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NON-LINEAR DEFORMATION AND STABILITY OF REINFORCED CONCRETE COLUMN UNDER THE LONG-TIME LOAD

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I. Cypinas

1. Introduction

If concrete stress exceeds 0.30...0.45 of the compressive strength, the stress-strain relation becomes essentially non-linear. That is true for instantaneous deformation as well as for sustained deformation of concrete, in the case of creep. Non-linear behaviour of material is substantially important for creep stability analysis when the singular point on the deformation path of a structure must be detected.

Energy criteria of stability, usually adopted for instantaneous loading, become inapplicable for long-time deformation. In this case the energy criterion should be replaced by more general Liapunov's stability concept [1], that is based on the perturbation analysis of an ideal solution in the whole time period. However, the point is that the comprehensive creep analysis of concrete structure is practicable only by means of numerical procedures. But the numerical solution of a problem is not ideal in itself because of inherent approximation and round-off errors. On the other hand, the strict numerical analysis of perturbations would be the time-consuming and tedious task. More realistic way of creep stability analysis will be to choose a number of representative loading histories, compute the corresponding deformation paths and check the system stability detecting in each computational step possible singular points of the numerical process.

There is a number of analytical investigations of reinforced concrete creep stability [1], but few works deal with non-linear creep. The problem is that there is a lack of comprehensive analytical representation of non-linear creep in the world literature.

Vast experimental research of concrete creep have been carried out in the former Soviet Union [2]. Corresponding theoretical developments are summarised in [3]. Remarkable contribution to the non-linear creep problem has been marked by the reference [4] where the problem was treated in connection with the long-time strength of concrete.

Experimental investigation of creep stability has been carried out in many research institutions of the Soviet Union. Experimental results of Soviet researchers are presented in references [5-9], published in the nineteen sixties and early seventies. Less successful was analytical representation creep and numerical implementation of creep deformation and stability analysis (see [10]). The results of experimental investigations in the Soviet Union were summarised in reference [11] where comprehensive analytical representations of non-linear long-time and instantaneous concrete deformations were presented. No later publications have appeared that amend or supplement the reference [11].

Simplified effective modulus and mean stress methods are used in practical creep calculations. The so-called age adjusted effective modulus method (AAEM) developed by Z. P. Bažant [1] is compared in [12] with these two above-mentioned; AAEM is recommended for deflection calculations. The AAEM method is applied to creep stability analysis of concrete-filled steel columns [13]. The tension zone was neglected in this article. Deficiency of all these methods is that they do not account for real stress history of concrete and thus the realistic picture of loss of stability cannot be revealed.

Geometrically and materially non-linear deformation and stability analysis of reinforced concrete column is described in this article. Numerical solution was obtained on a PC by means of the Newton-Raphson procedure implementing the arc-length method. Non-linear constitutive relations in the integral form were used for concrete, the smeared crack approach [14] was used for the tension zone of the cross-section. Linear stress-strain relation was used for steel reinforcement.

2. Analytical representation of concrete creep and shrinkage

Many publications deal with the analytical representation of concrete creep and shrinkage. CEB FIP recommendations implemented in Eurocode-2 [15], ACI design aid [16], and BPKX model [17], proposed by Z. P. Bažant are among them. In reference [18] the problems of long-time fracture and non-linear creep of concrete are linked together. ACI formulae deal with only linear creep.

The creep strain at the time t due to the constant stress σ applied at the time t_0 is represented in the form of

$$\varepsilon(\sigma,t)=\frac{F^{ins}(\sigma)}{E(t_0)}+\sigma\, C_0(t,t_0)+F^{cr}(\sigma)\, C_1(t,t_0) \quad (2.1)$$

where t_{cr} - modulus of elasticity at the time t_0, $F^{ins}(\sigma)$ and z_{i-1}, λ_{i-1} - predefined non-linear stress functions for instantaneous and creep deformation. The linear $C_0\ (t,\ t_0)$ and non-linear $C_1\ (t,\ t_0)$ creep functions are not identical. In the linear case we simply have $F^{ins}(\sigma)\equiv\sigma$, $F^{cr}(\sigma)\equiv 0$ and the third member of the equation (2.1) vanishes.

