TENSION STIFFENING BOND MODELLING OF CRACKED FLEXURAL REINFORCED CONCRETE BEAMS

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Abstract: This paper deals with the analysis of cracked flexural reinforced concrete structures with special highlighting of modelling the interaction between concrete and reinforcement. A new approach based on the bond stress distribution through the transfer length between the zero-slip and the cracked sections is proposed. Since the cracking phenomenon of concrete occurs, the fracture energy changes in order to appeal to the interaction between concrete and steel. The increment of stresses is evaluated by the bond-slip distribution by means of one-dimensional problem. Besides, the 2D non-linear description of components behaviour, concrete and steel are considered.

On numerical modelling level, the interaction property is obtained from a variety of fundamental pull out and push out tests, for the most part this phenomenon does not very well represent the bending members. For this object, this study presents a numerical approach, which can compute the distribution stresses at the steel-concrete interface near flexural crack in reinforced concrete beams. Finally, predictions made by the non-linear finite element analysis program and the non-linear material models for concrete, reinforcing bars and bond slip are in good agreement with the experimental results.

Keywords: concrete, reinforcement, bent beams, tension stiffening, bond models, cracked flexural reinforced concrete beams.

1. Introduction

The stress transfer between steel and surrounding concrete is a fundamental characteristic of reinforced concrete structures. The properties of this interaction depend on several factors, such as friction, mechanical interaction and chemical adhesion. In flexural reinforced concrete structures, deformed bars depend primarily on the mechanical interlocking for bond properties, while the contribution of chemical adhesion and friction are negligently considered. These suppositions led the researchers to express various relationships of analytical and experimental concepts. Recently, the development of analytical expressions to predict the bond stress-slip responses of flexural reinforced concrete structures is an important topic of research. Therefore, many analytical and numerical models have been developed to yield the bond stress-slip response of tensile reinforced concrete members (Redzuan 2004; Kwak, Kim 2004; Khalfallah 2006; Banholzer et al. 2005; Mutlu, Bobet 2005; Ashraf 2005; Dominguez 2005).

In general, the interface models are mainly used for the analysis of stress transfer problems between reinforcing bars and surrounding concrete. When the interface continuum is assumed to be imperfect “cohesive models”, then the discontinuity in the displacement field is considered. In this case, the transition zone between steel and concrete is taken into account in the analysis of specific properties of concrete matrix and steel bars. In structural mechanic concepts, the interface behaviour can be represented by shear stress-slip functions. The bond force resulting at the interface zone of steel bars and surrounding concrete has a great influence on the development and propagation of flexural cracks in reinforced concrete beams. Further, the bond effect is the main contributing factor of the tension stiffening in cracked reinforced concrete structures (Monti, Spacone 2000; Lackner, Mang 2003; Khalfallah 2003).

The analysis of cracked reinforced concrete members has remained among of the difficult tasks in the fracture mechanics. Various projects of research have already been proposed in order to quantify and characterize the mechanism of bond forces development around the deformed steel bar. To improve the accuracy of the reinforced concrete structures responses, formulations are based on theoretical investigations, and the effect of bond is implemented in the numerical analysis (Eligehausen et al. 1983; Cosenza et al. 1997; Focacci et al. 2000; Campione et al. 2005).

Naturally, the steel-concrete bond is always existing in reinforced concrete structures. And other phenomena can generate the adherence effect, such as interlocking due to the roughness of the bar surface, the bearing area, the spacing and face angle of ribs. It is shown that the bond strength increases with the project ribs area of the reinforcement and it is so important (Zuo, Darwin 2000).
For this reason, the presence of bond and the interaction between concrete and steel bars renders its deportment on account of this effect, absolute evidence in finite element non-linear analysis of composite structures.

This approach is specially applied to predict the bond-slip behaviour for cracked flexural reinforced concrete beams near the first crack and between adjacent cracks simultaneously. The obtained results show an agreement through the global response of beams when the perfect bond and cohesive bond models are considered.

2. The proposed model

2.1. General

The bond slip effect included in finite element analysis of reinforced concrete structures can be taken in two ways: linkage elements or cohesive elements. Khalfallah and Hammed have distinguished the limitations of the each type use. For each type of bond modelling, it appeared limitations and difficulties in numerical procedure or in rheological law of bond, indifferently. For these difficult tasks, an analytical procedure, which can simulate the bond-slip effect of cracked flexural reinforced concrete structures, is presented in this work.

2.2. Proposed tension stiffening bond model of cracked flexural members

In the open literature, they have already presented many works on pull-out and tensile tests, where bond-slip models are included to calibrate the contribution of bond-slip in the global response. In this case, various models and numerous methodologies have been published by Gan (2000), Banholzer et al. (2005), and Dominguez (2005), but few works have been presented treating the cracked flexural reinforced concrete members by Au, Bai (2007) and Nayal, Rasheed (2006). For this reason, a particular importance has been given to this kind of work. This paper presents a procedure that has been inspired by the pull-out test idea accompanied by relative specifications of bending problem phenomena.

