PROBABILITY-BASED DESIGN OF SPUN CONCRETE BEAM-COLUMNS

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Abstract. The attention of engineers is turned to the application of precast spun concrete columns reinforced by high-strength steel bars for office and administrative buildings. The paper discusses a possibility of using the reliability index approach to designing beam-columns of building frames and analyses resistance criteria for beam-columns of annular cross sections as compression members with bending moments and bending members with compressive forces. First and second order effects of beam-columns are considered. The article also presents time-dependent resistance safety margin and its stationary equivalent and investigates the unsophisticated applied models for probability-based design of frame beam-columns. The design of a beam-column of the braced frames is illustrated providing a numerical example.

Keywords: spun concrete, beam-columns, frames, second order effects, resistance safety margin, reliability index design.

1. Introduction

The economically and structurally rational precast spun concrete columns reinforced by longitudinal high-strength steel bars uniformly distributed throughout their annular cross sections may be treated as sustained beam-columns of office and administrative buildings (Kudzys and Kliukas 2009). High-strength reinforcing steel bars increase the ductility and capacity of eccentrically loaded columns due to the redistribution of ultimate compressive stresses of steel and concrete components of beam-columns (MacGregor 1988; Kudzys et al. 1993; Kliukas et al. 2010; Židonis 2009; Juocevičius and Vaidogas 2010).

The Standards EN 1990 (2002) in Europe and ASCE/SEI 7-05 (2006) in the USA require that the load carrying structures of buildings shall be designed with the appropriate degrees of reliability. However, these standards are based on limit state concepts and respectively on the methods of partial safety factors design and load and resistance factors design. Therefore, some contradictions in reliability approaches presented in these Standards and the International Standard ISO 2394 (1998) on the reliability for structures exist. Practically, the reliability degree of load-carrying structures designed by various native semi-probabilistic and full-probabilistic limit state concepts can be markedly different in their values. This difference depends on the conditionality of mechanical and statistical uncertainties evaluated and integrated in design models.

The quantitative reliability indices of particular members (sections) and structural members (columns, beams) may be objectively defined and predicted only by full-probabilistic approaches and models. However, the recommendations and directions devoted to the issues of design features of probability-based approaches presented in the design codes of reinforced concrete structures Eurocode (EN 1992-1 2004) and ACI Committee 318-05 (2005) are not fully formulated. Therefore, any possibility of engineering and wish for an objective prediction of the probabilistic parameters of building structures, including beam-columns, are rather hardly interpreted and used in design practice (Vaidogas and Juocevičius 2008; Jankovski and Atkovičius 2008).

In spite of the fairly developed concepts of probabilistic reliability design, it is difficult to apply probability-based approaches in structural safety predictions both of complicated and not complicated members and their systems. These approaches may be acceptable to building engineers only under the indispensable condition that the probabilistic performances of members may be considered in design practice using unsophisticated and easy perceptible manners.

The main task of this paper is to present new methodological formats on the probability-based reliability predictions of the beam-columns of reinforced concrete frames subjected to recurrent single and joint extreme service and climate actions. The paper considers the possibility of using reliability index design approaches based on the transformed conditional intersection in engineering practice and failure probabilities of members and the stationary processes of their resistance safety margins using equivalent recurrent extreme service actions instead of their sustained and extraordinary components.
2. Resisting Compressive Forces and Bending Moments

2.1. Compression with a Bending Moment

Beam-columns are represented in design state taking into account their particular members (normal sections). Therefore, the survival probabilities of beam-columns may be objectively assessed and predicted only knowing mechanical and statistical parameters of their normal sections.

Modelling the stress-strain state of spun concrete beam-columns must assess the structural features of annular cross sections reinforced by high-strength steel bars. According to the interaction diagram $N_R - M_R$ of eccentrically compressed spun concrete members, segments AB and B–C characterize their fully and partially compressed cross sections (Fig. 1).

When the action effects of building structures are provoked only by gravity loads, beam-columns are under compression with a small bending moment. Modelling strain and stress distributions in concrete and high-strength steel bars and the bearing capacity of eccentrically loaded sections may be based on a plain cross-section hypothesis and bi-linear concrete strain-stress relation (Fig. 2).

When eccentricity ratio $e/r_s \leq 1.0$, the ultimate internal resisting compressive force of annular cross sections may be expressed as

$$ R_N = N_R = (k_c f_{cc} A_c + k_s \sigma_{sc} A_s) r_s / (e + r_s), \quad (1) $$

where the response factors characterizing contributions of concrete and steel components to the ultimate resistance of beam-columns are defined as

$$ k_c = 1 - (0.3 e / r_s) / (1 + 10 \rho), \quad (2) $$

$$ k_s = 1 - 0.34 e / r_s, \quad (3) $$

$$ f_{cc} = \alpha_c \alpha_2 f_c = (1 - 0.1 N_{EP}/N_E) (0.85 - 1.7 \rho) f_c, \quad (4) $$
is the compressive strength of spun concrete in beam-columns the cylinder strength of which is $f_c$; $N_E$ and $N_{EP}$ are the applied total force and its quasi-permanent component;

$$ \sigma_{sc} = 452(1.36 + 4 \rho) \text{ MPa}, \quad (5) $$
is ultimate compressive stress in reinforcing bars; $\rho = A_s / A_c$ is reinforcement ratio; $A_c$, $A_s$ and $r_s$ are geometrical parameters taken from Fig. 2.

