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DETERMINING INTEGRATED WEIGHTS OF ATTRIBUTES

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1. Introduction

In Multiple Attribute Decision-Making (MADM) a decision maker (DM) is often faced with the problem of choosing the alternatives based on multidimensional and inconsistent attributes. The problems of MADM arise in many actual situations [1–4], eg when evaluating the efficiency of technological decisions. To solve this problem such attributes as the time spent, cost, labour expenditures, quality indices of a structure analyzed, etc are taken into account [5, 6]. To evaluate the efficiency of investments in construction such criteria as the duration of the investment project, the risks taken, repay, flows of money, etc are considered [7].

MADM problems have been investigated by many authors but the questions remain in many areas [8–12].

Whatever the approach used in solving MADM problems, the weights of the attributes showing their relative value in solving a particular problem should be determined first. A number of papers [1–14] deal with the ways of determining the weights of attributes and the associated problems.

The above papers may be divided according to subjective or objective approaches used. The weight of attributes was determined on the basis of DM privileged information as well as applying the eigenvector method [14], weighted least square [15], and Delphi methods [1] LINMAP (Linear Programming Techniques for Multidimensional Analysis of Privileged) [16], mathematical programming models [17], etc. It should be noted that the latter technique allows the weights to be determined by computer-aided handling of mathematical models with no privileged information available about any DM including the entropy method [2], multiple objective programming [18], etc.

Subjective and objective approaches have a number of advantages and disadvantages. A subjective approach to determining the weights yields a subjective DM resulting in ranking the alternatives for a MADM problem characterized by more arbitrary values. Since any of the above approaches can not be considered perfect, an integrated approach for determining the weights of attributes may turn out to be more effective. Recently some papers appeared [9, 10, 12] dealing with the problem of integrating subjective (ie qualitative data) and objective (ie quantitative data) information in MADM problems. However, the models obtained are too complicated to be applied. The authors recognize that the problems of integrating subjective and objective information have not been sufficiently studied yet and further research in this area is needed.

The aim of the present paper is the development of a new sufficiently simple method of calculating the integrated values of attribute weights. To determine the attribute weights a two-objective program model based on the integration of a subjective [14] and objective [2, 5] approach is suggested.

2. An integrated subjective and objective approach

Let us consider the entropy method to determine the objective weights of attributes.

Let $S = \{S_1, S_2, \dots, S_m\}$ be a discrete set of alternatives (versions), while $R = \{R_1, R_2, \dots, R_n\}$ – a set of attributes (criteria) and $X = [x_{ij}]_{m \times n}$ – a solution matrix, where x_{ij} is a numerical value of R_j in S_i ($i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$). Let us also assume that all the values of the original matrix range from 0 to 1 for any criterion to make the measurement uniform. This may be achieved by normalizing any element of matrix $X = [x_{ij}]_{m \times n}$ into the element of matrix $\bar{P} = [b_{ij}]_{m \times n}$ from

the formula:

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}}, \quad i = \overline{1, m}; \quad j = \overline{1, n}. \quad (1)$$

If analysed factor is minimized, then is used formula:

$$p_{ij} = \frac{1/x_{ij}}{\sum_{i=1}^m 1/x_{ij}}, \quad i = \overline{1, m}; \quad j = \overline{1, n}. \quad (2)$$

In consequence, matrix \bar{P} is obtained:

$$\bar{P} = \begin{matrix} & \begin{matrix} x_1 & x_2 & \dots & x_n \end{matrix} \\ \begin{matrix} a_1 \\ a_2 \\ \dots \\ a_n \end{matrix} & \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1n} \\ p_{21} & p_{22} & \dots & p_{2n} \\ \dots & \dots & \dots & \dots \\ p_{m1} & p_{m2} & \dots & p_{mn} \end{bmatrix} \end{matrix}. \quad (3)$$

Then the entropy level E_j of every criterion of efficiency is defined by the formula:

$$E_j = -k \sum_{i=1}^m p_{ij} \ln p_{ij}, \quad (i = \overline{1, m}; \quad j = \overline{1, n}) \quad (4)$$

where $k=1:\ln m$.