At the member scale, every structural component is modelled separately. The reinforcement is usually described by means of the embedded truss model in beam and frame analyses. In this case, the effect of the steel-concrete interaction is commonly referred to as 'tension stiffening phenomenon’. This term stems as the capacity of the intact concrete to carry tensile forces between adjacent cracks result in a higher stiffness compared to the respective steel bars. The tension stiffening models have been taken into account and various approaches were conducted. Most of these belong to one of two categories: (1) adding a supplementary terms of stiffness to the finite element formulation of the stiffness matrix, (2) the use of specific distribution of bond stress-slip relationship and it is in this way that this work is registered accordingly.

In composite structures it is impossible to separate the following 3 effects: cracking, tension stiffening and bond behaviour. Cracking in concrete will develop and propagate in the direction normal to the major principal strain starting from the section, where the first crack originates. After cracking, concrete can still partially contribute to the structural response due to the bond effect. In this phase of member behaviour, concrete less its rigidity that corresponds to transfer of stresses between concrete and reinforcing bars by link effect. Afterwards, the concrete will attempt its rigidity as soon as going away from the crack face.

The phenomenon, which results among the length characterized by the transfer of stresses between steel and concrete, is defined as the tension stiffening effect. The behaviour improves the softening response by introducing the tension stiffening models, which cause decreasing of tensile concrete stress, as the cracking intensifies. Most of tension stiffening models is based on one-dimensional modelling of problems (Yankelevsky et al. 2008). On numerical level, the tension stiffening effect can be taken into account by increasing the element stiffness. An increase in the tension stiffness of concrete can be explained by using a stress-strain relationship, including a softening branch of the curve beyond the strength of concrete value.

However, the interface environment is considered in this approach as material having a local behaviour, described by the bond-slip relationship. In this study, a parabolic variation of bond stresses is assumed with maximum bond stress value occurring at the middle section between zero-slip and cracked sections. This assumption can simplify the mathematical formulation for computing the bond forces at each node of the finite element (Fig. 1). The relationship used is expressed by:

\[
\tau(x) = 4\tau_{\text{max}} \frac{x}{s_0} \left(1 - \frac{x}{s_0}\right). \quad (1)
\]

To accurate the response of reinforced concrete beams, non-linear constitutive laws of material such as concrete, reinforcing bars and steel-concrete continuum are considered and integrated in the developed finite element analysis program. The behaviour of concrete in tension is assumed linearly elastic until the tensile strength has been reached. Beyond this value, the tensile stress decreases with increasing the principal tensile strain until the ultimate value, which is deducted using the fracture mechanic concept.

![Fig. 1. Bond distribution used in bending member, where \(\tau_{\text{max}}\) is the maximum bond value at \(x = s_0/2\) measured from the uncracked section](image-url)
In the same, the behaviour of reinforcing bars is modelled as linear elastic with linear strain hardening in the elastoplastic range. Bond stresses in reinforced concrete structures arise from the change in steel along the transfer length, and the slip is determined using the local bond–slip relationship. In this study, the simple trilinear bond stress-slip model (Fig. 2) is selected and introduced in the numerical program.

![Fig. 2. Local bond stress-slip used](image)

To present the effect of flexural bond, two different cases can be distinguished, as:

**Case 1. Bond near cracked face**

Let consider a free reinforced concrete body subjected to pure bending moment \( M \), as shown in Fig. 3.

![Fig. 3. Free body under bending effect](image)

At each section \( x \) measured from the zero-slip face, concrete and reinforcing bars carry the bending moment conjointly. When the primary flexural crack forms in reinforced concrete members, the steel stress at the cracked face increases, as the tensile force carried by concrete has completely transferred to the reinforcing bars. Here, bond forces take place along the transfer length in Fig. 4 and resist to this increment of steel stresses.

![Fig. 4. Bond stress near cracked section](image)

The resulting force of bond between cracked and zero-slip sections can be expressed by:

\[
\frac{\pi \phi^2}{4} (\sigma_{s,2} - \sigma_{s,1}) = \int_0^{\delta_0} \pi \phi \tau(x) \, dx = \frac{2}{3} \pi \phi \tau_{\text{max}} S_0,
\]

where \( \phi \) is the bar diameter, \( \sigma_{s,1} \) and \( \sigma_{s,2} \) are the steel stresses at the zero-slip (un-cracked section) and cracked sections, respectively.

If the local bond stress-slip is assumed, the maximum value of bond stress \( \tau_{\text{max}} \) and associated slip \( S_0 \) can be then established:

\[
\tau_{\text{max}} = f_b (\delta_0),
\]

which \( \delta_m = u_{m,s} - u_{m,c} \).