In non-linear analytical expressions offered by the Eurocode-2 [15] instantaneous strain is assumed to be linear and the second term in (2.1) is omitted. All creep curves irrespective of stress level are similar. The similarity of creep curves, however, is not supported by the experimental evidence, it is only the simplification of a problem (see [3]).

The creep strain due to variable stress is usually represented by a hereditary integral. The creep formula results in the following integral expression:

$$\varepsilon(t)=\int_{t_0}^{t}\frac{dF^{ins}[\sigma(t')]}{E(t')}+\int_{t_0}^{t}C_0(t,t')\,d\sigma(t')$$

$$+\int_{t_0}^{t}C_1(t,t')\,dF^{cr}[\sigma(t')] \quad (2.2)$$

Here we can see that linear creep law is generalised by the replacement in the first and third terms $d\sigma$ by the dF^{ins} and dF^{cr} respectively.

In reference [11] the analytical expressions for functions in equations (1.1) and (1.2) and the values of material constants recommended for design purposes are presented. Non-linear stress functions F^{ins} and F^{cr} are taken in the form

$$F^{ins}=\sigma(1+v_k\eta^m),\quad F^{cr}=\sigma v_c\eta^n \quad (2.3)$$

where $\eta=\sigma/f_c$ and v_k, v_c, m, n are constants depending on strength of concrete. The quantity f_c is characteristic cubic strength, "normative" according to Russian terminology.

The creep compliance function here appears in the form of

$$C_0(t,\ t_0)=C(\infty,28)\,\Omega(t_0)\,f(t-t_0) \quad (2.4)$$

where

$$\Omega(t_0)=c+d\,\exp(-\gamma\, t_0)$$

$$f(t-t_0)=1-k\,\exp[-\gamma_1(t-t_0)]$$

and $C(\infty,28)$ is the ultimate value of creep deformation of concrete loaded at the age of 28 days. The latter quantity depends on the strength class, slump of the concrete mix, notional size $M_0 = A/V$ (A - area of a cross-section, V - volume) of the structural member and the relative humidity of the surrounding atmosphere. The values of γ, γ_1, d depend on the notional size of a member and constants are $c = 0.5$, $k = 0.8$. The non-linear part of the function is

$$C_1(t,t_0)=\exp(-f(t-t_0))\,C_0(t,t_0) \quad (2.5)$$

Development of concrete strength with time is described by the formula

$$f_c(t)=\left[1+\frac{23\,(t-28)}{(55+f_{c28})\,(t+11)}\right]f_{c28} \quad (2.6)$$

where f_{c28} is the cubic strength class of concrete. The modulus of elasticity is tabulated in [11] as a function of $f_c(t)$.

The shrinkage strain at time t is estimated by the formula

$$\varepsilon_{sh}(t,t_d)=\varepsilon_{sh}(\infty,t_d)\{1-\exp[-\alpha_s(t-t_d)]\} \quad (2.7)$$

where $\varepsilon_{sh}(\infty,t_d)$ is the ultimate shrinkage value of concrete which started to dry at the age t_d. The quantity $\varepsilon_{sh}(\infty,t_d)$ depends on the strength class, slump of the concrete mix, notional size of the member and the relative humidity of the surrounding atmosphere. The parameter α_s depends on the notional size of a member.

