A GAME THEORY APPROACH FOR OPTIMUM STRATEGY OF THE OWNER AND CONTRACTOR IN DELAYED PROJECTS

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Received 20 Feb 2016; accepted 10 May 2016

Abstract. Delay is one of the problems occurring between owners and contractors. Deviation from base plans can be found using project control methods, continuous assessment of the schedule, determining progress percentages and earned value parameters. In such circumstances, conflicts might arise between contractor and owner as restoring project’s original schedule needs added expenditures by the contractor. Moreover, continuation of the previous procedure and late completion of the project will cause the owner damage.

In this research, a mathematical model using game theory has been presented. The model investigates the behaviour and strategies of the parties involved in a delayed project through bargaining. It helps owners and contractors gain deeper understanding of the given delay problem, get a fairly accurate analysis of their situation and consider possible strategies in facing with such circumstances without spending a long and inconclusive time. The points which both parties can agree rationally proposed with a numerical example.

Results of the model indicate that parameters involved in the problem are effective in changing the range width of negotiation. In special cases, such as disproportionate delay penalty, these parameters even make it neutral in negotiation. Step by step analysis of the model showed which features can threaten negotiation.

Keywords: construction management, delay, construction projects, game theory, bargaining, conflict.

Introduction

Dynamic relationship between project and environment, human and resources greatly raise the probability of interest conflict between project’s different parties.

Conflict management means providing solutions for settlement of these conflicts, within a short period of time. Usually, prolongation of this process results in more costly decisions in presence of other available options. This conflict settlement process covers a wide range of measures such as negotiation, mediation, arbitration and litigation (Kassab et al. 2006). Owner and contractor of a project are usually the parties which these conflicts refer to (Chen et al. 2014). A very common source of conflict occurrence between owner and contractor is a delayed project. Delay in a project can be delineated as the period of time a given project is completed after the due date. Both the owner and contractor intend to avoid delays for various reasons. If a project is completed late, the owner will lose the operation profit or will have a loss because of the social and political reasons due to the delayed opening of the project. Moreover, the contractor might raise claims owing to inflation resulting from the delayed completion of the project and the increased overhead costs. On the other hand, the contractor will also have a loss because of delay penalty, losing the chance of undertaking new projects and also the increase in overhead expenses during the delay. Therefore, either end users prefer to reduce delay. Therefore, the two sides are seeking to reduce delays, but with the least cost each of them pays. This is exactly the situation where the interest of the parties is placed in conflict.

Therefore, the aim of this paper is reconciliation between the involved parties to return the project to the originally fixed schedule as much as possible and with the least possible delay. However, since the failure in timely completion of projects causes damage to both the contractor and the owner, the present research tries to find the equilibrium point in the costs imposed on each side.

Considering the researches, despite probable condition of conflicts between stakeholders of a project and also significant number of disputes existing in construction projects, least of the cases in reality are preferred to be resolved in courts (Kassab et al. 2010). Negotiation, partnering, mediation, arbitration and dispute review boards are the chief alternative dispute resolution tech-
niques when parties try to manage a dispute with the minimum cost (Mitropoulos, Howell 2001; Kassab et al. 2006). Among these tools, negotiation is usually a default way of settling a dispute (Marzouk, Moamen 2009; Chow et al. 2012; Lu et al. 2014).

Negotiation together with dispute, have been topics of different researches. Fenn et al. (1997) gives taxonomy of conflict and dispute in construction. Marzouk and Moamen (2009) present a framework that is developed to assist contractors during negotiation process with owners. Yiu et al. (2011b) investigated effect of negotiators’ conditions in negotiation. Lu et al. (2014) have studied the situation of negotiators in construction claim negotiations. Cheung and Chow (2011) and Chow et al. (2012) have studied withdrawal of one of the negotiating parties. Some of the researchers have studied and anticipated the controversies happened in construction projects (Chou 2012; Cheung, Pang 2013). Cheung et al. (2009) and Yiu et al. (2011a) have investigated the appropriate tactics in construction dispute negotiation. A number of articles also have focused on presentation of methods to resolve disputes in construction projects. El-adaway and Kandil (2010) have simulated disputes in construction projects using logical algorithms. Ng et al. (2007) proposed a conflict management system using system dynamic. Yousefi et al. (2010) have presented systematic methods to resolve disputes in construction projects.

Although some of the above-mentioned studies propose conflict resolution recommendations, but generality of the concept may make their model inapplicable. Also, relationship between parties and their interactions for problem solving is usually neglected. Therefore, this paper investigates the interactions between contractor and owner in a delayed project through game theory approach.

As a conflict management tool, in different researches game theory has been a tool dealing with problem of dispute between decision-makers (Peldschus 2008; Barron 2013). But, it has not been mentioned in considerable number of articles in construction management area so far. Using the game theory and presenting a unique model, Ho and Liu (2004) have investigated the financial claims of the contractors who have won tenders with opportunistic bids. Kassab et al. (2010) have used the game theory to solve construction project conflicts between the involved sides under uncertainty, too. Furthermore, Ho (2006), Tserng et al. (2012, 2014) and Javed et al. (2014) have applied the game theory to facilitate the conditions of public-private partnership contracts. Shen et al. (2007), Chen et al. (2012) and Lv et al. (2013) have also given some suggestions in order to facilitate the bargaining process in BOT contracts.