The mean and variance of response $R_N$ by Eq. (1) are

$$ R_{Nm} = (k_{cm} f_{ccm} A_{cm} + k_{sm} \sigma_{scm} A_s) r_s / (e_m + r_s), \quad (6) $$

$$ \sigma^2 R_N = \left( \frac{\partial R_N}{\partial f_{cc}} \right)^2 \sigma^2 f_{cc} + \left( \frac{\partial R_N}{\partial A_c} \right)^2 \sigma^2 A_c + \left( \frac{\partial R_N}{\partial A_s} \right)^2 \sigma^2 A_s + \left( \frac{\partial R_N}{\partial e} \right)^2 \sigma^2 e + \left( \frac{\partial R_N}{\partial \rho} \right)^2 \sigma^2 \rho + 2 \left[ \left( \frac{\partial R_N}{\partial A_c} \right) \frac{\partial R_N}{\partial A_s} \right] \sigma A_c \times \sigma A_s + \left[ \left( \frac{\partial R_N}{\partial \sigma_{sc}} \right) \frac{\partial R_N}{\partial \rho} \right] \sigma \sigma_{sc} \times \sigma \rho, \quad (7) $$

where $k_{cm}$, $k_{sm}$, $f_{ccm}$ and $\sigma_{scm}$ are taken from Eqs. (2)–(5);

$$ \delta f_c = 0.088 + 0.00003(70 - f_c)^2, \quad (8) $$

$$ \delta f_{cc} = \delta^2 f_c + 0.082 \delta f_{cc} = \left( \delta f_{cc} \times f_{ccm} \right)^2, \quad (9) $$

$$ \delta A \approx \delta l = (1.2 - r_s) / 150(r_2 - r_1), \quad \sigma^2 A_c = (\delta A \times A_{cm})^2, \quad (10) $$

$$ \sigma^2 \sigma_{sc} = (\delta \sigma_{sc} \times \sigma_{scm})^2 = (0.105 \times \sigma_{scm})^2, \quad (11) $$

![Fig. 1. Interaction $N_R - M_R$ diagram of eccentrically loaded annular cross sections](image1)

![Fig. 2. Modelling strain and stress distributions in concrete and high strength steel bars based on a plain cross-section hypothesis](image2)
It must be noted that the coefficient of variation in spun concrete strength in thin-walled members increases and is equal to $\delta f_{cc} = \left(\delta^2 f_c + \delta^2 f_{cr}\right)^{1/2}$ where components $\delta f_c$ and $\delta f_{cr}$ indicate the effects of concrete mix quality and reinforcement presence on its value.

2.2. Bending with Compressive Forces

According to Vadlūga (1979), the ultimate internal resisting bending moment $M_R$ of the beam-columns of annular cross sections (Fig. 3) may be expressed and presented as

$$R_M = M_R = 1.2 r_i \left( A_s f_{st} + N_E \right) \times \left[ 1 - \frac{A_s f_{st} + N_E}{A_s f_{cc} + A_s \left( f_{st} + f_{sc} \right)} \right]. \quad (13)$$

For design practice, the mean of this resistance may be written in the form

$$R_{Mm} = T_2m T_3m / T_1m,$$  \quad (14)

where

$$T_1m = A_{cm} f_{ccm} + A_s \left( f_{sm} + f_{scm} \right), \quad (15)$$

$$T_2m = 1.2 r_i \left( A_s f_{sm} + N_{Em} \right), \quad (16)$$

$$T_3m = A_{cm} f_{ccm} + A_s f_{scm} - N_{Em}, \quad (17)$$

when $f_{ccm}$ is defined from Eq. (4); $f_{sm} = 500$ MPa, $f_{scm} = 600$ MPa; $N_{Em}$ joint compressive force provoked by service and climate actions.

The variance of resistance by Eq. (13) is expressed as

$$\sigma^2 R_M = \left[ \frac{T_2m(T_1m - T_3m)}{T_1m} \right]^2 \left( k_{cm}^2 \sigma^2 f_{cc} + \left( f_{ccm} \sigma A_c \right)^2 + \right.$$  

$$A_s^2 \sigma^2 f_{sc} + \left[ \frac{A_s T_3m(1.2 r_i T_1m - T_{2m})}{T_1m} \right]^2 \sigma^2 f_{st} +$$

$$\left[ \frac{1.2 r_i T_3m - T_{2m}}{T_1m} \right]^2 \sigma^2 N_E, \quad (18)$$

where $k_{cm}$, $k_{sm}$, $f_{ccm}$ and $\sigma_{scm}$ are taken from Eqs. (2)–(5); $\sigma^2 f_{cc}$ by Eq. (9); $\sigma^2 A_c$ by Eq. (10);

$$\sigma^2 f_{st} = \left( \delta f_s \times f_{sm} \right)^2, \quad \sigma^2 f_{sc} = \left( \delta f_s \times f_{scm} \right)^2. \quad (19)$$

where $\delta f_s = \left( \delta^2 f_s + \delta^2 f_{sc} \right)^{1/2}$ when components $\delta f_s = 0.09$ and $\delta f_{sc} = 0.12$ define statistical deviations and uncertainties of right-angled epures of steel bar stresses.

3. First and Second Order Effects

The first order mean value of the eccentricity of the applied total compressive force $N_{Em}$ is equal to

$$e_{om} = M_{OEm} / N_{Em} \quad (20)$$

but not less as $r_2 / 15$ and $20$ mm (EN 1992-1 2004).

