895	$A_1 \succ A_4 \succ A_2 \succ A_3$
$\lambda \to +\infty$	0.4370	0.5210	0.5513	0.4897	$A_1 \succ A_4 \succ A_2 \succ A_3$

Step 5. End.

As a whole, Vietnam (A_1) is still the most suitable country for investment. With the changing of the parameter λ, the ranking results are still the same, which implies that the presented approach is robust to variations resulting from the parameter. And decision makers can select different parameters according to actual needs.

4.3. Comparative analyses

4.3.1. Comparison with entropy-based approach

To handle the investment decision making problems mentioned above, Su et al. (2019) proposed an entropy-based method.

Step 1. Entropy is used to measure the uncertainties for the assessed countries, where the entropy formulas for PHFE are defined as below (Su et al., 2019):

$$E^1\left(h\left(\gamma_i \middle| p_i\right)\right) = -\frac{1}{\ln 2}\sum_{i=1}^{|h|}\left[\gamma_i \ln\gamma_i + (1-\gamma_i)\ln(1-\gamma_i)\right]p_i \; ; \tag{36}$$

$$E^2\left(h\left(\gamma_i \middle| p_i\right)\right) = \frac{1}{\left(\sqrt{e}-1\right)}\sum_{i=1}^{|h|}\left[\gamma_i e^{1-\gamma_i} + (1-\gamma_i)e^{\gamma_i} - 1\right]p_i \; ; \tag{37}$$

$$E^3\left(h\left(\gamma_i \middle| p_i\right)\right) = 1 - 2\left|\sum_{i=1}^{|h|} p_i\gamma_i - 0.5\right| . \tag{38}$$

Step 2. Let $W = \left(0.2, 0.2, 0.2, 0.2\right)^T$ be the weight vector (Su et al., 2019). The weighted entropies for the countries $A_i\left(i = 1,2,3,4\right)$ can be derived:

$$E_k\left(A_i\right) = \sum_{j=1}^{5} w_j E^k\left(h_j\right), \; k = 1,2,3 . \tag{39}$$

Step 3. The final entropies for the four countries are shown in Table 5.

Table 5. The final entropies for the assessed countries

	A_1	A_2	A_3	A_4	Ranking results
$E_1\left(A_i\right)$	0.797	0.900	0.770	0.772	$A_3 \succ A_4 \succ A_1 \succ A_2$
$E_2\left(A_i\right)$	0.747	0.911	0.711	0.719	$A_3 \succ A_4 \succ A_1 \succ A_2$
$E_3\left(A_i\right)$	0.528	0.748	0.472	0.500	$A_3 \succ A_4 \succ A_1 \succ A_2$

Therefore, larger entropy implies more uncertainty for the country, and A_3 is the most appropriate country. Obviously, it differs from that derived using the approach presented in this paper. The reasons for the difference are as below:

1) Different approaches to determine attribute weights are adopted. Su et al. (2019) used subjective approach for evaluating attribute weights, and it may not produce compelling results. Besides, Su's et al. approach cannot handle the situation that weight information is incompletely known. While in the proposed method, the CRITIC approach is extended for obtaining attribute weights in the setting of PHFSs, which can provide an approach combining subjective and objective analysis.
2) Different decision making methods are used. Su et al. (2019) used entropy to measure uncertainties of the assessed countries. However, their method neglects the difference between attributes, and may yield unreasonable results.

4.3.2. Comparison with PHFWA operator and entropy weight method

The entropy weight approach is widely utilized for ascertaining attribute weights, and is defined as below (Zeleny, 1982):

$$w_j = \frac{1 - E_j}{\sum_{j=1}^{n}\left(1 - E_j\right)}, \; j = 1, 2, \cdots, n, \tag{40}$$

here, E_j represents the entropy for attribute C_j. In this part, the PHFWA operator and entropy weight method are employed for the investment decision making above.

Step 1. To derive attribute weights, Eqs (36)–(38) are utilized, and the weight vectors obtained by entropy weight method are in the following (Su et al., 2019):

$$w^1 = \left(0.27, 0.144, 0.248, 0.183, 0.155\right)^T;$$

$$w^2 = \left(0.277, 0.151, 0.216, 0.191, 0.165\right)^T;$$

$$w^3 = \left(0.242, 0.169, 0.215, 0.187, 0.187\right)^T,$$

where the weight vectors w^1, w^2 and w^3 are derived from E^1, E^2 and E^3, respectively.

