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STRATEGIC QUALITY ASSURANCE

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Abstract. The purpose of this paper is to provide a strategic (game) approach to Quality Assurance. Unlike previous approaches that presume non-motivated sources of risk, we assume in this paper that risk may arise strategically due to other motivations. For example, problems associated to supply risks received by a producer-buyer. As a result, strategic quality assurance problems are formulated in terms of random payoff game which we solve while using the traditional approach to risk specification imbedded in quantile risks (Type I and Type II errors in statistics or producers and consumers risks). Technically, the approach devised consists in solving risk constrained (random payoff) games which involve strategic partners, potentially in conflict. The approach devised is then applied to a number of problems spanning essentially mutual sampling (quality assurance) between a buyer and supplier and strategic quality control in supply chains where potential conflict and information and power asymmetry is an inherent part of the operational problem to be dealt with. In such circumstances, contracts agreements might be violated if the parties do not apply strategic control tools to assure that what was intended is actually performed.

Keywords: risk, motivations, strategic.

1. Introduction

Uncertainty, Statistics and Stochastic Models have, for the most part, presumed that the underlying uncertainty is neutral! In other words, uncertainty and risk are not motivated. In reality, this may not be the case. For example, multiple parties interacting with broadly varying motivations; information asymmetries; Power and asymmetric relationships between interacting agents; a conflictual environment etc. have contributed to "endogenous uncertainty and risks". The purpose of this paper is to deal with the control of such risks through strategic quality control. For early references, see Reyniers, 1992, Reyniers and Tapiero (1995a, 1996b) as well as Tapiero (1995, 1996, 2001, 2004, 2004a, 2005, 2005a). Unlike previous approaches to quality control which have presumed risk neutral participants, this papers considers as well participants risk specifications imbedded in the traditional approach of quantile risks (Type I and Type II errors in statistics as well as the producer's and consumer's risk in industrial quality control). The approach devised consists then in solving risk constrained (random payoff) games which involve strategic partners, potentially in conflict. This approach devised is then applied to a number of problems spanning environmental quality control, supply chains and other problems where conflict is an inherent part of the problem to be dealt with. For our purposes we apply Nash equilibrium to the strategic quality assurance games we define (Nash, 1950, Thomas, 1986, Owen, 1982).

Statistics and control have traditionally been concerned with the control of uncertainty, seeking to monitor it, to predict it, limit its effects and whenever possible to control it. Quality control, stochastic control and general decision making under uncertainty are some of the fields which are involved, in one way or the other, in an attempt to deal with these problems which have plagued our profession whenever it has been confronted with uncertainty. The relationship between statistics, conflict and control as well as the role of statistical sampling in improving the control of conflict has to a large measure been neglected. Statistic's failure to deal with conflict arose from its presumption that "uncertainty is not motivated". In other words, randomness is not motivated by any special purpose. Interpreting uncertainty and reducing its effects is then based on the presumption that our measurements and our acts are independent of the origins of such uncertainty. Information asymmetry and strategic conflict induce therefore a greater need for controls, to assure that "what is intended will occur". For example in contracts in general and in particular insurance contracts binding clauses may be designed not only as a means of exchange but as a means to induce post contract behaviour which is compatible with a contract's intentions. Similarly, strategic audits have always a number of messages they convey; a control, a signal to the audited on the firm's intentions and of course to collect information which is needed to reach an economic decision. The control of exchanges between such parties should therefore keep in mind parties' intentionality imbedded in their preferences, the exchange terms as well as the information each will use in respecting or not the intended terms of their exchange.

2. Strategic Quality Assurance

Strategic Quality Control recognizes explicitly that agents' motivations and the pursuit of self interest as well as the cost and the origin of the information gathered matter in determining the quality control approach to apply. For example, the traditional formulation of sampling plans in terms of risk considerations based on Neyman - Pearson theory in Hypothesis Testing may be limiting, avoiding issues which are specific to cooperation, cheating and generally to opportunistic behaviour (Tapiero, 1996). Our framework generalizes this hypothesis testing approach to quality control by considering as well the conflict inherent between the supplier and the producer. For simplicity, assume that lots of size *N* are delivered by a supplier to a buyer (a producer of finished products). To assure contract compliance, both the supplier and the buyer can use a number of sampling programs, each with stringency tests of various degrees (spanning the no sampling case and thereby accepting the lot as is, to the full sampling case and thereby inspecting each individual unit). Let j = 1,2,…n be the alternative sampling programs used by the buyer and i = 1,2,…, m be the alternative sampling programs used by the supplier. Correspondingly, we denote by $\left(\alpha_{p,i},\beta_{p,i}\right);\ \left(\alpha_{S,j},\beta_{S,j}\right),\ i=1,...n$ and $j=1,...m$ the probabilities of rejecting a good lot and accepting a bad one by a producer (indexed p) and a supplier (indexed S), under each alternative sampling programs selection. These risks are summarized in the matrix below.