The indices \( s \) and \( c \) of \( u_m \) are the extensions of steel and concrete respectively for the median section. The relation (2) can be applied to each section localized at \( x \):

\[
\frac{\pi \phi^2}{4} (\sigma_s - \sigma_{s,1}) = \frac{4}{3} \int_0^{\delta_0} \pi \phi \tau(x) \, dx = \frac{2}{3} \pi \phi \tau_{\text{max}} S_0.
\]

The steel stress is expressed as:

\[
\sigma_s(x) = \sigma_{s,1} + \frac{16}{3} \frac{\tau_{\text{max}}}{S_0 \phi} \left( \frac{x^2}{2} - \frac{x^3}{3} \delta_0 \right).
\]

The strain of steel corresponds:

\[
\varepsilon_s(x) = \frac{\sigma_s(x)}{E_s}.
\]

Then the total extension of steel is:

\[
u_{m,s} = \int_0^{\delta_0} \varepsilon_s(x) \, dx = \frac{\sigma_{s,1} S_0}{2 E_s} + \frac{1}{4} \tau_{\text{max}} S_0^2.
\]

The finite element analyses show that the extension of concrete at reinforcing level due to the compressive force affecting the cracked section has not a significant effect, compared with this one caused by the reinforcing bars.

**Case 2. Bond between adjacent cracks**

In this section, two distinct cases can be selected basing on the variation of moment between cracked sections, as:

1. **Constant moment**

The distribution of bond stresses along the transferring range is anti-symmetric (Fig. 5) and the corresponding extension of steel bars is evaluated as analogous to the above manner:

\[
u_{m,s} = \frac{\sigma_{s,1} S_0}{4 E_s} + \frac{1}{16} \tau_{\text{max}} S_0^2.
\]

This \( \sigma_{s,2} \) is the steel stress at the second cracked section.

2. **Varying moment**

In this case, the deduced bending moments at the cracked sections are different and consequently the stresses between the transfer length are not symmetrical (Fig. 6).
\( \sigma_{s,2} \) and \( \sigma_{s,3} \) are the steel stresses in cracked sections.

The maximum extensions of steel in each region can be evaluated using the relation (8), as:

\[
u_{m,s,1} = \frac{\sigma_{s,1} S_1}{4E_s} + \frac{1}{16} \frac{\tau_{\max}}{\sigma_{s,1}} S_1^2,
\]

\[
u_{m,s,2} = \frac{\sigma_{s,1} S_2}{4E_s} + \frac{1}{16} \frac{\tau_{\max}}{\sigma_{s,1}} S_2^2.
\]

The approach presented is applied in the next section to calibrate the flexural bond modelling of reinforced concrete beam response.

3. Numerical applications

To show the ability of the described procedure for simulating bond slip behaviour of cracked flexural reinforced concrete beams, the specimen beam tested by Gaston et al. (1972) is chosen. The properties of materials, the characteristics of bond and geometries are summarized in Tables 1, 2 and 3, respectively.

In the studied case, the concrete was modelled by 8-node serendipity plane stress element with 3*3 Gauss integration points.

The reinforcements are modelled by 3-node truss element and the bond-slip effect is taken into account with the imperfect bond element based on the shear bond distribution, as described above. Firstly, to show the size effect, three meshes are selected for 12, 60 and 108 elements. Only one half of the beam is considered taking the advantage of the symmetry in the geometry with integration of the bond slip phenomenon in the analysis (Fig. 7).
The results in Fig. 8 indicate that the discretization has not a notable influence on the global response; load-mid span deflection of the beam established by this simulation exhibits, in general, a satisfactory behaviour with the experimental response that is essentially independent of the finite element mesh size.

To quantify the contribution of bond-slip effect, two different analyses, such as the perfect bond model and the cohesive bond-slip model were performed (Fig. 9). The analysis of responses shows a well correlation between different curves until the threshold of cracked reinforced concrete that is evaluated about 16 kN of the applied load. Beyond this limit, each model exhibits a proper response and the relative contribution of every bond model is well observed. It is evident that the perfect bond model shows a very satisfactory agreement with the experimental response in which the mechanical parameters of materials and the interface continuum were well considered. The response curve of the perfect bond model approaches relatively opposite to the experimental according to that of the bond slip model.

The results obtained by this simulation and those obtained by Yang, Chen (2005) seem very logical. The curves corresponding to the perfect bond model (this study) and to the strong model (Yang, Chen 2005) approach via the experimental curve, while it is the contrary case, opposite to the results obtained by Kwak, Kim (2002).