The second order eccentricity of this force may be expressed as a magnification of the first order eccentricity resulting from linear-elastic analysis with redistribution in which the internal moments are modified with external actions and without a more explicit calculation of rotation capacity (EN 1990 2002; Kargaudas and Adamukaitis 2010).

According to EN 1992-1 (2004), the additional action effects of reinforced concrete beam-columns may be estimated by the flexural stiffness method. This stiffness of slender compression elements with constant cross sections may be represented as

$$(EI) = K_c E_c I_c + E_s I_s, \quad (21)$$

where

$$K_c = 0.25(1 + \varnothing M_{OG}/M_{OE}), \quad (22)$$

is the factor for effects of cracking and creep deformations on the overall behaviour of members; $M_{OG}$ and $M_{OE}$ are bending moments caused by permanent and total actions; $\varnothing = 1.2–2.0$ is the basic creep coefficient of concrete the value of which depends on its strength class and dimensions of a cross section of members; $E_c$, $E_s$ and $I_c$, $I_s$ are the tangent module of elasticity and the second moments of the areas of concrete and steel bars.

The mean and variance of flexural stiffness by Eq. (21) may be expressed as

$$\langle (EI) \rangle_m = K_{cm} E_{cm} I_{cm} + E_s I_s, \quad (23)$$
\[
\sigma^2(EI) = (K_{cm}E_{cm})^2 \sigma^2 I_c + (K_{cm}E_{cm})^2 \sigma^2 E_c + \\
(E_{cm}E_{cm})^2 \sigma^2 K_c,
\]
where
\[
K_{cm} = 0.25/[1 + (\varnothing M_{OGm}/M_{OEM})],
\]
\[
\sigma^2 K_c = \left[0.25\varnothing M_{OGm}/(M_{OEM} + \varnothing M_{OGm})\right]^2 \times
\]
\[
\left(\sigma^2 N_G + \sigma^2 N_E\right),
\]
\[
E_{cm} = 20(0.1f_{cm})^{0.3},
\]
\[
\sigma^2 E_c = (0.15E_{cm})^2,
\]
\[
I_{cm} = \pi \left(\frac{a}{4} - \frac{n_m}{4}\right)^4,
\]
\[
\sigma^2 I_c = [I_{cm}^2]^{12} = 1/[150(\varnothing I - n_m)],
\]
\[
I_c = \pi \left(\frac{a}{4} - \frac{\varnothing I}{4}\right)^4.
\]

Buckling resistance \(N_B = \pi^2 (EI)/l_c^2\) can be used as a mechanical parameter in the second order analysis of members the mean and variance of which are
\[
N_{Bm} = \pi^2 (EI)/l_{cm}^2,
\]
\[
\sigma^2 N_B = \pi^2 \sigma^2 (EI)/l_{cm}^2 + \left[2\pi^2 (EI)/l_{cm}^2 \right] \sigma^2 l_{cm}, \quad (33)
\]
where \((EI)_m\) and \(\sigma^2 (EI)\) are defined by Eqs. (23) and (24); \(l_{cm} = 0.75h_s\) and \(l_{cm} = h_s\) are the effective length of the columns of buildings with in situ and precast floor beams; \(h_s\) is a storey height of a building.

For beam-columns without transverse loads, the second order eccentricity may be defined by the equation
\[
e = e_0 \left[1 + \frac{\pi^2/c}{\left(N_B/N_E - 1\right)}\right],
\]
where parameter \(c = 8-12\) depends on the distribution of moments and \(\pi^2/c\) is normally a reasonable simplification. Therefore, the mean and variance of this eccentricity are
\[
e = e_0 \left[1 - \frac{1}{1 - N_{Em}/N_{Bm}}\right],
\]
\[
\sigma^2 e = \frac{\left[N_{Em}e_0\right]^2}{\left(N_{Bm} - N_{Em}\right)^2} \sigma^2 N_B + \\
\frac{\left[N_{Em}e_0\right]^2}{\left(N_{Bm} - N_{Em}\right)^2} \sigma^2 N_E,
\]
where \(N_{Bm}, \sigma^2 N_B\) and \(N_{Em}, \sigma^2 N_E\) are the statistics of buckling resistance and the total compressive force of a beam-column.

Design limit states of beam-columns include their loss of equilibrium and large deformations leading them to the second order effects. The model of the structural response of beam-columns reinforced by high-strength steel bars may be based on linear-elastic (Fig. 2) and non-linear (plastic) (Fig. 3) resistances.

4. Resistance Safety Margins

According to probability-based approaches, the resistance safety margin of the particular members of the beam-columns of buildings is a non-stationary resistance performance process presented as:
\[
Z(t) = g[X(t), \theta] = \theta R - \theta G - \theta E, \quad (37)
\]
where \(X(t)\) and \(\theta\) are the random vectors of basic and additional variables representing mechanical parameters and their model uncertainties. These parameters include resistance \(R\) by Eq. 1 or \(R\) by Eq. (13) and action effects \(E\), \(Q\), \(Q\), \(E\), \(c\) caused by permanent, \(G\), floor sustained, \(Q\), and extraordinary. \(Q\), live loads and climate actions (snow, \(S\), wind, \(W\), loads) (Fig. 4).

Live floor loads vary in time and space in a random manner (JCSS 2000). The sustained part of these loads in civil and engineering buildings contains the weight of furniture and heavy equipments respectively. An intermittent extraordinary load represents all kinds of live floor loads which are not covered by the sustained load. It is caused by the crowded rooms during special events and by mobile equipment during processes in civil and engineering buildings respectively.