3. Incremental form of constitutive relation

The time period in the integral constitutive relation (2.2) can be divided into number of small time intervals and the equation (2.2) can be represented as a finite sum and rearranged in the incremental form. One can denote

$$\Delta\sigma_i = \sigma(t_i) - \sigma(t_{i-1}),\quad \Delta C_{i,k} = C(t_i, t_k) - C(t_{i-1}, t_k)$$

The strain increment during the time interval $\Delta t_i = t_i - t_{i-1}$ will be

$$\Delta\varepsilon_i = \sigma_0\,\Delta C^0_{i,0} + F^{cr}_0\,\Delta C^1_{i,0} +$$
$$\frac{1}{2}\Delta\sigma_i\left(\frac{1}{E_{i-1}}\frac{dF^{ins}_{i-1}}{d\sigma} + \frac{1}{E_i}\frac{dF^{ins}_i}{d\sigma}\right) +$$
$$\frac{1}{2}\sum_{k=1}^{i-1}\Delta\sigma_k\left(\Delta C^0_{i,k-1} + \Delta C^0_{i,k}\right) +$$
$$\frac{1}{2}\sum_{k=1}^{i-1}\Delta F^{cr}_k\left(\Delta C^1_{i,k-1} + \Delta C^1_{i,k}\right) +$$

$$\frac{1}{2}\Delta\sigma_i\left(C^0_{i,i-1} + \frac{dF^{cr}_{i-1}}{d\sigma}C^1_{i,i-1}\right) \tag{3.1}$$

where $\sigma_0 = \sigma(t_0)$, $E_i = E(t_i)$, $F_i = F(\sigma(t_i))$.

The linear version of such equation was earlier derived by the author in reference [19].

The total increment of concrete strain may be written in a concise form as

$$\Delta\varepsilon_i = \frac{\Delta\sigma_i}{E'_i} + \Delta\varepsilon'_i + \Delta\varepsilon^{sh}_i \tag{3.2}$$

where $\Delta\varepsilon^{sh}_i = \varepsilon_{sh}(t_i, t_d) - \varepsilon_{sh}(t_{i-1}, t_d)$. The first term of this equation expresses the creep strain due to stress increment during the current time interval and comprises the last line of the equation (3.1). The second term accounts for influence of the preceding stress history and represents the first four lines of the equation (3.1). The advantage of incremental equation (3.2) is that it can be simply inverted in regard of variable $\Delta\sigma_i$. This equation is used to derive the incremental stiffness relations of a finite element.

Until the appearance of cracks, the linear creep law for concrete in tension is adopted. The cracked tension zone of concrete is modelled using the averaged crack opening [19]. The strain-softening concept is applied to describe the interaction of cracked concrete and tensile reinforcement. The total tensile strain of concrete is represented as a sum

$$\varepsilon = \varepsilon^{cr} + \xi + \varepsilon^{sh} \tag{3.3}$$

where ε^{cr} – linear creep strain, ε^{sh} – shrinkage strain, and ξ is averaged tensile strain due to cracking of concrete. The latter is assumed to be independent of time and depends on the concrete stress only. Distinctive feature of the cracking model is that unloading of concrete from the falling branch of a stress-strain curve is essential. Analytical relations and computational procedures that describe strain-softening of cracked concrete and unloading are described by the author in reference [19].

4. Computer implementation of the method. Solution of the global equations

Let z be the n-dimensional vector of nodal displacements and λ is the load-scaling parameter that represents the imposed load in form of $\lambda\hat{P}$ where $\hat{P}$ is constant reference vector. Solution of time-independent non-linear problems is based on the load-scaling concept. The extension of this concept over the time domain is not a straightforward task.

The natural way is identifying the time t with the loading parameter and establishing a certain relation $\lambda = \lambda(t)$. The governing equations of concrete creep and shrinkage problems then can be written in the form of

$$\Psi(z, \lambda) = P(\lambda) \tag{4.1}$$

where P is the vector of the nodal forces equivalent to the time-varying external loads and shrinkage of concrete. The vector P is independent of nodal displacements z. Solving these equations, the deformation path of a structure can be traced and the limit point found out in $(n+1)$ - dimensional z–λ space.