As noted above, the entropy index ranges [0,1], therefore:

$$0 \leq E_j \leq 1, \quad (j = \overline{1, n}). \quad (5)$$

The variability of j -th criterion within the problem solved, ie a number of technological solutions in construction, is determined by:

$$d_j = 1 - E_j, \quad (j = \overline{1, n}). \quad (6)$$

If all the criteria of efficiency are equally important, ie any subjective or expert evaluation has not been made, then the objective weights of the criteria of efficiency may be obtained by the formula as follows:

$$q_j = \frac{d_j}{\sum_{j=1}^n d_j}, \quad (j = \overline{1, n}).$$

or:

$$d_j = q_j \sum_{j=1}^n d_j. \quad (7)$$

In cases, when the subjective weights values are determined by DM specialists or expertise (ie multidimensional efficiency values expressed as weights coefficient $\bar{q}_j(j = \overline{1, n})$, then the objective complex weights may be defined as follows:

dimensional efficiency values expressed as weights coefficient $\bar{q}_j(j = \overline{1, n})$, then the objective complex weights may be defined as follows:

$$q_j^0 = \frac{\bar{q}_j d_j}{\sum_{j=1}^n \bar{q}_j d_j}, \quad (j = \overline{1, n}).$$

Using formula (6), we get:

$$q_j^0 = \frac{\bar{q}_j q_j \sum_{j=1}^n d_j}{\sum_{j=1}^n \bar{q}_j q_j \sum_{j=1}^n d_j}, \quad (j = \overline{1, n}).$$

or:

$$q_j^0 = \frac{\bar{q}_j q_j}{\sum_{j=1}^n \bar{q}_j q_j}, \quad (j = \overline{1, n}). \quad (8)$$

In further calculations the above value is applied to TOPSIS, SAW, LINMAP, ELEKTRE and other methods determining the rationality of alternatives.

Let us consider a MADM problem with a matrix of the attributes evaluated pairwise which is provided by a rather qualified DM specialist. Pairwise comparison based on the expert evaluation allows to determine the values of the efficiency criteria. The data needed to determine the above values is obtained by comparing the pairs of criteria as to their "priority intensity". To state the priorities a scale of values, suggested by T.Saaty [14], may be used.

Based on the expert-filled questionnaires a table in the form of a questionnaire is developed containing the mean values of the criteria suggested by experts. Then, further mathematical calculations are made.

Let us assume that m alternatives described by n criteria are considered. The priority intensity is denoted by b_{ij} ; $i, j = \overline{1, n}$. This means the relationship of the expert estimation of the values of i -th and j -th criteria. Assume that all the criteria have been pairwise compared and their numerical priority values determined. The results obtained have been presented as matrix B :

$$B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{bmatrix} \quad (9)$$

Taking into consideration that the constant with the unknowns can not be calculated very accurately by hand (some error is inevitable) there may be cases when the system of equations (18) will be unsolvable or have endless ensemble of decisions. For this purpose, an error coefficient f is introduced into the above system of equations. Taking into account that $\sum_{j=1}^n q_j^* = 1$ a system of equations is of the form as follows:

$$\begin{cases} q_1^*(q_1q_1 - q_1) + q_2^*(q_1q_2 - q_2) + q_3^*(q_1q_3 - q_3) + \dots + q_n^*(q_1q_n - q_n) + f = 0 \\ q_1^*(q_2q_1 + q_2^*(q_2q_2 - q_2) + q_3^*(q_2q_3 - q_3) + \dots + q_n^*(q_2q_n - q_n) + f = 0 \\ q_1^*(q_3q_1 + q_2^*(q_3q_2 - q_2) + q_3^*(q_3q_3 - q_3) + \dots + q_n^*(q_3q_n - q_n) + f = 0 \\ \dots \\ q_1^*(q_nq_1 + q_2^*(q_nq_2 - q_2) + q_3^*(q_nq_3 - q_3) + \dots + q_n^*(q_nq_n - q_n) + f = 0 \\ q_1^* + q_2^* + q_3^* + \dots + q_n^* = 1 \end{cases} \quad (19)$$

Further, the values of q_j^* are used when applying the techniques such as TOPSIS, SAW, LINMAP, ELECTRE, etc to determine rationality of the alternatives.

4. Sample calculation

To illustrate the technique developed some variants of purchasing an office for a company are considered. Suppose, that a client (DM) needs to purchase an office for a firm. There are four variants of office location. Four attributes are considered :

- 1) R_1 – price (10,000 \$),
- 2) R_2 – office area (m²),
- 3) R_3 – distance from home to work (km),
- 4) R_4 - office location (in points).

The criteria R_2 and R_4 are maximized, while R_1 and R_3 are minimized. The data concerning office purchasing for a firm is presented in Table 1.

Table 1. Data on office purchasing

Variants \ Criteria	Criteria			
	R_1	R_2	R_3	R_4
S_1	3,0	100	10	7
S_2	2,5	80	8	5
S_3	1,8	50	20	11
S_4	2,2	70	12	9
	min	max	min	max

Conforming with Table 1 a solution matrix takes the form of:

$$X = \begin{bmatrix} 3,0 & 100 & 10 & 7 \\ 2,5 & 80 & 8 & 5 \\ 1,8 & 50 & 20 & 11 \\ 2,2 & 70 & 12 & 9 \end{bmatrix} \quad (20)$$

Suppose, that the experts provided a matrix B of pairwise evaluation of the criteria as follows:

$$B = \begin{bmatrix} 1 & 1/3 & 1/2 & 1/5 \\ 3 & 1 & 2 & 1/2 \\ 2 & 1/2 & 1 & 1/2 \\ 5 & 2 & 2 & 1 \end{bmatrix} \quad (21)$$

The subjective weight of the criteria of efficiency was determined by using expert pairwise evaluation as a subjective approach. The entropy method was used as an objective approach to determine the objective weights of the criteria. The calculated weights are presented in Table 2.