$$\begin{pmatrix} \left(\alpha_{p,1},\beta_{p,1}\right);\ \left(\alpha_{S,1},\beta_{S,1}\right) & \dots\ \dots & \left(\alpha_{p,1},\beta_{p,1}\right);\ \left(\alpha_{S,m},\beta_{S,m}\right) \\ \left(\alpha_{p,2},\beta_{p,2}\right);\ \left(\alpha_{S,1},\beta_{S,1}\right) & & \left(\alpha_{p,2},\beta_{p,2}\right);\ \left(\alpha_{S,m},\beta_{S,m}\right) \\ \dots & & \dots \\ \left(\alpha_{p,n},\beta_{p,n}\right);\ \left(\alpha_{S,1},\beta_{S,1}\right) & \dots\ \dots & \left(\alpha_{p,n},\beta_{p,n}\right);\ \left(\alpha_{S,m},\beta_{S,m}\right) \end{pmatrix}$$

Explicitly, if the alternative quality control (sampling programs are given by binomial test programs $\left(n_{p,i},k_{p,i}\right)$ for the producer and $\left(n_{S,j},k_{S,j}\right)$ for the supplier, we have then the following risk for the producer (buyer):

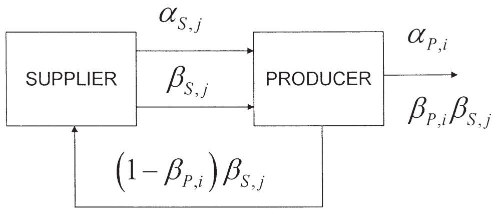
$$\alpha_{p,i}=1-\sum_{\ell=0}^{k_{p,i}}\binom{n_{p,i}}{\ell}(\theta_1)^{\ell}(1-\theta_1)^{n_{p,i}-\ell};$$
$$\beta_{p,i}=\sum_{\ell=0}^{k_{p,i}}\binom{n_{p,i}}{\ell}(\theta_2)^{\ell}(1-\theta_2)^{n_{p,i}-\ell}, \tag{1}$$

where θ_1 is a proportion of acceptable defectives (or the AQL) while θ_2 is the proportion of unacceptable defectives in a lot (or the LTFD). The probability that a lot is good (i.e. with the standard proportion defectives) is given by π however. The probability that a lot is defective is thus $1-\pi$. For the supplier, the corresponding risks are given by:

$$\alpha_{S,j}=1-\sum_{\ell=0}^{k_{p,j}}\binom{n_{S,i}}{\ell}(\theta_1)^{\ell}(1-\theta_1)^{n_{S,j}-\ell};$$
$$\beta_{S,j}=\sum_{\ell=0}^{k_{p,j}}\binom{n_{S,j}}{\ell}(\theta_2)^{\ell}(1-\theta_2)^{n_{S,j}-\ell}, \tag{2}$$

Of course for simplification purposes, we can approximate (1) and (2) by a normal distribution which will be considered in section 4.

For example, assuming that the supplier fully samples and prunes all non conforming units, then the probabilities for the supplier will necessarily be equal to $\alpha_{S,j}=1,\beta_{S,j}=0$ for all inspection programs j. If the buyer knew for sure that this were the case, he would then always use a costless no-inspection alternative. In a similar fashion, assume that the supplier accepts a bad lot with some probability (the resulting consumer risk). This probability will of course be a function of the actions taken by the buyer as well. In other words, a bad lot is accepted and reaches a final consumer if it is also accepted by the buyer (producer). The risk probabilities corresponding to each combination of the producer and the supplier selecting a sampling strategy, leads then to the matrix with entries: $\alpha_{p,i}(1-\alpha_{S,j});\alpha_{S,j}$ and $\beta_{p,i}\beta_{S,j};\beta_{S,j}$ for type I and type II errors. As a result, if in a game the producer selects a sampling strategy i with probability x_i while the supplier selects sampling strategy j with probability y_j and if both the producer and the supplier specify average risk specifications $(\bar{\alpha}_p, \bar{\beta}_p)$ and $(\bar{\alpha}_S, \bar{\beta}_S)$ respectively, then we have the following risk constraints:



$$\sum_{j=1}^{m}\sum_{i=1}^{n} x_i y_j \alpha_{p,i}(1-\alpha_{S,j}) \le \bar{\alpha}_p;$$
$$\sum_{j=1}^{m}\sum_{i=1}^{n} x_i y_j \beta_{p,i}\beta_{S,j} \le \bar{\beta}_p; \qquad (3)$$