The major principal stress distributions are shown in Figs 10–11. It is possible to appreciate how the bond modelling influences the transfer of forces between steel and surrounding concrete. The interface properties were more influenced by tensile forces that they allowed to the structure to profit of characteristics of interface elements using its total energy of dissipation.

When the damage arises in some nodes, then there is gradual energy dissipation during the cracking phenomenon, and the corresponding forces are transferred with the redistribution of principal stresses. The damage effect propagates to undamaged zone proportionally to the increase forces in concrete. Beyond the formation of the first crack, a discharge takes place in the interface in the vicinity of the crack planes; here the interface is already damaged and the transmission changes forces between concrete and steel.

Fig. 8. Mesh size effect on the global response

Fig. 9. Effect of bond models

Fig. 10. Major principal stress distribution of perfect bond model
The plotting of the stress fields along the beam (Figs 10–11) shows the distribution of principal stresses at the first cracking load and at the ultimate load, respectively. More, the figures describe the damaged diffuse within the beam due to the tensile stresses. This damage initiates in the middle of the section and propagates towards the support of the beam. Of the same, the damage is more pronounced in the vicinity of the applied load due to the compressive stresses.

The cracking process of the beam using the perfect bond model, taken as example, which was fully modelled using the smeared crack model. The finite element simulation shows that a few flexural cracks first appear near the mid-span at a load about \( F = 20 \text{kN} \) (red colour). In the constant moment case, for load of about 30–40 kN (Fig. 13), these flexural cracks propagate upwards into the upper half of the beam and the cracks become gradually curved towards the loading points (Khalfallah et al. 2004).

In the literature, the predicted cracking process, in general, agrees well with experimental observations, as shown on the test crack pattern of Vecchio, Shim (2004). It may be noted that the shear crack near the support is not modelled yet, because in the current model only cracks on the tensile face of the beam are allowed to initiate, and they are not allowed to propagate downwards to avoid crack intersection.

4. Conclusions

This work aims at the calibration of the tension stiffening bond model of cracked flexural reinforced concrete beams. The interaction was modelled by means of a non-linear local bond stress-slip relationship with the consideration of the fracture energy related to cracking of concrete. The analysis of obtained results can lead to the following conclusions:

1. The inclusion of tension stiffening is important even in structures dominantly affected by bending moments in crack formation and propagation range. For this assumption, its integration via bond-slip modelling in numerical analysis becomes a necessity to improve certainly the predictions of existing models.

2. The proposed approach predicts the stress field in the concrete and along the steel bars (local behaviour) and may provide force-elongation relationship (global behaviour). Addicting, in bending members, the bond between steel-concrete depends on the nature of the applied bending moment, as constant or varying moment.

3. The numerical results showed that the maximum concrete tensile value between successive cracks in a constant moment occurs at the middle plane between cracks, while it localized at the side of section having a lower value of bending moment in varying moment case.

4. The structural damage diffuses firstly according to the length and secondly following the height of the beam.

It is clear that the procedure need a refinement to improve well the RC response through the constitutive laws of materials or the numerical representation, such as the 3D modelling.

References


**SUPLEIŠJUSIŲ GELŽBETONINIŲ SIŲJ ARMATŪROS IR BETONO SANKIBOS MODELIAVIMAS**

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**Santrauka**

Striaipsnyje atlikta supleisjusiųjų lenkiančiųjų gelžbetoninių elementų analizė armaturos ir betono sąveikos modeliaišmo aspektu. Pasibaigus naujas modelis, pagrįstas sukibimo įtempių pasiskirstymu sąveikos zonas ilgiiu nuo nulinių praslydimo iki plyšio pločio. Supleisjusios betonui dėl jo sąveikos su armatuira keičiasi įvairiom energijai, įtempių didėjimas sukibimo ir praslydimo zonoje įvertinamas taikant vienmatį (1D) modelį. Sąveikos komponentų (betono ir armatuuros) elgsena aprašoma pasitelkiant dvimaičius (2D) modelius. Atliekant skaitinių modeliavimą, betono ir armatuuros sąveikos parametrai dažniausiai nustatomi pagal klasikinias ištraukimo ir ištūrimo bandymus. Tačiau lenkiančiųjų elementų tarp naudoti parametrai netinka. Pasibaigus skaitiniams supleisjusiųjų lenkiančiųjų gelžbetoninių sių skaičiavimo algoritmas, kurių taikant gali būti nustatyta įtempių pasiskirstymas pleno ir betono sąveikos paviršiuje pločioje ties plyšio. Skaičiavimui rezultatai, gauti taikant netiesinis analizēs būtų galbūt naudojami programą kartu su betono, armaturos ir sankibos modeliais, gerai sutapau su eksperimentinio tyrimo rezultatais.

**Reiškiniai žodžiais:** betonas, armatūra, sija, armaturos ir betono sąveika, sankibos modeliai, supleisjusios gelžbetoninių sių.

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