In a time-independent limit point problem the falling branch of deformation path appears and decrease of the loading parameter λ is observed. The time, however, is an irreversible quantity. In the case of non-decreasing relation $\lambda = (t)$ the falling branch of the deformation path will not exist and limit point in its usual sense will not appear. In this case, the loss of stability will be characterised by the infinite rate of deformation $\partial z/\partial t$ at a certain critical time t_{cr} (Fig 1).

The governing equation (4.1) can be solved using a step-by-step Newton-type procedure. Each step comprises the prediction and correction stage. Consider the i-th step of the procedure when the solution at the previous step, z_{i-1}, λ_{i-1} , is already known. In

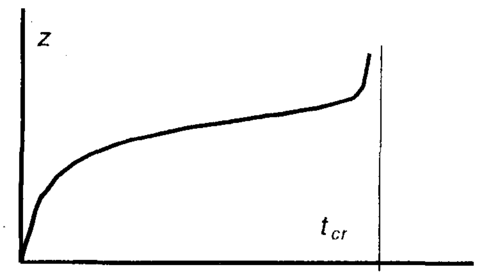

Fig 1. The critical time of the non-linear system

the prediction stage, the displacement increment will be obtained from the matrix equation

$$\frac{\partial \Psi_{i-1}}{\partial z^T} \Delta z_i = \left(\frac{\partial P_{i-1}}{\partial \lambda} - \frac{\partial \Psi_{i-1}}{\partial \lambda} \right) \Delta \lambda_i \qquad (4.2)$$

$$\Psi_i = \Psi(z_i, \lambda_i), \quad P_i = P(\lambda_i)$$

The first term of the right-hand side of the equation represents the direct increase of the nodal load vector. The second term represents the influence of the previous concrete stress history.

Equation (4.2) gives a linear approximation Δz_i of an exact displacement increment. Updated solution $z_{i-1} + \Delta z_i$ must be refined in the correction stage. The correction procedure may be obtained representing the equation (4.1) in the linearised form. The j-th update of the solution would be governed by the equation

$$\frac{\partial \Psi^{j-1}}{\partial z^T} \delta z^j_{(r)} = P^{j-1} - \Psi^{j-1} \qquad (4.3)$$

where

$$\Psi^j = \Psi(z^j, \lambda^j), \quad P^j = P(\lambda^j)$$

Right-hand side of the above equation represents the vector of unbalanced forces $r^{j-1} = P^{j-1} - \Psi^{j-1}$. This equation corresponds to the time-controlled correction procedure. More appropriate iterative correction procedure can be obtained using the so-called arc-length method proposed by E. Ramm [20]. The iteration path follows the normal plane to the tangent increment Δz_i obtained from the equation (4.2).

For the solution of the problem, the non-linear equation solver and a complex computer program, modelling non-linear behaviour of concrete, both in compression and tension zones, is elaborated. Incremental constitutive relations (3.1) and (3.2) for non-linear creep are implemented in the layer model of the reinforced concrete cross-section. The distinctive feature of the program is that the stress histories for all layers of the structural members are stored in the computer memory. The updated Lagrangian formulation of geometrically non-linear problem is used.

The program is written in Fortran and comprises four levels: 1) non-linear modelling of an individual concrete layer of the reinforced concrete cross-section, 2) evaluating the quasi-elastic incremental stiffness parameters and stress resultants of a cross-section, 3) computing incremental stiffness matrices and stress resultants of a finite element, 4) the solution of global non-linear equations for the whole structure using the arc-length method.

Three-node beam finite element is used in the third level of the algorithm. The third node is required to represent the non-uniformity of axial deformation of the element. The axial deformation depends on the concrete stress that varies along the member axis, while the stress resultant in both concrete and reinforcement is constant. The variable stiffness of the cross-section is also accounted for in the finite element model. The description of the finite element is given in full detail in author's article [19].