$$\sum_{j=1}^{m} y_j \alpha_{S,j} \le \bar{\alpha}_S; \quad \sum_{j=1}^{m} y_j \beta_{S,j} \le \bar{\beta}_S \ . \qquad (4)$$

$$\tilde{C}_{p,ij} = c_p n_{p,i} + \begin{cases} 0 & wp \quad (1-\alpha_{S,j})(1-\alpha_{p,i})\pi + \alpha_{S,j}\pi + (1-\beta_{S,j})(1-\pi) \\ R_i & wp \quad (1-\alpha_{S,j})\alpha_{p,i}\pi \\ U_i & wp \quad \beta_{p,i}\beta_{S,j}(1-\pi) \\ T_i & wp \quad (1-\beta_{p,i})\beta_{S,j}(1-\pi) \end{cases} \qquad (5)$$

$$\tilde{C}_{S,ij} = c_S n_{S,j} + \begin{cases} 0 & wp \quad (1-\alpha_{S,j})\pi \\ Q_j & wp \quad \alpha_{S,j}\pi \\ V_j & wp \quad \beta_{p,i}\beta_{S,j}(1-\pi) \\ W_j & wp \quad (1-\beta_{p,i})\beta_{S,j}(1-\pi) \\ \bar{Q}_j & wp \quad (1-\beta_{S,j})(1-\pi) \end{cases} \qquad (6)$$

In these expressions we have an inspection cost only for the supplier and the buyer if the supplier accepts a good lot (with probability $(1-\alpha_{S,j})\pi$) while the buyer-producer will do so if upon reception of the good lot he also accepts the good lot (probability $(1-\alpha_{S,j})(1-\alpha_{p,i})\pi$), or the supplier rejects a good lot and therefore attend to it at the cost Q_j (with probability $\alpha_{S,j}\pi$) or if the supplier also rejected a bad lot and attended to it wholly at a cost $\bar{Q}_j > Q_j$, assuring therefore that it is good for sure (with probability $(1-\beta_{S,j})(1-\pi)$). If the supplier accepts a good lot but the buyer rejects the lot (with probability $(1-\alpha_{S,j})\alpha_{p,i}\pi$) at a cost R_i which is sustained by the buyer only—since it is in fact a good lot. When both the buyer and the supplier accept a bad lot (with probability $\beta_{p,i}\beta_{S,j}(1-\pi)$), the final cost is incurred by the ultimate customer who penalizes both the firm and the supplier at costs (U_i, V_j). Finally, when the supplier accepts a bad lot and the buyer rejects it (with probability $(1-\beta_{p,i})\beta_{S,j}(1-\pi)$) then the costs sustained by the supplier equals W_j which is much larger than the transfer cost sustained by the buyer, equal to T_i. Of course, the cost parameters $R_i, U_i, T_i, Q_j, V_j, W_j, \bar{Q}_j$ can be given specific values as a function of the sampling strategies and the costs associated to the risks assumed by the supplier and the buyer in the various circumstances. Examples to these effects will be considered subsequently. The expected costs are:

$$\hat{C}_{p,ij} = c_p n_{p,i} + R_i(1-\alpha_{S,j})\alpha_{p,i}\pi + \left(U_i\beta_{p,i} + T_i(1-\beta_{p,i})\right)\beta_{S,j}(1-\pi), \qquad (7)$$

$$\hat{C}_{S,ij} = c_S n_{S,j} + Q_j\alpha_{S,j}\pi + \left(V_j\beta_{p,i}\beta_{S,j} + W_j(1-\beta_{p,i})\beta_{S,j} + \bar{Q}_j(1-\beta_{S,j})\right)(1-\pi), \qquad (8)$$

If we minimize the expected costs subject to the risk constraints, the strategic quality assurance programs are given by solving the following mathematical programs:

$$\underset{x_i, n_{p_i}}{Min} \sum_{i=1}^{n}\sum_{j=1}^{m} x_i y_j \hat{C}_{p,ij}; \quad \underset{y_j, n_{S_j}}{Min} \sum_{i=1}^{n}\sum_{j=1}^{m} x_i y_j \hat{C}_{S,ij}$$
$$\text{Subject to: } (1),(2),(3),(4),(7),(8) \text{ and} \qquad (9)$$
$$\sum_{i=1}^{n} x_i = 1, \ \sum_{j=1}^{m} y_j = 1, \ 0 \le x_i \le 1, \ 0 \le y_j \le 1.$$

This problem can be solved under a number of specific assumptions regarding the participants behaviors and of course assuming the information each has regarding the other. In the following section we shall consider a number of special cases. First, we consider the case of full cooperation in minimizing inspection costs, thereby generalizing traditional approaches to quality assurance focusing on the selection of the sample inspection programs that meet a set of consumers risks (type II) and minimize the producers' risks (type I). Subsequently, we consider a number of situations where there might be non-cooperation between the buyer and the supplier.