The probability distribution of resistance, \(R\), and permanent action effect, \(E\), of reinforced concrete members is close to normal distribution (Ramsay et al. 1979; Ballung 1979; Ellingwood 1981; ISO 2394 1988; EN 1990 2002; EN 1992-1 2004; JCSS 2000). According to these international standards, Gaussian or lognormal distributions may be assumed for sustained actions and Weibull, gamma or exponential distributions may be used for extraordinary actions representing non-stationary processes.

Analogically to traditional standard approaches of design codes, the probability-based design needs simplifications of safety margin processes of particular members.

The annular extreme sum of sustained and extraordinary live action effects may be modeled as a stationary rectangular wave renewal process described by Type 1 (Gumbel) distribution (Rosowsky and Ellingwood 1992). The mean and coefficient of variation as well as variance in this process may be defined as
\[
E_{Qm} = E_{Qm}(t_1) + E_{Qm}(t_2) = E_{Qm}(t_1) + E_{Qm}(t_2) = \\
\frac{E_{Q,k}}{1 + k_{0.95}\delta E_{Q_k}} + \frac{E_{Q,k}}{1 + k_{0.95}\delta E_{Q_k}}, \quad (38)
\]
\[ \delta E_Q = \frac{\sigma E_Q}{E_{Qm}} = \left( \frac{\sigma^2 E_{Q_1} + \sigma^2 E_{Q_2}}{E_{Qm}} \right)^{1/2} \]

\[ \left[ \delta E_{Q_1} \times E_{Q_1,m} \right]^2 + \left[ \delta E_{Q_2} \times E_{Q_2,m} \right]^2 \]

\[ \frac{\sigma^2 E_Q = \left( \delta E_Q \times E_{Qm} \right)^2}{E_{Qm}} \]

where \( E_{Q,k} \) and \( E_{Q,k} \) are the characteristic (nominal) values of live load components; \( k_{0.95} \) and \( k_{0.95} \) are the characteristic fractile of their probability distributions; \( \delta E_{Q_1} \) and \( \delta E_{Q_2} \) are the coefficients of variation in these components.

When \( E_{Q,m} = 18.55 \text{kN}, \delta E_{Q_1} = 0.9, \delta E_{Q_2} = 0.5, \) \( E_{Q,m} = 18.55 + 12.0 = 30.55 \text{kN} \) and \( E_{Q,k} = 65 \text{kN}, \) then according to Eq. (39), the coefficient of variation in the stationary extreme floor action, \( E_Q, \) is equal to

\[ \delta E_Q = \left[ (18.55 \times 0.9)^2 + (12.0 \times 0.5)^2 \right]^{1/2} / 30.55 = 0.58. \]

Values \( E_{Q,m} = 0.47 E_{Q,k} \) and \( \delta E_Q = 0.58 \) were suggested by Rosowsky and Ellingwood (1992).

Gumbel cumulative distribution is quite appropriate for the annual extreme snow and wind loads (Ellingwood 1981; ICSS 2000; Vrouwenvelde 2002). The mean values, coefficients of variation and variances in these load effects may be expressed as

\[ E_{sm} = E_{sk} \left[ 1 + k_{0.98} \delta E_s \right], \delta E_s = 0.3 - 0.7, \]

\[ E_{wm} = E_{wk} \left[ 1 + k_{0.98} \delta E_w \right], \delta E_w = 0.2 - 0.5, \]

\[ \sigma^2 E_s = (\delta E_s \times E_{sm})^2, \]

\[ \sigma^2 E_w = (\delta E_w \times E_{wm})^2. \]

Besides, a coincidence of annual extreme wind and snow loads is impossible. However, the recurrence number of joint extreme floor and climate action effects during the working life \( t_a \) of beam-columns may be defined as

\[ n_{Q,c} = n_a d_Q + d_{cl} \lambda_{cl}, \]

where \( d_Q = 1 - 3 \) days, \( d_{cl} = d_s = 14 - 28 \) days and \( d_{cl} = d_w = 8 - 12 \) hours are the durations of extreme action effects; \( \lambda_Q = 1 \) and \( \lambda_{cl} = 1 \) are their renewal rates.

Therefore, \( n_{Q,c} = 2.06 - 4.25 \) and \( n_{Q,c} = 0.18 - 0.48. \)

Thus, extreme events arise from extraordinary floor and climate conditions that usually are not considered explicitly in the limit state design of buildings. The mean and variance of stationary bivariate annual extreme action effects are

\[ E_{cm} = E_{Qm} + E_{Sm} \text{ or } E_{cm} = E_{Qm} + E_{Wm}, \]

\[ \sigma^2 E_c = \sigma^2 E_Q + \sigma^2 E_S \text{ or } \sigma^2 E_c = \sigma^2 E_Q + \sigma^2 E_W. \]

The combination of short episodic annual extreme action effects of beam-columns belongs to exceptional
The instantaneous survival probability of beam-columns may be represented by the convolution integral as

$$ z_n = F_n(x) = \int_0^\infty F_n(x-y) f(y) dy $$

where $F_n(x)$ is defined by Eq. (58), $P_n$ is the probability of a series system the bounded index of which is $n$, and $\sigma^2 F_n$ is the variance of system $S_n$. The system $S_n$ is characterized by these elements. Therefore, the resistance failure probability of a beam-column $R$ is given by Eq. (54) or (42).