Step 2. The decision information is aggregated using the PHFWA operator (Xu & Zhou, 2017):

$$PHFWA\left(h_1, h_2, \cdots, h_n\right) = \underset{i=1}{\overset{n}{\otimes}} w_i h_i = \bigcup_{\gamma_{1_l} \in h_1, \gamma_{2_l} \in h_2, \cdots, \gamma_{n_l} \in h_n} \left\{ \left[1 - \prod_{i=1}^{n} \left(1 - \gamma_{i_l}\right)^{w_i} \right] \middle| \prod_{i=1}^{n} p_{i_l} \right\}. \tag{41}$$

And the score function of PHFE can be derived: $s\left(h\left(\gamma_i \middle| p_i\right)\right) = \sum_{i=1}^{|h|} \gamma_i p_i$.

Step 3. After aggregating the attribute values and calculating the scores of alternatives, the decision results can be derived as Table 6 (see Su et al., 2019 for details).

Table 6. The final entropies for the assessed countries

	A_1	A_2	A_3	A_4	Ranking results
$s_{E^1}(A_i)$	0.767	0.642	0.776	0.809	$A_4 \succ A_3 \succ A_1 \succ A_2$
$s_{E^2}(A_i)$	0.766	0.638	0.774	0.809	$A_4 \succ A_3 \succ A_1 \succ A_2$
$s_{E^3}(A_i)$	0.760	0.639	0.774	0.804	$A_4 \succ A_3 \succ A_1 \succ A_2$

Therefore, Russia (A_4) with the highest score is the most suitable country, which differs from that derived using the approach presented in this paper. The causes leading to such difference are as below:

1) The entropy weight approach cannot capture the interrelationship between attributes. For example, in the problem above, the occurrence of financial risk (C_4) will affect political stability (C_1), and the imperfect legal and regulatory (C_3) may induce credit

risk (C_2). All these imply that there is correlation between attributes and the proposed method can handle this situation well.

2) The PHFWA operator does not consider the influence of different decision attributes. Direct aggregation for attribute values is impracticable (Liu et al., 2017). In addition, the computation burden is serious when adopting the PHFWA operator. While in the proposed method, the cost attribute and benefit attribute can be differentiated. Then a reasonable decision result can be derived.

4.3.3. Comparison with TOPSIS method

In this part, a TOPSIS-based method is used for handling the MADM (Ding et al., 2017).

Step 1. Probabilistic hesitant fuzzy positive and negative ideal solutions can be obtained, respectively:

$$A^{+}=\left\{\left\langle C_j,\max_{1\le i\le 4}\left\{h_{ij}^{\sigma(k)}\left(\gamma_l p_l\right)\right\}\right\rangle\middle| j=1,2,\cdots,5,k=1,2,3\right\}=\left\{\left\langle C_j,h_j^{+}\right\rangle\middle| j=1,2,\cdots,5\right\}=$$

$$\left\{\begin{array}{l}\left\langle C_1,\{0.8,0.16,0.06\}\right\rangle,\left\langle C_2,\{0.56,0.35,0.09\}\right\rangle,\left\langle C_3,\{0.64,0.24,0.06\}\right\rangle,\\ \left\langle C_4,\{0.63,0.27,0.12\}\right\rangle,\left\langle C_5,\{0.48,0.35,0.08\}\right\rangle\end{array}\right\};$$

$$A^{-}=\left\{\left\langle C_j,\min_{1\le i\le 4}\left\{h_{ij}^{\sigma(k)}\left(\gamma_l p_l\right)\right\}\right\rangle\middle| j=1,2,\cdots,5,k=1,2,3\right\}=\left\{\left\langle C_j,h_j^{-}\right\rangle\middle| j=1,2,\cdots,5\right\}=$$

$$\left\{\left\langle C_1,\{0.48,0,0\}\right\rangle,\left\langle C_2,\{0.4,0.14,0\}\right\rangle,\left\langle C_3,\{0.42,0.08,0\}\right\rangle,\left\langle C_4,\{0.3,0.08,0\}\right\rangle,\left\langle C_5,\{0.4,0.16,0\}\right\rangle\right\}.$$