3. Strategic Quality Assurance with and without Conflict

3.1. Strategic Economic Inspection

Assume that the economic costs outlined in the equations above are given by:

$R_i = c_p n_{p,i} + c_P(N - n_{P,i})$,

$U_i = c_p n_{p,i} + (1-\xi)u(N)$ and

$T_i = c_p n_{p,i} + c_{PR}(N)$ as well as

$Q_j = c_S n_{S,j} + c_S(N - n_{S,j}) + c_{SG}(N, \theta_1)$,

$V_j = c_S n_{S,j} + \xi u(N)$, $W_j = c_S n_{S,j} + c_{SR}(N, \theta_2)$ and

$\bar{Q}_j = c_S n_{S,j} + c_S(N - n_{S,j}) + c_{Sb}(N, \theta_2)$. For the supplier, we have $c_S(N - n_{S,j}) + c_{SG}(N, \theta_1)$ denoting the cost of producing a good lot and the sampling plan rejects this lot (with probability $\alpha_{S,j}\pi$). c_S is the supplier unit inspection cost while c_P is the producer's cost. When the supplier rejects a good lot, then the cost incurred to attend to defective units is $c_{SG}(N, \theta_1)$. When a bad lot is accepted by both the supplier and the producer there is a consumers cost which is equal $u(N)$, a proportion of which ξ is assumed by the supplier and its complement $1-\xi$ assumed by the producer. The cost $c_{SR}(N, \theta_2)$ is assumed when the suppliers accepts a defective lot which is detected and rejected by the producer (at a cost of $c_{PR}(N)$). As a result, the expected economic cost of both the supplier and the producer are:

$$\hat{C}_{S,ij} = c_S n_{S,j} + c_S(N - n_{S,j})\left[\alpha_{S,j}\pi + (1-\beta_{S,j})(1-\pi)\right] + c_{SG}(N,\theta_1)\alpha_{S,j}\pi + \left[\left(\xi u(N)\beta_{p,i} + c_{SR}(N,\theta_2)(1-\beta_{p,i})\right)\beta_{S,j} + c_{Sb}(N,\theta_2)(1-\beta_{S,j})\right]\cdot(1-\pi) \quad (10)$$

and

$$\hat{C}_{p,ij} = c_p n_{p,i} + c_P(N - n_{P,i})(1-\alpha_{S,j})\alpha_{p,i}\pi + \left[(1-\xi)u(N)\beta_{p,i} + c_{PR}(N)(1-\beta_{p,i})\right]\beta_{S,j}(1-\pi) \quad (11)$$

The problem they face and specified by equation (9) is a strategic economic assurance problem with risk constraints (3) and (4).

To simplify our presentation say that the supplier and the producer set their type II risks to $\left(\bar{\beta}_S, \bar{\beta}_p\right)$ and consider (for simplicity) two alternative sampling programs (one light, the other extensive), we have then two equations in *(x,y)* given by:

$$\begin{aligned} &xy\beta_{p,1}\beta_{S,1} + x(1-y)\beta_{p,1}\beta_{S,2} + \\ &(1-x)y\beta_{p,2}\beta_{S,1} + (1-x)(1-y)\beta_{p,2}\beta_{S,2} = \bar{\beta}_p \\ &y\beta_{S,1} + (1-y)\beta_{S,2} = \bar{\beta}_S. \end{aligned} \quad (12)$$

And therefore the randomized sampling parameter can be defined in terms of type II (consumer) risks only, or:

$$x^* = \frac{\bar{\beta}_p - \bar{\beta}_S\beta_{p,2}}{\bar{\beta}_S\left(\beta_{p,1} - \beta_{p,2}\right)};\ y^* = \frac{\bar{\beta}_S - \beta_{S,2}}{\beta_{S,1} - \beta_{S,2}} \quad (13)$$

If the supplier and the producer operate in a conflict, minimizing their respective costs, then we have, applying Nash's equilibrium solution for nonzero sum two persons game:

$$\begin{aligned} &\underset{n_{P1},k_{P1},n_{P2},k_{P1}}{Min}\ x^*y^*\left[\hat{C}_{p,11} - \hat{C}_{p,12} - \hat{C}_{p,21} + \hat{C}_{p,22}\right] + \\ &x^*\left(\hat{C}_{p,12} - \hat{C}_{p,22}\right) + y^*\left(\hat{C}_{p,21} - \hat{C}_{p,22}\right) + \hat{C}_{p,22} \\ &\underset{n_{S1},k_{S1},n_{S2},k_{S2}}{Min}\ x^*y^*\left[\hat{C}_{S,11} - \hat{C}_{S,12} - \hat{C}_{S,21} + \hat{C}_{S,22}\right] + \\ &x^*\left(\hat{C}_{S,12} - \hat{C}_{S,22}\right) + y^*\left(\hat{C}_{S,21} - \hat{C}_{S,22}\right) + \hat{C}_{S,22} \end{aligned} \quad (13')$$