In another study, Ho (2005) has proposed a model for the projects with costly bid preparations. He also studied projects with heterogeneous bidders by game theory (Ho et al. 2014). Based on game theory, Unsal and Taylor (2011) and Asgari et al. (2013) have also presented some recommendations for solving problems related to subcontractors and Hsueh and Yan (2011) proposed a profit-sharing model for joint ventures. Moreover, game theory have been applied to problem of staff payments (Wu et al. 2011), and renewal of construction objects (Antuchevičienė et al. 2006).

Even some researchers have scrutinized occurrence of delays in projects with game theory point of view. Estévez-Fernández (2012) has used game theory in order to share penalties and rewards in projects. Castro et al. (2007) have applied this theory in setting PERT networks too. Up to now, no research has dealt with the possible strategies in the case of delays in construction projects and how to agree on the completion date in such projects yet.

In the present paper, transactions between contractor and owner in DBB (Design- Bid- Build) projects facing delay thoroughly analyzed. This gives the reader a vivid understanding about the structure of the costs imposed to parties in different scenarios of a delayed project. Furthermore, using game theory a systematic approach is presented the basic rule of which is proposing recommendations to parties in order to solve the problem without time consuming bargaining.

This paper is organized as follow: a literature review of dispute resolution and problem of delay in construction projects is first presented as introduction to define the scope of this research. Then, with practical and academic aspects, a mathematical model of the transactions between the parties in a delayed project is presented to help practitioners understand the circulating terms between owners and contractors. This model is analyzed with game theory that is a common tool when conflict occurs between decision makers. This model offers a framework that assists parties to get round the problem of delay and avert long litigations. Finally, applicability of the proposed framework is shown with a numerical example.

1. Mathematical model based on game theory

In this study, by using game theory and based on real experiences, an analytical model will be provided to study the delay in construction projects which in turn paves the way for investigation of the challenges occurring between involved parties. Here, the model is called “game of the delayed projects”. Thus, in Sections 3.1 and 3.2 transactions occurring between owner and contractor are clarified.

In this model, which is very common in the projects, a project with initial budget and contract completion date \((T_u)\) begins at the initial time. Having computed the completion percentage of the activities and earned value of the project at the present time \((T_p)\), it will be clear that the project will be operationalized with delay at \((T_p)\) which \((T_p < T_u)\). In addition, the expenses for completion of the project will be increased. On the other hand, the contractor will be able to compensate the whole or part of the predicted delay by appropriately managing
or increasing the efficiency and resources (human and machineries). In such situation, the contractor begins to claim for the compensation of the delays and increasing the costs. At this point, the conflict between parties of a contract begins. Other researchers (Ho, Liu 2004) have already modelled the contractors’ claims in a game. But, in the present paper contractor’s claim is specifically compensation of the delay and negotiation topic is the agreement with the owner on project’s completion time. In other words, the contractor’s claim will be investigated for one certain case in this article.

It should be noted that in the present article we investigate the delay while the project is being executed and before the end of initial contract duration. The reason is that the sooner existing problems are settled, the dispute originated interaction costs are reduced (Kassab et al. 2010). In other words, it is advised to minimize the costs originating from delays before becoming a disaster.

1.1. Description of the game

The game tree (extended form) is outlined at first. Next, recommendations are provided in order to quickly reach an agreement on solving the problem of delay in projects. Figure 1 illustrates the game for the projects facing delay. C and O are symbols of contractor and owner, respectively. Then, the game of the projects facing delay is described.

This model is based on the general conditions of Design-Bid-Build projects in Iran and it was found useful for delayed projects based on the positive comments received from the experts in the industry—irrespective of the project’s delivery system. In another word, a conflict resolution model is presented here which can be extended by other researchers based on requirements of projects with any governing general conditions.

The game begins by the owner at $T_n$. In this stage, owner’s strategies include his request from the contractor to compensate the delayed time $T_d$, or not to negotiate on it $T_c$. If the owner does not offer a negotiation proposal to the contractor in order to solve the problem of delay, the project may progress in the same trend, or the contractor himself may manage to optimize so that he can reduce his own overhead expenses (top of Fig. 1). If the owner requests the contractor to compensate the total time of delay $T_d$, next decision, to accept or not to accept this request, should be made by the contractor. Also, the owner will be able to make decision if the contractor doesn’t accept to balance the total delay $T_d$. The options among which the owner should select include: agreement on compensation of a part of the delay $T_d$, contract termination or contract cancelation (bottom of Fig. 1). In fact in this stage, the owner decides whether the contract continues with any delay which should be specified in the future, or it should be cancelled or terminated. In order to reach agreement on compensating a part of delay, in this stage the owner proposes that the contractor compensate the $pT_d$ length of delay. Here, the contractor may accept the proposal and compensate a part of delay through agreement with the owner, or reject it for some reasons. The owner may decide to cancel or terminate the contract if the contractor doesn’t accept the proposal.