5. Numerical results

Pinned-end column under the constant long-lasting eccentrically applied axial force was analysed. The structural parameters of the column were taken the same as for the worked example presented in reference [11]. The effective length of the column is assumed l_0 = =15.0 m, the cross-section is $b \times h$ = 0.4×0.5 m, area of steel reinforcement is A_s = 12.32 cm² (Fig 2).

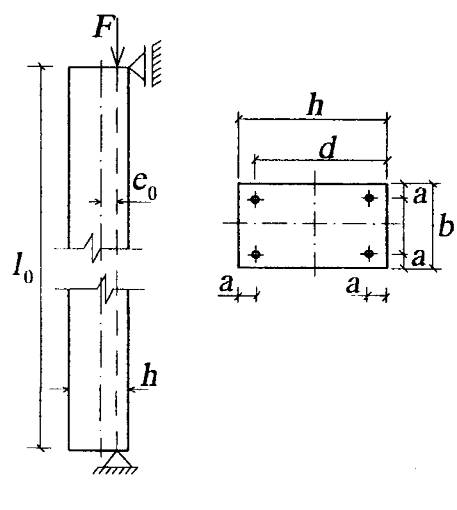

Fig 2. Eccentrically compressed column

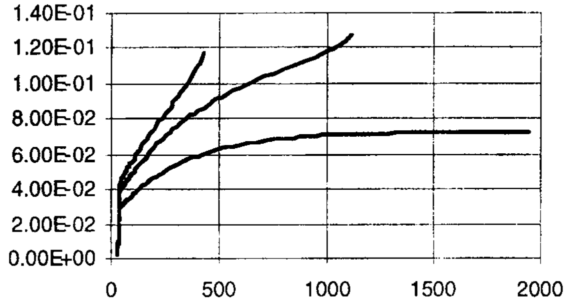

Fig 3. Displacements at the middle section of the column as functions of time, days: top curve — F=2.90 MN, middle — F=2.75 MN, lower — F=2.40 MN

Material parameters comply with the Soviet design code [21]. The characteristic yield strength and modulus of elasticity for steel are correspondingly $f_{yk} = 390$ MPa and E_s = 200000 MPa. The cubic strength of concrete and mean secant modulus of elasticity for concrete correspondingly f_{c28} = 40.0 MPa and E_{cm} = 36000 MPa. The axial load is applied at the ends of the column with the eccentricity e_0 = =0.0375 m. The slump of concrete mix is 2 cm, the relative humidity of surrounding atmosphere is W = =70 %. The notional size of a column, $M_0 = A/V$, m^{-1} in which A – the area of member surface exposed to drying, and V - the volume of a member, is M_0 = = 9.0 m^{-1}.

The load was imposed at the concrete age t_0 = 28 days, the start of concrete drying t_d = 28. For the sake of computational stability the load was assumed to be growing linearly until the time t = 40 days and then remained constant.

In order to keep numerical efficiency, the time steps were varied according to a geometrical progression, while the increments of a step parameter λ were kept constant. The time relating to the i-th step was assumed $t_i = t_0 q^i$, where t_0 is initial time and q is a time step parameter. Denoting the structure life period as a final time moment $t_N = t_0 q^N$ one can obtain the equality

$$\sqrt[i]{t_i/t_0} = \sqrt[N]{t_N/t_0}$$

hence,

$$t_i = t_0 \exp\left(\lambda_i \ln \frac{t_N}{t_0}\right) \tag{5.1}$$

where i = 1,..., N and $\lambda_i = i/N$

To identify the buckling phenomenon, several values of the acting force F were tried. Computed displacements of the middle section of the column are plotted in Fig 3 against the time variable. The critical time t_{cr} = 1118 days for load value F = 2.75 MN and t_{cr} = 426 days for load value F = 2.90 MN was obtained. For load F = 2.70 MN and less the loss of stability had not been reached within the assumed life span t_N = 10000 days of the structural member.

