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A NEW STRESS-STRAIN RELATIONSHIP FOR CRACKED TENSILE CONCRETE IN FLEXURE

G. Kaklauskas

1. Introduction

At the end of the nineteenth century, in testing small mortar prisms reinforced with steel wires it has been observed [1] that their tensile load-deformation response was well above the bare steel bar response. In 1908, Mörsch [2] explained that cracked concrete has the ability to decrease strain in reinforcement due to tensile stresses in the concrete between the cracks. This phenomenon was later called tension stiffening. Sometimes tension stiffening is confused with tension softening [3]. The latter is a property of plain concrete, while tension stiffening is a property of reinforced concrete. Due to bond with reinforcement, the cracked concrete between cracks carries a certain amount of tensile force normal to the cracked plane. The concrete adheres to the reinforcement bars and contributes to overall stiffness of the structure. Bond behaviour is a key aspect since it controls the ability of reinforcement to transfer tensile stresses to concrete. In the real, discrete cracking the cracks are spaced at final distances and the concrete between cracks interacts with the embedded steel bars. The stresses in the bars are highest in the cracks and decrease in the direction from the crack space. The stress distribution in a bar embedded in concrete with more or less regularly spaced cracks resembles a periodic function with peaks in the cracks and minimums between the cracks.

Based on a variety of assumptions, many constitutive models for reinforced concrete in tension have been proposed [4-15].

In approach based on experimental results, average stress-strain relationships for concrete in tension have been defined from several types of tests of reinforced concrete members subjected to tension, eg [8, 15, 16]. Tensile concrete behaviour in reinforced concrete has been often modelled by a relationship shown in Fig 1. An ascending straight line with a slope of $E_c$, concrete modulus of elasticity, reaching the tensile strength, $\sigma_{cr}$, models behaviour of non-cracked concrete. Its descending part models tension stiffening effect and is characterised by parameters $\alpha$ and $\beta$ which are related to tensile strength, $\sigma_{cr}$, and the corresponding strain $\varepsilon_{cr}$ respectively. Scanlon and Murray [17] used a saw-toothed and Lin and Scordelis [18] a curved diagram for the descending branch. Vebo and Ghali [19] used a trilinear piece-wise stress-strain relationship for concrete in tension. Although tension stiffening is most frequently assigned to the concrete, however, it can be handled by a change in the modulus assigned to the steel [6].

Different fixed values of parameter $\beta$ which controls the tension stiffening have been specified by different investigators. Lin and Scordelis [18], Scanlon and Murray [17], Gilbert and Warner [6] adopted a value of 10. Damjan and Owen [20] proposed values of 5 to 10 for shear-type cracking and 20 to 25 for flexural cracking. Mehlhorn [21] and Cope [22] used $\beta$ values which fall into the interval indicated above.

Based on experimental investigation of reinforced concrete panels subjected to pure shear Vecchio and Collins [8] proposed the following relationship for the cracked concrete:

![Fig 1. Average stress-strain relationship for concrete in tension](image-url)
\[ \sigma_t = \frac{\sigma_{cr}}{1 + \sqrt{200\epsilon_t}}, \quad (1) \]

where \( \epsilon_t \) is strain of tensile concrete.

This relationship was obtained from relatively heavily reinforced specimens, with a reinforcement percentage of 1.9% in at least one direction. Assessed \[ \beta \] for Eq (1) is about 20.

Carreira and Chu [23] proposed a relationship of the same general form as the stress-strain relationship in compression:

\[ \sigma_t = \frac{\beta_t f_t\left(\epsilon_t / \epsilon_t^*\right)}{\beta_t - 1 + \left(\epsilon_t / \epsilon_t^*\right)^2}, \quad (2) \]

where \( \beta_t \) is an empirical factor.

An analysis has shown [24] that parameter \( \beta \) has a great influence on numerical results particularly for lightly reinforced members. If to neglect tension stiffening in calculation of flexural members, deflections might be overestimated by 100 percent, particularly in the serviceability range of loads [6].

Accurate experimental investigations both on tensile [25, 26] and flexural [5, 27, 28] reinforced concrete members have shown that tension stiffening is significantly affected by such factors as reinforcement ratio, bar diameter, concrete strength, cover, and the distribution of reinforcement.