1.2. Responses of the game

As Figure 1 illustrates, six final results have been devised for this game. In fact, these final results are six probable scenarios according to general conditions of DBB contracts prevalent in Iran. A brief presentation about each of these scenarios is given here. We will give complete explanation in Sections 1.2.1 to 1.2.6.

Scenario 1: When both contractor and owner do not negotiate about the reduction of the delayed time and project may progress in the same trend as it was. We call it “Continuation of the current trend”.

Scenario 2: In this scenario and without owner’s request, contractor himself intends to reduce the delays in order to decline his overhead expenses. We call it “Delay reduction by the contractor”.

Scenario 3: It occurs when owner asks the contractor to compensate total time of delay $T_d$ and also contractor accepts. We call it “Elimination of the total time”.

Fig. 1. Game tree for the projects facing delay
Scenario 4: This scenario occurs when both sides agree on eliminating a part of the total delay \((p \cdot T_d)\). We call it “Agreement on compensation of a part of delay”.

Scenario 5: In this scenario and according to general conditions of DBB contracts in Iran, contract termination occurs when both parties intend to end the project before its completion. This scenario is very similar to the case that FIDIC’s Red Book refers “Contract termination”. Here, we also call this scenario “Contract termination”.

Scenario 6: In this scenario and according to general conditions of DBB contracts in Iran, if a contractor fails to fulfill his obligations – so that impose very exorbitant cost to owner – conditions for unilateral termination of the contract would be provided in favour of the owner. More details about this scenario are provided in Section 1.2.6. Here, we call this scenario “Cancellation of the contract”.

1.2.1. Continuation of the current trend

If the owner decides not to negotiate and also the contractor continues the same trend, the owner’s following payoffs will happen if the project completes with delay in \(T_u\):
1. The loss due to deprivation from the profits of timely utilization of the project. We name it \(L_{Td}\).
2. The fine which the contractor is entitled to pay to the owner due to delay with a length of \(T_d\) against the completion date of project \((T_c)\), which is called \(F_{Td}\).

On the other hand, contractor’s payoffs due to this decision made by the owner include:
1. Paying the delay fine \((F_{Td})\).
2. Increasing cost of project completion till \(T_u\) which is called \(IC_{Tu}\).

The logical assumption is that \(IC_{Tu}\) Cost is due to contractor’s poor productivity or inexact evaluation of the activities. In other words, the cost of \(IC_{Tu}\) will not have any payoff for the owner and the contractor himself decides to take steps in order to reduce such costs by using various strategies and changes in management approaches. Therefore, the payoff originated from owner’s decision not to negotiate for the parties is \(\{-L_{Td} + F_{Td} - IC_{Tu} - F_{Td}\}\).

1.2.2. Delay reduction by the contractor

If the owner decides not to negotiate, the contractor may compensate a part of the delay with the length of \(s \cdot T_d\) so that \((0 < s < 1)\) in order to reduce his overhead expenses. Thus, the owner cannot manage to utilize the project on time with an amount of \(L_{Td}\) loss. So, the owner demands the \(F_{Td}\) delay fine from the contractor.

In order to compensate the aforementioned delay, the contractor has to bear the \(CC_{sTd}\) crash cost. Also, the contractor is required to pay the increased costs for completion of the project during the time of \(T_u - s \cdot T_d\), which is called \(IC_{sTd}\). Therefore, the payoffs for both parties from contractor’s decision for reducing the delay length, without owner’s request, will be \(\{-L_{Td} + F_{Td} - IC_{sTd} - IC_{Tu} - F_{Td}\}\).

1.2.3. Compensation of the total delay by the contractor

If the owner requires the contractor to compensate the total delay time, the next decision to accept the request or not should be made by the contractor. The payoff from acceptance of removing the \(T_d\) delay for the contractor includes:
1. Increased Costs of project completion till \(T_c\), which are called \(IC_{Tc}\). The reason for this increased cost is poor performance of the contractor till the time of evaluation.
2. The costs of crashing the length of the remained activities of the project till \(T_d\) which are called \(CC_{Td}\).

On the other hand, the above decision will result in zero payoff for the owner. Thus, the payoff from decision made for compensation of the total delay for both parties will be \(\{0, CC_{Td} - IC_{Tc}\}\).

1.2.4. Agreement on compensation of a part of delay

If the contractor does not accept to remove the total delay \((T_d)\), the owner will be in the position of making a decision. One of the options is to agree on compensating a part of \(T_d\).

In order to get to the agreement on compensating a part of delay, the owner proposes the contractor to compensate the length of \(p \cdot T_d\) so that \((0 < p < 1)\). If the contractor agrees to remove a part of delay with the amount of \(p \cdot T_d\), and his or her payoff includes:
1. Increased cost of project completion in \(T_u - p \cdot T_d\), which is called \(IC_{pTd}\). It should be paid by the contractor.
2. The crash costs of the remained activities of the project up to \(p \cdot T_d\) which is called \(CC_{pTd}\).

The owner’s payoff because of contractor’s decision includes the loss of being deprived from the profits due to on time utilization of the project which is called \(L_{pTd}\).

In fact, in this stage the owner withdraws the delay fine due to the agreement with the owner on compensation of a part of delay. Therefore, the payoff from this decision for both parties (owner and contractor) will be \(\{-L_{pTd} + CC_{pTd} - IC_{pTd}\}\).