$$\begin{aligned} &Subject\ to: (13)\ and \\ &\left[x^*y^*\left(\alpha_{p,1} + \alpha_{p,2}\right) + y^*\alpha_{p,2}\right]\left(\alpha_{S,2} - \alpha_{S,1}\right) + \\ &x^*\left(\alpha_{p,1} - \alpha_{p,2}\right)\left(1 - \alpha_{S,2}\right) + \alpha_{p,2}(1 - \alpha_{S,2}) \le \bar{\alpha}_p \\ &y^*\left(\alpha_{S,1} - \alpha_{S,2}\right) + \alpha_{S,2} \le \bar{\alpha}_S \end{aligned} \quad (14)$$

An explicit expression of the sampling optimization problem can thus be calculated while the Type I risk constraints are:

$$\begin{aligned} &\frac{\left(\bar{\beta}_p - \bar{\beta}_S\beta_{p,2}\right)\left(\bar{\beta}_S - \beta_{S,2}\right)\left(\alpha_{p,1} + \alpha_{p,2}\right)\left(\alpha_{S,2} - \alpha_{S,1}\right)}{\bar{\beta}_S\left(\beta_{p,1} - \beta_{p,2}\right)\left(\beta_{S,1} - \beta_{S,2}\right)} + \\ &\frac{\left(\bar{\beta}_p - \bar{\beta}_S\beta_{p,2}\right)\left(\alpha_{p,1} - \alpha_{p,2}\right)\left(1 - \alpha_{S,2}\right)}{\bar{\beta}_S\left(\beta_{p,1} - \beta_{p,2}\right)} + \\ &\frac{\left(\bar{\beta}_S - \beta_{S,2}\right)\alpha_{p,2}\left(\alpha_{S,2} - \alpha_{S,1}\right)}{\left(\beta_{S,1} - \beta_{S,2}\right)} + \alpha_{p,2}(1 - \alpha_{S,2}) \le \bar{\alpha}_p \\ &\frac{\left(\bar{\beta}_S - \beta_{S,2}\right)\left(\alpha_{S,1} - \alpha_{S,2}\right)}{\left(\beta_{S,1} - \beta_{S,2}\right)} + \alpha_{S,2} \le \bar{\alpha}_S \end{aligned} \quad (15)$$

To simplify our analysis, say that we consider two alternatives, no sampling and sampling *n* and *m* by the producer and the supplier respectively.

In this case $\beta_{S,1} = 1, \beta_{p,1} = 1$ and $\alpha_{S,1} = 0, \alpha_{p,1} = 0$ simplifying thereby (13) while the costs are:

$$\begin{aligned} &\hat{C}_{p,11} = (1-\xi)u(N)(1-\pi);\ \hat{C}_{p,12} = \hat{C}_{p,11}\beta_{S,2} \\ &\hat{C}_{p,21} = c_p n + \chi_1 + \chi_2;\quad \hat{C}_{p,22} = c_p n + \chi_1(1-\alpha_{S,2}) + \chi_2\beta_{S,2}, \end{aligned} \quad (16)$$

where for notational convenience

$\chi_2 = \left[(1-\xi)u(N)\beta_{p,2} + c_{PR}(N)(1-\beta_{p,2})\right](1-\pi)$ and $\chi_1 = c_P(N-n)\alpha_{p,2}\pi$. For the supplier we have similarly:

$$\hat{C}_{S,11} = \xi u(N)(1-\pi);$$
$$\hat{C}_{S,12} = c_S m + c_S(N-m)\left(\alpha_{S,2}\pi + (1-\beta_{S,2})(1-\pi)\right) + c_{SG}(N,\theta_1)\alpha_{S,2}\pi + \left[\xi u(N)\beta_{S,2} + c_{Sb}(N,\theta_2)(1-\beta_{S,2})\right](1-\pi)$$
$$\hat{C}_{S,21} = \left(\xi u(N)\beta_{p,2} + c_{SR}(N,\theta_2)(1-\beta_{p,2})\right)(1-\pi);$$
$$\hat{C}_{S,22} = c_S m + c_S(N-m)\left[\alpha_{S,2}\pi + (1-\beta_{S,2})(1-\pi)\right] + c_{SG}(N,\theta_1)\alpha_{S,2}\pi + \left[\left(\xi u(N)\beta_{p,2} + c_{SR}(N,\theta_2)(1-\beta_{p,2})\right)\beta_{S,2} + c_{Sb}(N,\theta_2)(1-\beta_{S,2})\right]\cdot(1-\pi) \quad (17)$$