Fig 4 shows the distribution of concrete stress over the height of the cross-section. The concrete stress is growing with time and redistribution of internal forces between the concrete and steel reinforcement is observed: the concrete stress is diminishing and the reinforcement stress is growing. The extreme values of concrete stress MPa are indicated in the figure. It is remarkable that the critical load of the column computed by approximate formulas of the reference [11] is F_{cr} = 2.30 MN. The formula of the Soviet design code [21] yields more conservative value of the critical load F_{cr} = 2.115 MN.

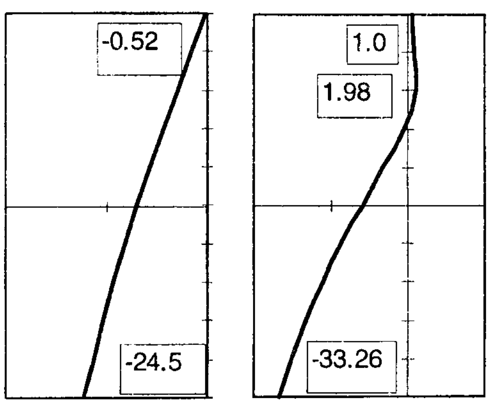

Fig 4. Distribution of concrete stress MPa over the height of the middle section when F=2.90 MN: left - at the time moment t = 41.2 days, right - at the time moment t = 426.4 days

6. Conclusions

1. Incremental constitutive relations for non-linear creep, based on the code-type recommendations, are constructed.

2. The finite element that models materially and geometrically non-linear time-dependent deformation and cracking of the tension zone of a reinforced concrete member has been successfully implemented in the Fortran program.

3. The arc-length algorithm was employed for the global analysis of a structure. The non-linear numerical simulation of structural behaviour reveals the non-linear creep buckling phenomenon.

4. The computer code can be used for interpretation of test results and verification of simplified methods used in everyday design practice.

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GELŽBETONINĖS KOLONOS NETIESINĖS DEFORMACIJOS IR PASTOVUMAS VEIKIANT ILGALAIKEI APKROVAI

I. Cypinas

Santrauka

Ankstesnėje autoriaus publikacijoje [19] paskelbtas geometriškai netiesinės gelžbetoninės konstrukcijos valkšnumo skaičiavimo skaitmeninis metodas įvertinant tempiamos betono zonos pleišėtumą, laikant, kad gniuždomas betonas neišeina iš tiesinio valkšnumo ribų. Šiame straipsnyje vertinamas gniuždomo betono netiesinis valkšnumas. Taikomas sluoksniuotasis skerspjūvio modelis.

Panaudotos netiesinio valkšnumo pareinamybės, kurios pateiktos Maskvos NIIŽB parengtose rekomendacijose betono valkšnumui ir susitraukimui apskaičiuoti [11]. Atsisakoma nuo valkšnumo kreivių panašumo hipotezės nepriklausomai nuo įtempimų lygio. Supleišėjusio tempiamo betono ir armatūros sąveika modeliuojama taikant

išsklidusio plyšio sąvoką, kai pagal (3.3) formulę sumuojamos tarp plyšių esančio betono deformacijos ir suvidurkintas plyšio atsivėrimo plotis.

Konstrukcijos deformuotasis būvis modeliuojamas geometriškai ir fiziškai netiesiniais strypiniais baigtiniais elementais. Sistemos deformacijų lygtys (4.2) sprendžiamos diskretiniais laiko prieaugiais, randant deformacijų trajektoriją jungtinėje laiko ir poslinkių erdvėje. Pastovumo klausimas sprendžiamas varijuojant apkrovos dydį ir nustatant konstrukcijos egzistavimo kritinį laiko momentą, kai sistemos kitimo pobūdis tampa singuliarus. Taikomas autoriaus sukurtas netiesinis baigtinis elementas ir autoriaus sudaryta netiesinių lygčių sprendimo programa.

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