1.2.5. Contract termination

When the owner negotiates with the contractor about compensation of the total or part of delay time, the owner can terminate the contract if the contractor doesn’t accept to do so. When the contract is terminated, the owner has to repeat a tender to choose a new contractor for completing the project. Also, his or her decision for termination of the contract will cause prestigious costs as jeopardizing owner’s reputation for inability in promotion of the objectives and completion of the project on time and also weakening his or her position against other contractors.

The total cost of holding another tender and the prestigious costs are considered as \(r\); in addition, as the completion time of the new contract \((T_c')\) isn’t less than the pre-
vious one \( \{ T_c \} \), the owner will be deprived of the profits from on time utilization of the project \( \{ L_{Tc} \} \).

In contractor’s point of view, the contract termination damages contractor’s prestige in the owner’s eyes; the cost incurred to the contractor in this regard is called \( m \). It should also be noted that the contractor bears the costs of \( IC_{Tn} \) because of his mismanagement and poor productivity. Therefore, the payoff of contract termination will be \( \{ -r - L_{Tc} , IC_{Tn} - m \} \) for both parties.

### 1.2.6. Cancellation of the contract

According to general conditions of contracts in Iran, when owner’s proposals on compensation of total or part of delay are not accepted, the owner can decide to cancel the contract. As mentioned above, when the owner decides to cancel the contract, he or she will be deprived of the profits from on time utilization of the project \( \{ L_{Tc} \} \). However, bank guarantees related to the contract \( \{ W \} \) will be confiscated in the interest of the owner and if the case is not compromised, the trial case for procedure of the activities of the project which have costs for the parties will start. The amount of \( R \) which is brought in the game tree as a part of owner’s payoff when the contract is cancelled is the total cost of trial and also holding the tender again together with prestigious costs will be borne by the owner. Based on Iranian experts’ opinions, occurrence of contract cancelation is a real disaster for contractor.

In addition, if the cancellation occurs, costs of \( M \) and \( IC_{Tn} \) and confiscation of the \( W \) will be incurred by the contractor. The amount of \( M \) refers to those costs borne by the contractor when the contract is cancelled or when the trial is restored and his or her relationship with the owner is wasted. Therefore, the payoff of cancellation of the contract will be \( \{ -r - L_{Tc} , IC_{Tn} - M - W \} \) for both parties. It should be noted that the payoff of decisions made by the owner in order to cancel or terminate the contract will be identical without the fact that they have been obtained before or after provision of \( pT_d \) proposal.

### 2. Game analysis

#### 2.1. Form 1: the percentage of reduced delay \( (p) \) should be assumed fixed

The analysis begins in the simplest manner. In other words, it is impossible to propose \( (p) \) mutually; \( (p) \) is fixed before negotiation and it is a known number for both sides. Although \( (p) \) can be variable, this is studied in the negotiation time. The problem will be solved by SPNE in the following 4 forms:

**Resolution 1: the owner negotiates.** The contractor doesn’t accept compensation of the total time, the owner proposes \( pT_d \), the contractor doesn’t accept and the owner terminates the contract.

It is clear if a contract is cancelled, the project completion time will be delayed more and in equal conditions, the owner will not tend to cancel the contract due to upcoming costs like cost of trial, wastage of the relationship with other contractors, etc. Then, the following relationship usually holds:

\[
R > r \quad \text{and} \quad L_{Tc} > 0 \Rightarrow L_{Tc}^\prime;
\]

\[
R - L_{Tc}^\prime + W < -r - L_{Tc}^\prime.
\]

At an earlier stage, contractor knows if the owner’s proposal is rejected, the owner will select the contract termination. Therefore, the following requirements (Eqn (3)) need to be established so that the contractor can decide not to accept it while knowing the value of \( p \):

\[
m < -CC_{pTd} + IC_{pTd} - IC_{Tn}.
\]

The above mentioned relationship means that the contractor’s cost of wasting relationship with the owner will be less than that of compensating owner’s proposed time. In other words, the contractor reaches to the conclusion that the continuation of the project involves more cost than its termination according to owner’s proposed time.

**On the other hand, the following relationship exists between the owner’s payoffs:**

\[
-r - L_{Tc} > -L_{sTd} + F_{sTd};
\]

\[
L_{Tc}^\prime < L_{sTd}.
\]

Equation (5) indicates heavy cost which the owner pays for completion of the project in this stage and consequently with the same contractor, in such a manner that the cost and time are much more and longer compared to that of the termination of the contract and substitution of the new contractor. However, it should be mentioned that contract cancellation could be equilibrium provided that one of the conditions required for cancellation occurs within the general conditions of the contract and the following relationship is met:

\[
-R - L_{Tc} + W > -r - L_{Tc}^\prime;
\]

\[
r > L_{Tc}^\prime - L_{Tc} - W + R.
\]

Equation (7) describes the significant increase in owner’s prestigious cost when the contract terminates under the mentioned conditions. Such large prestigious costs may arise from the following factors:

1. Wasting owner’s prestige against other contractors.
2. Losing the possibility of compensating partial costs of upcoming maintenance due to poor quality of the contractor’s performance.