$$ P(R < r) = \int_0^r f_R(x) dx $$

where $f_R(x)$ is the density function of the conventional resistance.

$$ P(R < r) = \int_0^r f_R(x) dx $$

The cumulative distribution function for the conventional resistance is defined by Eqs. (54) and (55). The variables $E_\sigma$ and $E_\theta$ are formed conditional probabilities (Kudzys et al. 1997), the variable $\sigma E_\sigma$ and $E_\theta$ are used to avoid the complicated intersections of recurrent survival probabilities characterized by these elements. Therefore, the estimation of the long term safety of structures is connected with some methodological and mathematical improvements. Improved computational methods are based on the extended probabilistic approaches (Bulte and Higashi 1997) the variable $\sigma E_\sigma$ and $E_\theta$ are used to avoid the complicated intersections of recurrent survival probabilities characterized by these elements. Therefore, the estimation of the long term safety of structures is connected with some methodological and mathematical improvements.

The variable $\sigma E_\sigma$ and $E_\theta$ are used to avoid the complicated intersections of recurrent survival probabilities characterized by these elements. Therefore, the estimation of the long term safety of structures is connected with some methodological and mathematical improvements.
The prediction of the probabilistic reliabilities of beam-columns should be based on the reliability index approach. Their generalized reliability index may be introduced as

$$\beta(t \geq t_n) = \Phi^{-1}[P(T \geq t_n)],$$

(63)

where \(\Phi(*)\) is the cumulative distribution function of standard normal distribution tabulated in statistic texts; \(P(T \geq t_n)\) is probability defined by Eq. (60). For persistent design situations during \(t_n = \) 50 years, the target value of reliability index \(\beta_p(T \geq t_n)\) for the beam-columns of residential, office and public buildings is equal to 3.8 (EN 1990 2002).

When the action effects of beam-columns are provoked only by permanent loads, \(G\), the resistance safety margin of these structural members may be expressed as \(Z = R_C = \theta_R R - \theta_G E_G\). In this case, the reliability index of beam-columns may be defined as

$$\beta_G = \Phi^{-1}[P(R_c > 0)] = R_{Gm}/\sigma_{Rc},$$

(64)

where the statistics of their conventional resistance, \(R_c\), is defined by Eqs. (51) and (52).

6. Numerical Illustration

6.1. The Parameters of Analysis

Consider the reliability indices \(\beta_N\) and \(\beta_M\) of the spun concrete beam-column of the braced two-storied frames of Reliability Class RC2 (Fig. 5) designed by directions EN 1990 (2002) and ASCE/SEI 7-05 (2006), (Kudzys and Kliukiwas 2009).

The means and variances of compressive forces and the first order bending moments are as follows:

\[N_{Gm} = N_{Gk} = 612\; \text{kN}, \; \sigma^2N_G = (0.1 \times 612)^2 = 3745\; (\text{kN})^2;\]

\[M_{OGm} = 28.8\; \text{kNm}, \; \sigma^2M_{OG} = (0.1 \times 28.8)^2 = 8.29\; (\text{kNm})^2;\]

\[N_{Qm} = 0.47N_{Qk} = 30.55\; \text{kN}, \; \sigma^2N_Q = (0.58 \times 30.55)^2 = 314\; (\text{kNm})^2;\]

\[M_{Qm} = 7.64\; \text{kNm}, \; \sigma^2M_{Q} = (0.58 \times 7.64)^2 = 19.64\; (\text{kNm})^2;\]

\[N_{Sm} = N_{Sk}/(1 + k_{0.9}\delta N_S) = 12.19\; \text{kN};\]

\[\sigma^2N_S = (0.5 \times 12.19)^2 = 37.2\; (\text{kN})^2;\]

\[M_{Qsm} = 1.52\; \text{kNm}, \; \sigma^2M_{Qs} = (0.5 \times 1.52)^2 = 0.58\; (\text{kNm})^2;\]

\[N_{Em} = 612 + 30.55 + 12.2 = 654.75\; \text{kN}, \; \sigma^2N_E = 3745 + 314 + 37.2 = 4096.2\; (\text{kN})^2;\]

\[M_{QEm} = 28.8 + 7.64 + 1.52 = 37.96\; \text{kNm}, \; \sigma^2M_{OE} = 8.29 + 19.64 + 0.58 = 28.51\; (\text{kNm})^2.\]

The geometrical parameters of the beam-column can be expressed as

\[l_{om} = 3.0\; \text{m}, \; \bar{d}_{l_o} = 0.1, \; \sigma^2l_o = (0.1 \times 3.0) = 0.09\; \text{m}^2;\]

\[r_2 = 0.15\; \text{m}, \; r_{im} = 0.09\; \text{m}, \; r_s = 0.12\; \text{m}, \; A_m = 0.04524\; \text{m}^2;\]

\[A_S = 0.00181\; \text{m}^2 (162 \times 12), \; A_{cm} = 0.0434\; \text{m}^2;\]

\[\rho_m = 0.0417, \; I_m = \pi (r_2^4 - r_{im}^4)/4 = 346 \times 10^{-6}\; \text{m}^4;\]

\[I_s = \pi (r_s^4 - r_{im}^4)/4 = 13 \times 10^{-6}\; \text{m}^4;\]

\[\bar{d}A_c \approx \bar{d}I = (1.2 - r_2)/[150(r_2 - r_{im})] = 0.1167;\]

\[\sigma^2A_c = (0.1167 \times 0.0434)^2 = 25.65 \times 10^{-6}\; \text{m}^4;\]

\[\sigma^2I = (0.1167 \times 346 \times 10^{-6})^2 = 0.00163 \times 10^{-6}\; \text{m}^8;\]

\[\sigma^2\rho = (A_s/A_{cm})^2 \sigma^2A_c = 24.65 \times 10^{-6}.\]