To simplify further, say that both the supplier and the producer reject a lot as soon as one unit is found to be defective and let $\theta_1 = 0, \theta_2 = \theta$. In this case $\alpha_{p,2} = 0,\ \beta_{p,2} = (1-\theta)^n$ while for the supplier $\alpha_{S,2} = 0; \beta_{S,2} = (1-\theta)^m$. As a result,

$$x = \frac{\bar{\beta}_p - \bar{\beta}_S\ (1-\theta)^n}{\bar{\beta}_S\left(1-\ (1-\theta)^n\right)},\ y = \frac{\bar{\beta}_S - (1-\theta)^m}{1-(1-\theta)^m}. \quad (18)$$

The costs of the producer are now:

$$\begin{bmatrix} \hat{C}_{p,11} = (1-\xi)u(N)(1-\pi) & \hat{C}_{p,12} = \hat{C}_{p,11}q^m \\ \hat{C}_{p,21} = c_p n + \chi_2; & \hat{C}_{p,22} = c_p n + \chi_2 q^m \end{bmatrix} \quad (19)$$

with

$$\chi_2 = \left[(1-\xi)u(N)(1-\theta)^n + c_{PR}(N)(1-(1-\theta)^n)\right](1-\pi)$$

and the supplier cost are:

$$\hat{C}_{S,11} = \xi u(N)(1-\pi);$$
$$\hat{C}_{S,12} = c_S m + c_S(N-m)\left(1-q^m\right)(1-\pi) + \left[\xi u(N)q^m + c_{Sb}(N,\theta_2)\left(1-q^m\right)\right](1-\pi)$$
$$\hat{C}_{S,21} = \left(\xi u(N)q^n + c_{SR}(N,\theta_2)\left(1-q^n\right)\right)(1-\pi);$$
$$\hat{C}_{S,22} = c_S m + c_S(N-m)\left(1-q^m\right)(1-\pi) + \begin{bmatrix} \xi u(N)q^{n+m} + \\ c_{SR}(N,\theta_2)\left(1-q^n\right)q^m \\ +c_{Sb}(N,\theta_2)\left(1-q^m\right) \end{bmatrix}(1-\pi). \quad (20)$$

Optimal sampling by the supplier and the producer can then be determined by minimizing the expected costs subject to the specified risk constraints for each. A solution can of course be found numerically. Explicitly, our problem is the *(n, m)* strategic sampling problem:

$$\underset{n}{Min}\ \left(\frac{\bar{\beta}_p - \bar{\beta}_S\ q^n}{\bar{\beta}_S\left(1-\ q^n\right)}\right)\left(\frac{\bar{\beta}_S - q^m}{1-q^m}\right)\left[\hat{C}_{p,11} - \hat{C}_{p,12} - \hat{C}_{p,21} + \hat{C}_{p,22}\right] + \frac{\bar{\beta}_p - \bar{\beta}_S\ q^n}{\bar{\beta}_S\left(1-\ q^n\right)}\left(\hat{C}_{p,12} - \hat{C}_{p,22}\right) + \frac{\bar{\beta}_S - q^m}{1-q^m}\left(\hat{C}_{p,21} - \hat{C}_{p,22}\right) + \hat{C}_{p,22}$$
$$\underset{m}{Min}\ \left(\frac{\bar{\beta}_p - \bar{\beta}_S\ q^n}{\bar{\beta}_S\left(1-\ q^n\right)}\right)\left(\frac{\bar{\beta}_S - q^m}{1-q^m}\right)\left[\hat{C}_{S,11} - \hat{C}_{S,12} - \hat{C}_{S,21} + \hat{C}_{S,22}\right] + \frac{\bar{\beta}_p - \bar{\beta}_S\ q^n}{\bar{\beta}_S\left(1-\ q^n\right)}\left(\hat{C}_{S,12} - \hat{C}_{S,22}\right) + \frac{\bar{\beta}_S - q^m}{1-q^m}\left(\hat{C}_{S,21} - \hat{C}_{S,22}\right) + \hat{C}_{S,22} \quad (21)$$