This occasion may rarely occur in projects but it may also occur in certain conditions. The final solution for resolution 1 – form 1 has been provided through SPNE method (Fig. 2).
Resolution 2: the owner negotiates, the contractor doesn’t accept compensation of the total time, the owner proposes $p_Td$ and the contractor accepts.

In this manner, the contractor knows that if the owner’s proposal is rejected, he or she will decide to terminate the contract. Therefore, the following requirements should exist that the contractor can decide not to accept it while he or she is aware of $p$ value:

$$-IC_{Tn} - m < CC_{pTd} - IC_{pTd}; \quad (8)$$

$$m > -CC_{pTd} + IC_{pTd} - IC_{Tn}. \quad (9)$$

Equation (8) means that the costs of wasting relationship with the owner are high for contractor. In other words, the contractor accepts to increase his or her costs to satisfy the owner in order not to waste the relationship with the owner:

$$-L_{pTd} > -L_{std} + F_{std}; \quad (10)$$

$$F_{std} < L_{std} - L_{pTd}. \quad (11)$$

Equation (10) describes that the delay fine which the owner can receive from the contractor is less than the production corresponding with time interval considered by the owner and if the fine were high, the owner would not be interested in agreeing on $p_Td$ time.

The final solution for resolution 2 – form 1 through SPNE Method is provided in Figure 3.

Resolution 3: the owner negotiates, the contractor accepts to compensate the total delay.

The contractor knows that the condition in this solution is in such a manner that if the total $T_d$ time isn’t agreed upon in the negotiation, the owner will decide to cancel or terminate the contract:

$$Td TC TnCC IC IC m > - - - ; \quad (12)$$

$$Td Tc IC IC m > + - ; \quad (13)$$

Equation (12) means that cost of wastage of the relationship with the owner is too high for the contractor. Consequently, contractor is ready to increase his or her cost in order not to waste the relationship with the owner, so he compensates total delay time incurred or the whole possible time. In other words, the following relationship
needs to exist so that the owner doesn’t accept contractor’s optimization option:

\[-L_{STd} + F_{STd} < 0; \]

\[F_{STd} < L_{STd}. \]  

Equation (15) means that the contractor’s delay fine is less than the profit which the owner can gain if the project is delivered at the time considered by the contractor \((sT_d)\).

The final Solution for resolution 3 – form 1 through SPNE Method is presented in Figure 4.

Resolution 4: the contractor doesn’t negotiate; the contractor compensates time with a value of \(pT_d\) to reduce his or her costs.

The owner knows that if the contractor enters into negotiation for the time \(pT_d\), he or she will not pay the delay fine when the time of \(pT_d\) is compensated. Of course, the owner knows that wastage of the relationship cost is high in contractor’s viewpoint:

\[I_{CTn} + m > -CC_{STd} + IC_{STd} + F_{STd}; \]

\[m > -CC_{STd} + IC_{STd} - I_{CTn} + F_{STd}; \]

\[-L_{STd} + F_{STd} > -L_{pTd}; \]  

\[F_{STd} > L_{STd} - L_{pTd}. \]  

Combining Eqns (17) and (19) gives:

\[L_{STd} - L_{pTd} < F_{STd} < I_{CTn} + m + CC_{STd} - IC_{STd}. \]

Equation (20) describes a very highly increased delay fine in comparison to the profits from productivity for the owner, so that it increases even more than his or her production during the time intervals of \(sT_d\) to \(pT_d\). Therefore, the owner does not intend to propose \(pT_d\) for compensation of delay time. In addition, the continuity of the previous process of the project can be equilibrium in certain conditions. It means that considering the conditions in Eqns (17) and (19), \(s = 0\) should happen.

The final solution for resolution 4 – form 1 through SPNE method is presented in Figure 5.

2.2. Form 2: the percentage of reduced delay \((p)\) should be assumed variable

In this condition, similar to what really occurs in the projects, we assume that it is possible to bargain and exchange proposals and agree on a determined \(p\). If both
2.2.1. Agreement procedure for determination of \( p_{Td} \)

In order to determine \( p \), the payoffs of the parties should be higher compared to other payoffs when agreement is made. A set of answers will be obtained through step by step evaluation of the agreement condition with canceling the contract, terminating it, accepting total time of \( T_d \) and compensation of a part of delay by the contractor. If no set of answers is obtained, the equilibrium point of the game will be determined by comparing payoffs of different conditions. The following points should be considered when analyzing the model in this case:

1. The owner can cancel the contract only when one condition of the requirements established in the contract is satisfied which provides the owner with the condition for contract cancellation. If none of these requirements is satisfied, the owner will not be entitled to cancelling the contract. Therefore, in step by step evaluation the ordered pair of the parties’ payoffs shouldn’t be compared with those of contract cancellation due to provision of invalid answers.

2. Clearly, contract cancellation will always have worse payoffs for contractor compared to its termination. As a result, if no agreement is reached, the contractor tries none of the contract cancellation conditions occurs and convinces the owner to terminate the contract.