Table 2. Weights of the criteria

Criteria \ Weights of criteria	R_1	R_2	R_3	R_4
	Subjective weights	0,0953	0,2303	0,1928
Objective weights	0,128	0,217	0,360	0,295

The values of an integrated weight are determined by solving a system of equations (19). Taking into account the data given in Table 2 a system of equation (19) may be written as:

$$\begin{cases} -0,1155q_1^* + 0,0207q_2^* + 3,43q_3^* + 0,0281q_4^* + f = 0 \\ 0,0294q_1^* - 0,1673q_2^* + 0,0829q_3^* + 0,0679q_4^* + f = 0 \\ 0,0246q_1^* + 0,0419q_2^* - 0,2904q_3^* + 0,0569q_4^* + f = 0 \\ 0,0614q_1^* + 0,1047q_2^* + 0,1733q_3^* - 0,1529q_4^* + f = 0 \\ q_1^* + q_2^* + q_3^* + q_4^* = 1 \end{cases} \quad (22)$$

The values of integrated weights as well as the values of objective complex weights of the criteria of efficiency (determined from formula (7)) are given in Table 3.

Table 3. The values obtained for integrated and objective complex weights of the criteria

Criteria \ Weights of criteria	R ₁	R ₂	R ₃	R ₄
Integrated weights	0.188	0.266	0.135	0.411
Objective complex weights	0,0473	0,1913	0,2772	0,4842
Integrated weights obtained by method [20]	0.1011	0.2403	0.2104	0.4482

When the values of the objective weights of the attributes are applied, their effect on the rationality of variants does not match that of subjective weights which may adversely affect the accuracy of the results obtained. The use of the integrated value of the attribute weights in rationality evaluation techniques eliminates the above negative effect. When a system of equations (22) is solved, the accuracy factor f acquires the value of 0,000132 indicating that the accuracy of the integrated attribute weights is not considerably affected.

The values of q_i^* in this case are slightly different from corresponding values obtained by another method [20]. It can be seen when rows 1 and 3 of Table 3 are compared. The calculations made revealed the need for further investigation in the area of the integrated weights of the criteria of efficiency.

By applying TOPSIS approach [2] the rationality of the alternatives of office purchasing has been determined. The calculations were based on the application of the integrated weights, objective complex weights and the integrated weights obtained by method [20]. The calculation results are given in Table 4.

Table 4. Results of calculating the rationality of the alternatives based on various types of attribute weights

Value of variant rationality \ Type of weights of criteria used	S ₁	S ₂	S ₃	S ₄
Integrated weights	0.504	0.347	0.595	0.605
Objective complex weights	0,531	0,425	0,557	0,643
Integrated weights obtained by method [20]	0.524	0.391	0.572	0.623

The analysis of the results obtained shows that the use of integrated weights made some corrections to ranking the alternatives depending on their rationality. These variations are more clearly illustrated by Fig 1.

The picture clearly shows that the application of integrated weights of attributes (criteria) made the difference between the efficiency of variant 3 and variant 4 less evident. In some cases, this may change decision making when choosing the best variant.

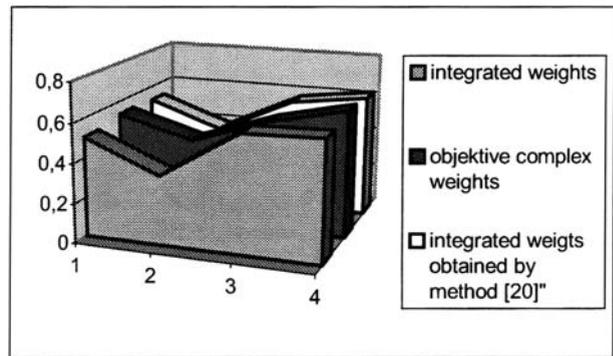


Fig 1. The dependence of variant rationality on the type of criteria weights used

5. Conclusions and recommendations

- A method to determine the integrated weights of attributes (criteria) was suggested.
- An integrated subjective and objective approach offered in this paper provides an alternative technique of determining the weights of attributes in MADM problems.
- The application of the integrated weights of attributes makes some corrections to rationality evaluation.
- Further investigation as well as the improvement of methods to determine the integrated weights of attributes are needed.