The parameters of spun concrete C50/60 are given by \(f_{ck} = 50\; \text{MPa}, \; \alpha_3 = 0.85 - 1.7\rho_m = 0.779,\)

\[\alpha_{cm} = 1 - 0.1N_{Gm}/N_{Em} = 0.9065, \; f_{cm} = 58\; \text{MPa}.\]

![Fig. 5. Compressive forces \(N_G, N_Q\) and \(N_S\) caused by permanent (a) and variable extreme floor (b) or snow (c) loads](image-url)
\[ \delta_{f,c} = 0.088 + 3(70 - f_{\text{cr}})^2 \times 10^{-5} = 0.10, \]
\[ \delta_{f,c} = (0.10^2 + 0.085)^{1/2} = 0.128; \]
\[ f_{\text{cr}} = \alpha_{\text{ccm}} f_{\text{cm}} = 40.96 \text{ MPa}, \]
\[ \sigma^2 f_{\text{cr}} = (\delta f_{\text{cr}})^2 + f_{\text{cm}}^2 = 27.49 \text{ (MPa)}^2; \]
\[ E_{\text{cr}} = 20(0.1 f_{\text{cm}})^{3/2} = 33.89 \text{ GPa}, \quad \sigma^2 E_{\text{c}} = (0.15 \times 33.89)^2 = 25.84 \text{ (GPa)}^2. \]

The parameters of reinforcing high-strength cold worked bars are
\[ f_{0.2k} = 800 \text{ MPa}, \quad \sigma_{\text{sm}} = 452(1.36 + 4\rho_m) = 690 \text{ MPa}, \]
\[ \sigma^2 \sigma_{\text{sm}} = (0.105 \times 690)^2 = 5249 \text{ (MPa)}, \]
\[ \delta_{f_{0.2}} = (0.09^2 + 0.12^2) = 0.15, \quad f_{\text{sm}} = 500 \text{ MPa}, \]
\[ \sigma^2 f_{\text{sm}} = (0.15 \times 500)^2 = 5625 \text{ (MPa)}^2, \quad f_{\text{sm}} = 600 \text{ MPa}, \]
\[ \sigma^2 f_{\text{sm}} = (0.15 \times 600)^2 = 8100 \text{ (MPa)}^2. \]

The statistics of additional random variables are:
\[ \theta_{N_m} = \theta_{N_m} = 1.0 \text{ and } \sigma_{\theta_{N}} = 0.05, \quad \sigma_{\theta_{N}} = 0.10 \text{ for action effects and } \theta_{K_m} = 0.99, \quad \sigma_{\theta_{K}} = 0.08 \text{ and } \theta_{K_m} = 1.0, \quad \sigma_{\theta_{K}} = 0.14 \text{ for the resistance of the compression and flexural members of the analysed annular cross sections.} \]

### 6.2. Second Order Effects

According to Eqs. (25) and (26), the mean and variance of the stiffness factor are
\[ K_{\text{cm}} = 0.25/(1 + 1.7 \times 28.8/37.96) = 0.1092, \]
\[ \sigma^2 K = \left[ \frac{0.25 \times 1.7 \times 28.8}{(37.96 + 1.7 \times 28.8)^2} \right] \times (8.29 + 28.51) = 9659 \times 10^{-8}. \]
Therefore, the statistics of the effective flexural stiffness of a beam column considering Eqs. (23) and (24) are
\[ (EI)_{\text{m}} = 0.1092 \times 3.389 \times 10^4 \times 3.46 \times 10^{-4} + 2 \times 10^3 \times 1.3 \times 10^{-5} = 3.88 \text{ MNm}^2. \]
\[ \sigma^2 (EI) = (0.1092 \times 3.389 \times 10^4)^2 \times 0.163 \times 10^8 + (0.1092 \times 3.46 \times 10^{-5})^2 \times 0.2584 \times 10^8 \]
\[ (3.389 \times 10^4 \times 3.46 \times 10^{-4})^2 \times 9659 \times 10^{-8} = 0.07247 \text{ (MNm}^2). \]

According to Eqs. (32) and (33), the mean and variance of buckling load are
\[ N_{\text{bm}} = \pi^2 \times 3.88/3.0^2 = 4.255 \text{ MN}, \]
\[ \sigma^2 N_B = \pi^2 \times 0.07247/3.0^2 + \left(2\pi^2 \times 3.88/3.0^2 \right) \times 0.09 = 0.8037 \text{ (MN)}^2. \]

The first order mean value of the eccentricity of joint compressive force is
\[ e_{\text{cm}} = 37.96/654.75 = 0.058 \text{ m}. \]
Thus, the statistics of the second order eccentricity according to Eqs. (35) and (36) are
\[ e_m = 0.058 \times \left[ \frac{1}{(1.0 - 0.6547/4.255)} \right] = 0.06855 \text{ m}, \]
\[ \sigma^2 e = \left[ \frac{0.6547 \times 0.058}{(4.255 - 0.6547)} \right] = 0.06855 \text{ m.} \]