3. In most projects, cancellation may have worse outcomes than termination for the owner. For this reason, the owners usually aren’t interested in contract cancellation. Thus, contract cancellation is generally considered as a threatening strategy inevitably chosen by the owner if the contractor doesn’t agree with him or her. Here, with comparing the payoffs of the cancellation and termination of the contract and also with evaluation of the characteristics of the project together with owner’s patience, the validity of his or her threat will be determined. If the threat is invalid, the payoffs of agreement and termination shouldn’t be compared.

First step: Evaluation of the possibility and validity of the threat for cancellation of the contract and determination of comparison criterion. Here, considering the above-mentioned points, the payoff which is the criterion for comparing with the agreement payoff will be determined by studying provision of contract cancellation and validity or invalidity of the threat.

Second step: Priority of negotiation on contract termination or cancellation.

1. Once cancellation of the contract hasn’t been possible or valid in previous step:

The payoffs from negotiation on \( p_{Td} \) are assumed to be more than the payoffs from contract termination:

\[
r + L_{Tc} > L_{pTd};
\]

\[
IC_{Tn} + m > -CC_{pTd} + IC_{pTd}.
\]

According to the Eqns (21) and (22), if there is a set of answers for in order to satisfy the requirements of both equations, an agreement is reached. Otherwise, the owner will decide on contract termination or contractor’s optimization after evaluation of his or her payoffs:

\[
p_1 = \left\{ x: r + L_{Tc} > L_{xTd} & IC_{Tn} + m > CC_{xTd} + IC_{xTd} \right\}.
\]

Focusing on the above equations, it could be concluded that these two equations have the same set of answers unless and are very small numbers or they may involve higher crash costs.

2. If it is possible and valid to cancel the contract, the following relationships will be held:

\[
R + L_{Tc} - W > L_{pTd};
\]
\[ IC_{Tn} + M + W > -CC_{pTd} + IC_{pTd} \]  

(25)

According to Eqns (24) and (25), if there is a set of answers for \( p \) in order to meet the conditions of both equations, agreement will be reached. Otherwise, the owner may choose between contract cancellation and contractor’s optimization through evaluation of his or her payoffs. It is observed that the set of answers from comparing with cancellation payoffs are more extended and the contractor is ready to choose bigger (\( p \))’s:

\[ p_2 = \left\{ x : R + L_{tc} - W > L_{sTd} \quad \text{and} \quad IC_{Tn} + M + W > -CC_{sTd} + IC_{sTd} \right\} . \]

(26)

Third step: comparison of agreement on \( pT_d \) with compensation of total time of \( T_d \): If there is a set of answers for \( p \) in second step, the termination and cancellation of the contract will not result in better payoffs for the owner as compared to when the delays are compensated with \( pT_d \) and termination and cancellation of the contract will not be valid threats. If so, the owner will be forced to negotiate if the contractor does not accept to compensate total time of \( T_d \):

\[ -L_{sTd} + F_{sTd} < -L_{pTd} ; \]

(27)

\[ +CC_{sTd} - IC_{sTd} - F_{sTd} < +CC_{pTd} - IC_{pTd} ; \]

(28)

\[ -CC_{pTd} + IC_{pTd} + CC_{sTd} - IC_{sTd} < F_{sTd} < L_{sTd} - L_{pTd} . \]

(29)

Equation (29) indicates the effective range of delay fine. It is in such a manner that both the contractor and owner get interested in an agreement. In other words, the amount of delay fine should be in such an amount that is more than the differences in costs of crashing the activities considered by the contractor \( sT_d \) to the range of agreed \( pT_d \) while it is less than the differences in profits utilized by the owner within the same time interval.

Sharing the set of answers of the second step with the set of answers in Eqn (29) if exists, will be the final set of answer in the variable \( p \) form. Otherwise, with \( F_{sTd} \) getting more increased or decreased, the contract will be directed toward termination or optimization of the contractor. If the delay fine is a lot, being aware of not being able to reach an agreement the contractor will not accept any proposal by the owner to prevent the fine. As a consequence, the owner will be forced to terminate or cancel the contract. On the other hand, if the delay fine is much little, the contractor will also optimize the project and compensate a part of delay with the least cost since it is not possible to reach an agreement. Thus, the owner decides either to terminate the contract or to accept contractor’s decision.

3. Numerical example

Having controlled the schedule of a project which must have been finalized within 48 months, the owner concluded that the project would finish with a 6 month delay. The contract value is 120 million dollars and it is predicted that the contractor will have to pay 6 million dollars extra to complete the project. Additional investigations show that maximum 5 months of the delay length can be compensated. The crash cost of the activities for each 15 days include 0.5, 0.9, 1.3, 2.2, 3.1, 4.5, 6.1, 7.3, 8.5 and 9.6 million dollars. It has been contended in the contract that the contractor is required to pay 800,000 dollars fine per month. The revenue from utilization of the project is 2 million dollars per month for the owner. The cost imposed on the owner and contractor in the cancellation is 16 and 14 million dollars, respectively.

3.1. Analysis of the example

As seen in Table 1 in which the parties’ payoffs have been computed in terms of different compensation times, it is clear that compensated time more than 70% is impossible; this is because they are more than the costs imposed on the contractor when the contract is cancelled. On the other hand, contractor’s least cost is 10.73 million dollars.