Inserting the cost entries for the producer for example, we obtain after some manipulations that:

$$\underset{n}{Min}\ \left(\bar{\beta}_p / \bar{\beta}_S - \ q^n\right)\left(\bar{\beta}_S - q^m\right)\left[(1-\xi)u(N) - c_{PR}(N)\right] + \left(\bar{\beta}_p / \bar{\beta}_S - \ q^n\right)\left(q^m\left[(1-\xi)u(N) - c_{PR}(N)\right] - \frac{c_p n}{1-\pi}\right) + \left(\bar{\beta}_S - q^m\right)\left[(1-\xi)u(N)q^n + c_{PR}(N)(1-q^n)\right] + \frac{c_p n}{1-\pi} + \left[(1-\xi)u(N)q^n + c_{PR}(N)(1-q^n)\right]q^m. \quad (22)$$

$$\underset{n}{Min}\ \bar{\beta}_S(1-\xi)u(N) + \frac{c_p n}{(1-\pi)} + \left[\bar{\beta}_S q^n - \bar{\beta}_p\right]\left[\frac{c_p n}{\bar{\beta}_S(1-\pi)} + c_{PR}(N) - (1-\xi)u(N)\right]. \quad (23)$$

3.2. Full cooperation to reduce inspection costs

If the buyer and the supplier fully cooperate in reducing the inspection cost then (ignoring the cost bi-matrix) we have an optimization problem seeking to minimize inspection and economic costs, subject to a set of risk constraints specified by both the supplier and the buyer. These costs consist of the sum:

$$\underset{x_i, n_{p_i}, y_j, n_{S_j}}{Min}\ \sum_{i=1}^{n}\sum_{j=1}^{m} x_i y_j\left(\hat{C}_{p,ij} + \hat{C}_{S,ij}\right)\ Subject\ to:$$

$$\sum_{j=1}^{m}\sum_{i=1}^{n} x_i y_j \alpha_{p,i}(1-\alpha_{S,j}) \le \bar{\alpha}_p;\quad \sum_{j=1}^{m}\sum_{i=1}^{n} x_i y_j \beta_{p,i}\beta_{S,j} = \bar{\beta}_p;$$
$$\sum_{j=1}^{m} y_j \alpha_{S,j} \le \bar{\alpha}_S;\quad \sum_{j=1}^{m} y_j \beta_{S,j} = \bar{\beta}_S. \quad (24)$$

For 2 alternatives, this problem is simplified since x and y are given by (15). Of course for more extensive sampling alternatives, we will obtain different results, reflecting the risks implied in the control procedure. The problems we will face will then be essentially numerical.

3.3. A non-cooperative problem with type II risks not fixed

We maintain for simplicity the CSP-1 sampling alternatives above. The corresponding expected cost values (equations 9) for the buyer and the supplier are as given below in table.

The following potential risk sensitive solutions (where only $0 \le x < 1$ and $0 \le y < 1$ are feasible) yield the following:

$x = 0, y = 0 \to \bar{\beta}_p = \bar{\beta}_S (1-\theta)^n ; \bar{\beta}_S = (1-\theta)^m$; feasible

$x = 1 \to \bar{\beta}_S = \bar{\beta}_p$; not feasible if $\bar{\beta}_S \ne \bar{\beta}_p$

$y = 1 \to 1 = \bar{\beta}_S$; not feasible

$$0 < x^* < 1 \text{ implies } (1-\theta)^n < \frac{\bar{\beta}_p}{\bar{\beta}_S} < 1 \qquad (25)$$

$$0 < y^* < 1 \text{ implies } (1-\theta)^m < \bar{\beta}_S < 1$$

In the case $x = 0, y = y^*$ we have the following condition for optimality:

$$y^* \le \frac{\bar{\beta}_S - (1-\theta)^m}{\left(1-(1-\theta)^m\right)} \quad \text{with}$$

$$U_1 - T_2 \le \frac{c_p n}{1-\pi} + (U_2 - T_2)(1-\theta)^n . \qquad (26)$$

Such a solution is optimal if *x=0* is optimal which occurs if for the buyer, the no sampling strategy is optimal, or:

$$U_1 - T_2 \le \frac{c_p n}{1-\pi} + (U_2 - T_2)(1-\theta)^n ;$$
$$U_1 - T_2 \le \frac{c_p n}{(1-\pi)(1-\theta)^m} + (U_2 - T_2)(1-\theta)^n \qquad (27)$$

which is reduced to :

$$U_1 - T_2 \le \frac{c_p n}{1-\pi} + (U_2 - T_2)(1-\theta)^n \text{ with}$$
$$\bar{\beta}_p = \bar{\beta}_S (1-\theta)^n , \bar{\beta}_p > \bar{\beta}_S . \qquad (28)$$