### 6.3. Beam-Column as a Compression Member

The mean values of the response factors of member components taking into account Eqs. (2) and (3) are
\[ k_{\text{cm}} = 1.0 \times 0.06855/(0.12 \times 1.417) = 0.8791, \]
\[ k_{\text{cm}} = 1.0 \times 0.06855/(0.12 \times 1.417) = 0.8058. \]

Then, according to Eqs. (6) and (7), the mean and variance of beam-column resistance are
\[ R_{\text{Nnm}} = R_{\text{Nm}} = (0.8791 \times 0.0434 \times 40.96 + 0.8058 \times 0.00181 \times 690)/0.12 = 0.0085/(0.12 \times 1.417) = 0.8791, \]
\[ k_{\text{cm}} = 1.0 \times 0.06855/(0.12 \times 1.417) = 0.8058. \]

According to Eqs. (51)–(54), the means and variances of conventional resistance, \( R_{\text{CN}} \), and extreme action effects are
\[ R_{\text{CNm}} = 0.99 \times 1.6351 - 1.0 \times 0.612 = 1.0067 \text{ MN}, \]
\[ \sigma^2 R_{\text{CN}} = 0.99^2 \times 0.04122 + 1.6351 \times 0.0064 + 1.0 \times 0.3745 \times 1.0 \times 0.612 + 0.03055 = 0.06222 \text{ (MN)}^2; \]
\[ N_{\text{om}} = 1.0 \times 0.03055 = 0.03055 \text{ MN}, \]
\[ \sigma^2 N_{\text{m}} = 1.0 \times 3.14 \times 10^{-6} + 0.03055 \times 0.0025 = 316 \times 10^{-6} \text{ (MN)}^2; \]
\[ N_{\text{sm}} = 1.0 \times 0.0122 = 0.0122 \text{ MN}, \]
\[ \sigma^2 N_{\text{m}} = 1.0 \times 3.72 \times 10^{-6} + 0.0122 \times 0.0025 = 38 \times 10^{-6} \text{ (MN)}^2; \]
\[ N_{\text{Qm}} = 0.03055 \times 0.0122 = 0.04275 \text{ MN}, \]
\[ \sigma^2 N_{\text{Qm}} = 316 \times 10^{-6} + 38 \times 10^{-6} = 354 \times 10^{-6} \text{ (MN)}^2. \]
According to Eq. (62), the coefficients of the correlation of the cuts of safety margins $Z_{QN}$, $Z_{SN}$ and $Z_{QSM}$ are $\rho_Q = 0.99495$, $\rho_S = 0.99944$ and $\rho_{QS} = 0.99434$.

The reliability parameters of beam-columns are presented in Table 1. Because coefficients $\rho_Q \approx \rho_S \approx \rho_{QS} \approx 1$, index $\beta_N = 3.78$ may be treated as the basic parameter the value of which is close to target reliability index $\beta_T = 3.80$ (EN 1990 2002).

### 6.4. Beam-Column as a Bending Member

According to Eqs. (15), (16) and (17), the means of bending resistance components are

$$
T_{lm} = 0.0434 \times 40.96 + 0.0181(500+600) = 3.7687 \text{ MN},
$$

$$
T_{2m} = 1.2 \times 0.12(0.00181 \times 500+0.6547) = 0.2246 \text{ MNm},
$$

$$
T_{3m} = 0.0434 \times 40.96 + 0.00181(600-0.6547) = 2.209 \text{ MNm}.
$$

Thus, according to Eqs. (14) and (18), the mean and variance of beam-column resistance are

$$
R_{MN} = M_{MN} = 0.2246 \times 2.209 / 3.7687 = 0.1316 \text{ MNm},
$$

$$
\sigma^2 R_{MN} = [0.2246(3.7687 - 2.209)]^{-2} \times \left[0.0434^2 \times 27.49 + 40.96^2 \times 25.65 + 6 \times 0.00181^2 \times 8100 + (0.00181 \times 0.012) / 3.7687 - 0.2246 \times 3.7687 \times 25.65 + (0.12 \times 0.012 \times 2.209 - 0.2246) / 3.7687 \times 27.49 + 0.00181 \times 0.012 \times 0.00181 \times 0.012 \times 0.00181 \times 0.012 \times 0.00181 \times 0.012] / 3.7687^2 = 0.0496 \times 10^6 = 121.4 \times 10^6 \text{ (MNm)}^2.
$$

The statistics of the second order bending moment components from Eqs. (53) and (54) are

$$
M_{CN} = 1.0 \times 0.612 \times 0.06855 = 0.04195 \text{ MNm};
$$

$$
\sigma^2 M_{CN} = (0.10 \times 0.04195)^2 = 0.176 \times 10^6 \text{ (MNm)}^2;
$$

$$
\sigma^2 (\theta_M M_{CN}) = 0.00035 \times 10^6 / 0.04195 \times 0.01 = 35.2 \times 10^6 \text{ (MNm)}^2;
$$

$$
\sigma^2 (\theta_M M_{SN}) = 0.002 \times 10^6 / 0.04195 \times 0.01 = 0.0309 \times 10^6 \text{ (MNm)}^2;
$$

$$
\sigma^2 (\theta_M M_{QSM}) = 0.01 \times 0.04195 \times 0.01 = 0.002 \times 10^6 \text{ (MNm)}^2;
$$