References

1. C. L. Hwang, M. J. Lin. Group Decision Making under Multiple Criteria: Methods and Applications, Springer-Verlag, New York, 1987. 400 p.
2. C. L. Hwang, K. Yoon. Multiple Attribute Decision Making, Springer-Verlag, Berlin Heidelberg-New York, 1981. 259 p.
3. Y. Sawaragi, K. Inoue, H. Nakayama Toward Interactive and Intelligent Decision Support Systems, Springer-Verlag, New York, 1987. 450 p.

4. E. K. Zavadskas, O. Kaplinski, A. Kaklauskas, J. Brzeziński. Expert System in Construction Industry. Trends, Potential and Application. Vilnius: Technika, 1995. 175 p.
5. Э. К. Завадскас. Системотехническая оценка технологических решений строительного производства. Л.: Стройиздат, Ленингр. отделение, 1991. 256 с.
6. L. Ustinovičius, V. Šarka, E. K. Zavadskas. Projektų daugiakriterinių sprendimų sintezės remiantis priimamo sprendimo sėkmės kriterijumi metodas // *Statyba*, VI t., Nr. 3. Vilnius: Technika, 2000, p. 193–201.
7. L. Ustinovičius, S. Jakučionis. Daugiakriterinių metodų taikymas vertinant senamiesčio pastatų renovacijos investicinius projektus // *Statyba*, VI t., Nr. 4. Vilnius: Technika, 2000, p. 227–237.
8. N. Bryson, A. Mobolurin. An action learning evaluation procedure for multiple criteria decision making problems // *European Journal of Operational Research*, 96(1996), p. 379–386.
9. W. D. Cook, M. Kress. A multiple-criteria composite index model for quantitative and qualitative data // *European Journal of Operational Research*, 78 (1994), p. 367–379.
10. G. S. Liang, M. J. Wang. Personnel selection using fuzzy MCDM algorithm // *European Journal of Operational Research*, 78 (1994), p. 22–33.
11. B. Malakooti, Y. Q. Zhou. Feed forward artificial neural networks for solving discrete multiple criteria decision making problems // *Management Science*, 40(1994), p. 1542–1561.
12. J. B. Yan, M. G. Singh. An evidential reasoning approach for multiple-attribute decision making with uncertainty // *IEEE Transactions on Systems, Man, and Cybernetics* 24 (1994), p. 1–18.
13. T. L. Saaty. *The Analytic Hierarchy Process*. McGraw-Hill, New York, 1980. 296 p.
14. T. L. Saaty. A scaling method for priorities in hierarchical structures // *Journal of Mathematical Psychology*, 15 (1977), p. 234–281.
15. A. T. Chu, R. E. Kalaba, K. Spingarn. A comparison of two methods for determining the weights of belonging to fuzzy sets // *Journal of Optimization Theory and Application*, 27 (1979), p. 531–538.
16. V. Srinivasan, A. D. Shocker. Linear programming techniques for multidimensional analysis of privileged // *Psychometrika*, 38 (1973), p. 337–369.
17. D. Pekelman, S. K. Sen. Mathematical programming models for the determination of attribute weights // *Management Science*, 20 (1974), p. 1217–1229.
18. Z. P. Fan. A new method for multiple attribute decision making // *Systems Engineering*, 12 (1994), p. 25–28 (in Chinese).
19. Л. Евланов. Теория и практика принятия решений. Москва: Экономика, 1984. 176 с.
20. Z. Fan, J. Ma, P. Tian. A Subjective and Objective Integrated Approach for The Determination of Attribute Weights // *Materials of 4th Conference of the International Society for Decision Support Systems*, 1977.

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INTEGRUOTŲ RODIKLIŲ REIKŠMINGUMŲ NUSTATYMO METODAS

L. Ustinovičius

Santrauka

Atlikta naudojamų rodiklių objektyvių bei subjektyvių reikšmingumų analizė. Nustatytas integruoto reikšmingumo skaičiavimo poreikis.

Subjektyvus reikšmingumas nustato rodiklių svarbumą, sprendžiant konkretų uždavinį. Objektyvus reikšmingumas įvertina nagrinėjamo rodiklio įtaką varianto racionalumui. Subjektyvios ir objektyvios reikšmingumo reikšmės dažnai nesutampa. Tai mažina varianto racionalumą. Rodiklio integruoto reikšmingumo panaudojimas leis nagrinėjamiems rodikliams turėti įtakos varianto racionalumui atitinkamai jų subjektyviems reikšmingumams.

Pasiūlytas rodiklių integruotų reikšmingumų nustatymo metodas – objektyvus rodiklių reikšmingumas nustatomas taikant entropijos metodą, subjektyvus rodiklių reikšmingumas nustatomas, taikant porinio palyginimo metodą. Integruotiems rodikliams skaičiuoti pasiūlyta ir suformuluota lygčių sistema.

Metodo galimybės buvo parodytos sprendžiant realų variantų lyginimo uždavinį (firmos patalpų pirkimas).

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