And finally to:

$$U_1 - T_2 \le \frac{c_p \ln\left(\bar{\beta}_S / \bar{\beta}_p\right)}{(1-\pi)\ln\left(1/(1-\theta)\right)} + (U_2 - T_2)\left(1 - \frac{\bar{\beta}_p}{\bar{\beta}_S}\right);$$
$$n = \frac{\ln\left(\bar{\beta}_S / \bar{\beta}_p\right)}{\ln\left(1/(1-\theta)\right)}, \qquad (29)$$

as a necessary condition for optimality. If we set $x = x^*, y = 0$, then of course, it is feasible and optimal if:

$$V_1 \le \frac{c_S m}{(1-\pi)} + \left(\bar{Q}_2 + \left(V_2 - \bar{Q}_2\right)(1-\theta)^m\right)$$
$$\left(W_1 + (V_1 - W_1)(1-\theta)^m\right) \le$$
$$\frac{c_S m}{(1-\pi)} + \left(\begin{array}{c} \bar{Q}_2 + \left(W_2 - \bar{Q}_2\right)(1-\theta)^m \\ + (V_2 - W_2)(1-\theta)^{n+m} \end{array}\right) \qquad (30)$$

and

$$m = \frac{\ln\left(\bar{\beta}_S\right)}{\ln(1-\theta)} . \qquad (31)$$

Finally, for an interior Nash solution, we obtain after some elementary manipulations:

	Supplier does not sample *(m=0), y*	Supplier samples *(m,1)* *1-y*
Buyer does not sample *(n=0),x*	$U_1(1-\pi)$ $V_1(1-\pi)$	$U_1(1-\theta)^m(1-\pi)$ $c_S m + \left(\bar{Q}_2 + \left(V_2 - \bar{Q}_2\right)(1-\theta)^m\right)(1-\pi)$
Buyer samples *(n,1),1-x*	$c_p n + \left(\begin{array}{c} U_2(1-\theta)^n + \\ +T_2\left(1-(1-\theta)^n\right) \end{array}\right)(1-\pi)$ $\left(W_1 + (V_1 - W_1)(1-\theta)^m\right)(1-\pi)$	$c_p n + \left[\begin{array}{c} T_2 + \\ (U_2 - T_2)(1-\theta)^n \end{array}\right](1-\theta)^m(1-\pi)$ $c_S m + \left(\begin{array}{c} \bar{Q}_2 + \left(W_2 - \bar{Q}_2\right)(1-\theta)^m \\ +(V_2 - W_2)(1-\theta)^{n+m} \end{array}\right)(1-\pi)$

$$0 \le x^* = \frac{(1-\theta)^m}{1-(1-\theta)^m}\left[\frac{\dfrac{c_p n(1-\theta)^{-m}}{1-\pi}}{\left[(U_1-T_2)-(U_2-T_2)(1-\theta)^n\right]}-1\right] \le 1,$$

$$0 \le y^* = \frac{(V_2-W_2)(1-\theta)^{n+m}}{(V_1-W_1)\left(1-(1-\theta)^m\right)+(V_2-W_2)(1-\theta)^{n+m}} \le 1 \qquad (32)$$

as well as:

$$(1-\theta)^n < \frac{\bar{\beta}_P}{\bar{\beta}_S} < 1; (1-\theta)^m < \bar{\beta}_S < 1, \bar{\beta}_S > \bar{\beta}_P \qquad (33)$$

To determine the optimal sampling quantities, we will of course introduce the Nash estimates for the sampling plans probabilities (x^*, y^*) into the Nash values and minimize with respect to (n^*, m^*). This is of course again, a problem we can easily solve numerically.

If the sampling quantities are to be determined as well in the game, then, we can define our bi-matrix game consisting of the following sampling alternatives:

$n \in (0,1,2,3,....,N)$ and
$m \in (0,1,2,3,....,N)$

and the Nash solution will involve determination of the probabilities $(x_0, x_1, x_2, ..., x_N)$ for the buyer and $(y_0, y_1, y_2, ..., y_N)$ for the supplier. In this approach, strategic quality assurance is performed not only to determine how much to sample but also how to mix sampling procedures so that "sampling and assurance" assume the dual and strategic purpose of controlling incoming products and providing a threat (or signal) which can be used by the parties as incentives in doing what they contracted to do.

4. Conclusion

Strategic quality control recognizes that parties' motivations (in a supplier-producer relationship for example) can impact the type and the process of control we apply in managing the relationship between these parties. The resulting solution depends of course on the kind of assumptions we are willing to make to reconcile the parties involved. For example, if a party has information that the other does not have and is therefore a leader in the strategic control game, the Nash or cooperative solution might not be appropriate. Of course, a Stackleberg solution may be applied (Stackleberg, 1934). In some situations, one might be a leader with respect to some variables and the other might lead with respect to other variables (as it is typically the case in supply chains and in situations where firms exchange goods and information). Here too, controls of some sort might be required. The purpose of this paper was to focus attention on these problems by providing a strategic framework and an approach we might profitably use when quality assurance and control involve potential conflicts.

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