Step 2. The attribute weights can be determined using the following mathematical model:

$$\begin{cases}\max\varepsilon\\ s.t.\ c\left(A_i\right)=\dfrac{\sum_{j=1}^{5}w_j d_1\left(h_{ij},h_j^{-}\right)}{\sum_{j=1}^{n}w_j d_1\left(h_{ij},h_j^{+}\right)+\sum_{j=1}^{n}w_j d_1\left(h_{ij},h_j^{-}\right)}\ge\varepsilon,\quad i=1,2,3,4\\ \quad 0.1\le w_1\le 0.25, 0.1\le w_2, 0.15\le w_3\le 0.25, 0.3\le w_4\le 0.4, 0.2\le w_5\le 0.3,\\ \qquad \sum_{j=1}^{n}w_j=1,\ \sum_{j=1}^{5}w_j=1,\ w_j\ge 0,\ j=1,2,\cdots,5\end{cases}$$

here, $d_1(\cdot,\cdot)$ denotes the normalized Hamming distance between PHFEs (Ding et al., 2017). $c(A_i)$ is the relative closeness coefficient for A_i. Then the weight vector can be derived:

$$w=\left(0.1,0.1,0.25,0.3,0.25\right)^{T}.$$

Step 3. Calculate $c(A_i)$ for alternative A_i:

$$c(A_1)=0.5434,\quad c(A_2)=0.3064,\quad c(A_3)=0.5812,\quad c(A_4)=0.4934,$$

which implies that $A_3\succ A_1\succ A_4\succ A_2$.

The decision results differ from that derived using the method presented in this paper. Both methods provide effective ways for handling MADM with partially known weight information. Nevertheless, the suggested approach has several advantages in the following:

1) The CRITIC approach is extended and can be utilized for ascertaining attribute weights, no matter whether weight information is incompletely known or not. Besides, there is strong correlation between attributes in MADM problem. However, Ding et al.'s approach does not take the correlation between attributes into account, and thus the misleading results may be derived.
2) A novel hybrid probabilistic hesitant fuzzy distance measure, which has several advantages over the existing distance measures, is provided. To assess the four countries, TOPSIS approach is utilized (Ding et al., 2017), where the distances between alternatives and ideal solutions are calculated using probabilistic hesitant fuzzy normalized Hamming distance (PHFNHD) measure. However, as mentioned above, the PHFNHD measure fails to meet the condition of distance measure. Furthermore, Ding et al. (2017) neglect the difference between attributes when determining the ideal solutions. And the sum of probabilities associated with ideal solutions is larger than 1, which is problematic. Therefore, unreasonable results may be derived.

Conclusions

PHFS can reflect different preferences of people and provide a novel research perspective for decision theory. This paper highlights several achievements in PHFMADM: First, the CRITIC approach, which takes the correlation between attributes into account, is extended for ascertaining attribute weights in the setting of PHFSs. It also offers an efficient way for tackling the situation that weight information is incompletely known. Second, the existing distances for PHFSs fail to meet the condition of distance measure, which impels us to search for new distance for PHFSs. Fortunately, the distance suggested in this paper can overcome the defects and possesses the advantages over the existing ones, and then the presented method is applied to assess the countries along B&R.

As for results of presented studies, they heavily depend on evaluation of people and are inevitably affected by decision bias. A limitation of this research is that probabilities for elements in PHFE are assumed to be known. However, it is not easy to determine them by subjective evaluation of decision makers. In future research, an approach that focuses on objectively determining the probabilities of elements in PHFE is presented and applied to evaluate the venture capital projects.

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Author contributions

Xiaodi Liu and Zengwen Wang conceived the study and were responsible for the design of the data analysis. Shitao Zhang and Yaofeng Chen were responsible for data collection. Shitao Zhang analyzed the data. Xiaodi Liu wrote the first draft of the article.

Disclosure statement

The authors declare that they have no competing financial, professional, or personal interests from other parties.

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