Table 1. Overall payoff for owner and contractor against the compensable time percentage

<table>
<thead>
<tr>
<th>Compensable time (%)</th>
<th>Crash cost</th>
<th>Project utilization revenue for employer</th>
<th>Delay fine</th>
<th>Increased cost due to poor productivity</th>
<th>Owner’s total cost</th>
<th>Contractor’s total cost (including delay fine)</th>
<th>Contractor’s total cost (without delay fine)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>12</td>
<td>4.8</td>
<td>6.0</td>
<td>7.20</td>
<td>10.80</td>
<td>6.00</td>
</tr>
<tr>
<td>10</td>
<td>0.9</td>
<td>11</td>
<td>4.4</td>
<td>5.94</td>
<td>6.60</td>
<td>10.84</td>
<td>5.94</td>
</tr>
<tr>
<td>20</td>
<td>0.9</td>
<td>10</td>
<td>4.0</td>
<td>5.89</td>
<td>6.00</td>
<td>10.79</td>
<td>5.89</td>
</tr>
<tr>
<td>30</td>
<td>1.3</td>
<td>9</td>
<td>3.6</td>
<td>5.83</td>
<td>5.40</td>
<td>10.73</td>
<td>5.83</td>
</tr>
<tr>
<td>40</td>
<td>2.2</td>
<td>8</td>
<td>3.2</td>
<td>5.78</td>
<td>4.80</td>
<td>11.18</td>
<td>5.78</td>
</tr>
<tr>
<td>50</td>
<td>1.3</td>
<td>7</td>
<td>2.8</td>
<td>5.72</td>
<td>4.20</td>
<td>11.62</td>
<td>5.72</td>
</tr>
<tr>
<td>60</td>
<td>4.5</td>
<td>6</td>
<td>2.4</td>
<td>5.67</td>
<td>3.60</td>
<td>12.57</td>
<td>5.67</td>
</tr>
<tr>
<td>70</td>
<td>6.1</td>
<td>5</td>
<td>2.0</td>
<td>5.61</td>
<td>3.00</td>
<td>13.71</td>
<td>5.61</td>
</tr>
<tr>
<td>80</td>
<td>7.3</td>
<td>4</td>
<td>1.6</td>
<td>5.56</td>
<td>2.40</td>
<td>14.46</td>
<td>5.56</td>
</tr>
<tr>
<td>90</td>
<td>8.5</td>
<td>3</td>
<td>1.2</td>
<td>5.50</td>
<td>1.80</td>
<td>15.20</td>
<td>5.50</td>
</tr>
<tr>
<td>100</td>
<td>9.8</td>
<td>2</td>
<td>0.8</td>
<td>5.44</td>
<td>1.20</td>
<td>16.04</td>
<td>5.44</td>
</tr>
</tbody>
</table>
Thus, he or she will voluntarily take measures to compensate up to 30% of the delayed time. Here, we analyze the problem by Nash-Harsanyi and Kalai-Smorodinsky solution and we will also study the parties’ agreement on crashing the activities in the interval of 30% to 70% of the maximum compensation time.

3.2. Analysis of the bargaining process

Various analytical models have studied the process of bargaining so far. Nash (1950) proves that the answers from Eqn (30) are unique solutions for bargaining between two players:

\[ \Omega = \max \left( x_1 - d_1 \right) \left( x_2 - d_2 \right), \]  

(30)

where \( x_1 \) and \( x_2 \) respectively represent player 1 and 2’s payoffs in feasible set of alternatives during bargaining. \( d_1 \) and \( d_2 \) also belong to disagreement point, which is where no agreement is reached.

Harsanyi and Selten (1972) provide Eqn (31) for an \( n \)-player bargaining in which players have different bargaining powers \( w_i \):

\[ z = \prod_{i=1}^{n} (x_i - d_i)^{w_i}. \]  

(31)

Kalai and Smorodinsky (1975) believe that the bargaining solution is located in the junction of line connecting disagreement and ideal points with Pareto front. Thus, they have proposed Eqn (32) which takes into account players’ aspiration levels:

\[ \frac{x_1 - d_1}{t_1 - d_1} = \frac{x_2 - d_2}{t_2 - d_2}. \]  

(32)

In the above equation, \( t_1 \) and \( t_2 \) respectively represent player 1 and 2’s best payoffs in bargaining.

With setting Table 2, payoffs of the owner and contractor will be determined in a relatively continuous strategy. The fourth column of Table 2 is the value for the objective function of Nash (1950) which has been obtained by Eqn (33):

\[ \Omega = \max \left( x_1 - d_1 \right)^{w_1} \left( x_2 - d_2 \right)^{w_2}. \]  

(33)

Table 2. Nash objective function value (with equal discount factor)

<table>
<thead>
<tr>
<th>Compensable time (%)</th>
<th>Contractor’s total cost (million $)</th>
<th>Owner’s total cost (million $)</th>
<th>Nash objective function value</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>7.13</td>
<td>5.40</td>
<td>8.53</td>
</tr>
<tr>
<td>40</td>
<td>7.98</td>
<td>4.80</td>
<td>8.21</td>
</tr>
<tr>
<td>50</td>
<td>8.82</td>
<td>4.20</td>
<td>7.82</td>
</tr>
<tr>
<td>60</td>
<td>10.17</td>
<td>3.60</td>
<td>6.89</td>
</tr>
<tr>
<td>70</td>
<td>11.71</td>
<td>3.00</td>
<td>5.46</td>
</tr>
</tbody>
</table>

In this equation, the discount factor for both parties has been regarded equally \( w_1 = w_2 = 0.5 \). Observing Table 2, the objective function will maximize in 30% of time. Therefore, this agreement between the owner and contractor will occur within an extended 75 days.