$$
\sigma^2 (\theta_M M_{Q}) = 0.002 \times 0.04195 \times 0.01 = 0.002 \times 10^6 \text{ (MNm)}^2;
$$

$$
\sigma^2 (\theta_M M_{SN}) = 0.01 \times 0.04195 \times 0.01 = 0.01 \times 10^6 \text{ (MNm)}^2;
$$

$$
\sigma^2 (\theta_M M_{QSM}) = 0.001 \times 0.04195 \times 0.01 = 0.001 \times 10^6 \text{ (MNm)}^2;
$$

$$
\sigma^2 (\theta_M M_{Q}) = 0.001 \times 0.04195 \times 0.01 = 0.001 \times 10^6 \text{ (MNm)}^2;
$$

$$
\sigma^2 (\theta_M M_{SN}) = 0.0001 \times 0.04195 \times 0.01 = 0.0001 \times 10^6 \text{ (MNm)}^2.
$$

### Table 1. Parameters of beam-columns as compression members

<table>
<thead>
<tr>
<th>$R_{CNm}$ (by 51), MN</th>
<th>$\sigma^2 R_{CN}$ (by 52), (MNm)$^2$</th>
<th>Extreme actions</th>
<th>Recurrent number n (by 45)</th>
<th>$N_{CM}$ (by 53), MN</th>
<th>$\sigma^2 N_C$ (by 54), (MNm)$^2$</th>
<th>$P(R_{CN} &gt; N_C)$ by (58)</th>
<th>$P(T \geq t_n)$ by (60)</th>
<th>$\beta_N (T \geq t_n)$ by (63)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0067</td>
<td>0.068 60</td>
<td>Q</td>
<td>50</td>
<td>0.030 55</td>
<td>316x10$^{-6}$</td>
<td>0.999 952</td>
<td>0.999 921</td>
<td>3.78</td>
</tr>
<tr>
<td></td>
<td></td>
<td>S</td>
<td>50</td>
<td>0.012 20</td>
<td>38x10$^{-6}$</td>
<td>0.999 966</td>
<td>0.999 963</td>
<td>3.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q + S</td>
<td>4.25</td>
<td>0.042 75</td>
<td>354x10$^{-6}$</td>
<td>0.999 942</td>
<td>0.999 932</td>
<td>3.82</td>
</tr>
</tbody>
</table>

### Table 2. Parameters of beam-columns as bending members

<table>
<thead>
<tr>
<th>$R_{CM}$ (by 51), MN</th>
<th>$\sigma^2 R_{CM}$ (by 52), (MNm)$^2$</th>
<th>Extreme actions</th>
<th>Recurrent number n (by 45)</th>
<th>$M_{CM}$ (by 53), MN</th>
<th>$\sigma^2 M_C$ (by 54), (MNm)$^2$</th>
<th>$P(R_{CM} &gt; M_C)$ by (58)</th>
<th>$P(T \geq t_n)$ by (60)</th>
<th>$\beta_M (T \geq t_n)$ by (63)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.089 65</td>
<td>0.000 496</td>
<td>Q</td>
<td>50</td>
<td>0.002 09</td>
<td>1.518x10$^{-6}$</td>
<td>0.999 957</td>
<td>0.999 938</td>
<td>3.84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>S</td>
<td>50</td>
<td>0.000 836</td>
<td>0.181x10$^{-6}$</td>
<td>0.999 967</td>
<td>0.999 965</td>
<td>3.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q + S</td>
<td>4.25</td>
<td>0.002 93</td>
<td>1.699x10$^{-6}$</td>
<td>0.999 949</td>
<td>0.999 944</td>
<td>3.86</td>
</tr>
</tbody>
</table>
These values exceed the generalized reliability index $\beta_M(T \geq t_n)$ = 3.78 not much. It shows that the variable actions of residential, office and public low rise buildings with reinforced concrete floors may only slightly decrease the structural safety of columns.

7. Conclusions

The resistance concept of spun concrete beam-columns may be based on their ultimate resisting compressive force or resisting bending moment values. Therefore, the structural safety of these members may be assessed and predicted by the generalized reliability index $\beta_M(T \geq t_n)$ or $\beta_N(T \geq t_n)$ from Eq. (63) respectively.

The non-stationary time-dependent resistance safety margin of building beam-columns is closely related to their action effects provoked both by live floor and climate actions. Instead of the sustained and extraordinary components of live floor loads, it is expedient to use their annual extreme sums in probabilistic design practice. The mean and variance of joint annual extreme action effects are expressed by Eqs. (38) and (40). Live and climate (wind and snow) annual extreme actions effects modelled by Type 1 (Gumbel) distribution help engineers with expressing the resistance safety margin of building beam-columns as a stationary process.

The reliability levels of beam-columns designed by limit state and probability-based approaches were compared. Regardless of methodological features, both limit state design methods EN 1990 and ASCE/SEI 7-05 lead to the same design results and are confirmed by the reliability index values. The reliability index approach based on the transformed conditional probability concept opens quite realistic design formats in the long term structural safety prediction of beam-columns and other structural members.

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TIKIMYBINIS SIJINIŲ KOLONŲ IŠ CENTRIFUGUOTOJO BETONO PROJEKTAVIMAS

A. Kudzys, R. Kliukas

S ant rau k a


Reikšminiai žodžiai: centrifugotasis betonas, rėmai, antrosios eilės įrąžos, ribinė atspario sauga, patikimumo indekso skaičiavimas.

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