It is essential to note that according to the previous assumption which owner withdraws the delay fine due to successful agreement in bargaining, the values of contractor’s total cost in Tables 2–4 should be obtained from the last column of Table 1.

Observing Table 3 and if the bargaining power of the owner and contractor is considered 0.8 and 0.2, respectively, the payoffs of the owner and contractor will be determined through an almost continuous strategy. According to this table, the maximum objective function occurs in 50% of the time. Thus, the agreement between the owner and contractor will be reached with an extension of 105 days.

Table 3. Harsanyi–Selten objective function value (with parties’ different bargaining power)

<table>
<thead>
<tr>
<th>Compensable time (%)</th>
<th>Contractor’s total cost (million $)</th>
<th>Owner’s total cost (million $)</th>
<th>Harsanyi–Selten objective function value</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>7.13</td>
<td>5.40</td>
<td>9.72</td>
</tr>
<tr>
<td>40</td>
<td>7.98</td>
<td>4.80</td>
<td>9.89</td>
</tr>
<tr>
<td>50</td>
<td>8.82</td>
<td>4.20</td>
<td>10.01</td>
</tr>
<tr>
<td>60</td>
<td>10.17</td>
<td>3.60</td>
<td>9.80</td>
</tr>
<tr>
<td>70</td>
<td>11.71</td>
<td>3.00</td>
<td>9.19</td>
</tr>
</tbody>
</table>

The objective function, related to the solution of Kalai–Smorodinsky (Eqn (34)), was considered by subtraction of both sides of the above-mentioned equation (we name it Kalai–Smorodinsky objective function), and the answer of bargaining in this solution must be equal to Zero:

\[ F(u) = (x_1 - d_1) (x_2 - d_2) - (x_2 - d_2) (x_1 - d_1). \]  

(34)

Setting Table 4, the owner and contractor’s payoffs should be determined through a relatively continuous strategy. The fourth column of this table shows the objective function value of Kalai-Smorodinsky which has been obtained by Eqn (34). Observing and interpolating

Table 4. Kalai–Smorodinsky objective function value against the compensable time percentage

<table>
<thead>
<tr>
<th>Compensable time (%)</th>
<th>Contractor’s total cost (million $)</th>
<th>Owner’s total cost (million $)</th>
<th>Kalai-smorodinsky objective function value</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>7.13</td>
<td>5.40</td>
<td>16.88</td>
</tr>
<tr>
<td>40</td>
<td>7.98</td>
<td>4.80</td>
<td>-0.50</td>
</tr>
<tr>
<td>50</td>
<td>8.82</td>
<td>4.20</td>
<td>-17.74</td>
</tr>
<tr>
<td>60</td>
<td>10.17</td>
<td>3.60</td>
<td>-42.52</td>
</tr>
<tr>
<td>70</td>
<td>11.71</td>
<td>3.00</td>
<td>-70.11</td>
</tr>
</tbody>
</table>
this table, the agreement between the owner and the contractor will be reached in 39% of the compensable time and with extension of 89 days.

**Conclusions**

Delays in construction projects may occur frequently. Current research provides a game theory model in order to clarify the interactions between parties in a delayed DBB project. The project was studied according to the general conditions of DBB contracts in Iran and based on the comments received from construction industry experts in this country. Different scenarios were devised, and then, different parameters which affected different parties’ costs were applied in the model. Finally, through step by step analysis, possible measures to facilitate parties’ compromise and delay reduction were presented.

The model enables owners and contractors to analyze their situation and their strategies correctly while understanding delay more deeply. Applying the model, it can be found that the parameters involved in the problem can affect negotiation in a wider range. It is shown that, the more an owner is capable of bargaining, the negotiation interval will extend toward more compensation time and when the interval reduces, large compensation time will be invalid. Moreover, if the delay fine does not match other parameters of the project, it cannot be effective in the result of the negotiation. From the owner’s eyes it means when there is a huge delay fine, the only criterion is the comparison of the payoff of agreement with that of cancellation or termination and when it is small, the contractor is not worried about the delay. Thus, the delay fine is not effective in negotiation range. However, choosing a proper delay fine can affect the negotiation range so that the owner with no threat and the contractor voluntarily can reach an agreement on compensation of delay with a logical time and cost.

With exact examination of the numerical example of this research, it can be seen that by increase of owner’s bargaining power, the agreement point moves toward shorter extension time and vice versa. Also, recommendations were offered for both parties’ strategies that empower them to reach reasonable and realistic results without investing more time. However, some assumptions are provided in the model which can help other researchers to develop the model more extensively. The suggestions presented in this article can be investigated in future studies as a case study.

**References**


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