An attempt was made by Prakhya and Morley [29] to include several parameters affecting the tension stiffening into the stress-strain curve of tensile concrete for analysis of flexural members. On the basis of simplified assumptions and by using some experimental data [5, 27] they have applied Eq (2) proposed by Carreira and Chu [23] by modifying the empirical factor \( \beta_t \):

\[ \beta_t = \left( \frac{100A_t}{b(h-x_{nt})} \right)^{0.366} \left( \frac{b(h-x_{nt})}{n\pi d_b} \right)^{0.344} \left( \frac{c}{s} \right)^{0.146}, \quad (3) \]

where \( A_t \) is the cross-section area of tensile reinforcement; \( b \) is the width; \( h \) is the total depth; \( c \) is the clear cover to the reinforcement; \( d_b \) is the reinforcement bar diameter; \( n \) is the number of bars; \( s \) is the reinforcing spacing, and \( x_{nt} \) is the neutral axis depth neglecting tension in the concrete.

The experimental data used for developing the constitutive model did not cover cases of small reinforcement ratios \( p \geq 0.45\% \) and for the number of variables included, the relationship lacks statistical justification. Besides, due to the simplified assumptions, the relationship "used in the layered approach will always underestimate the tension stiffening stresses" [29].

A number of tension stiffening models based on fracture mechanics principles has been proposed by Hildeborg [30], Sih and DiTommaso [31], Petersson and Gustavasson [32], Bązant and Oh [33], and Nallathambi et al [34]. Bązant and Oh [35] suggested a value of

\[ \beta = E_c / E_t + 1 \quad (4) \]

where the slope of the descending branch

\[ E_t = \frac{0.483E_c}{0.393 + \sigma_{cr}} \quad [\text{MPa}] \quad (5) \]

for practical values of \( \sigma_{cr} \) giving \( \beta \) between 5 and 8.

Recently a method [24, 36, 37] has been developed for determining the average concrete stress-strain relations in tension and compression from experimental moment-strain (curvature) diagrams of reinforced concrete beams. The stress-strain relations are computed incrementally from equilibrium equations for the extreme surface fibres. The computation is based on an idea of using the previously computed portions of the stress-strain relations at each load increment to compute the current increments of the stress-strain relations. The proposed method has been successfully applied [24] to accurately performed experimental data of Clark and Speirs [5]. Stress-strain relationships for tensile concrete were obtained for 14 beams with moderate reinforcement ratios.

Present research is dedicated to investigation of tension stiffening effect in lightly reinforced concrete beams using experimental data reported in literature. Average stress-strain relationships for cracked tensile concrete are derived for beams reinforced with plain and deformed bars. Based on these and previously obtained relationships [24] a new constitutive relationship for tensile concrete in flexure is proposed.

2. Figarovskij test results in flexure

Figarovskij [38] conducted experiments on lightly reinforced concrete beams with different reinforcement ratios using both plain and deformed bars. The experimental program was devoted to investigation of short-term and long-term deformations and deflections of reinforced concrete beams. Present research deploys experimental data of the first and third series, ie rectangular cross-section specimens reinforced with plain and defor-
med bars, respectively. The specimens were nominally 3.2 m long, 250 mm high and 180 mm wide and were tested under a four-point loading system which gave a constant moment zone and two shear spans of 1.0 m each. The measured cross-section dimensions and data on 100 mm concrete cube strength, $R_{10}$, and tensile steel yield strength as well as details on bottom reinforcement for each of the specimen are given in Table 1. The specimens were also reinforced with top reinforcement comprising of two 6 mm bars located at 15 mm from the top surface. Stirrups in the shear spans were provided to all the beams.

Tests of the beams were terminated prior to the yielding of reinforcement and the experimental results were presented in terms of moment-deflection, $(M - f)$, and moment-curvature, $(M - \kappa)$, diagrams for each of the specimen [38]. The latter were obtained from the average strain measurements taken in the zone of pure bending at two levels: the extreme compressive concrete surface and the centroid of tensile reinforcement.

Concrete tensile strength, $\sigma_{cr}$, and modulus of elasticity, $E_c$, necessary for analysis were determined from the following empirical formulae:

$$\sigma_{cr} = 0.233\sqrt{R_{15}} \text{ [MPa]} \quad (6)$$

$$E_c = \frac{5.5R_{15}}{27 + R_{15}} \times 10^4 \text{ [MPa]}, \quad (7)$$

where $R_{15}$ is 150 mm cube compression strength taken as a product of $R_{10}$ and conversion factor 0.91.

### 3. Derivation of stress-strain relationships for cracked tensile concrete in flexure

This section presents results of derivation of stress-strain relationships for tensile concrete from the experimental $M - \kappa$ diagrams by the method proposed [24,36,37]. The experimental $M - \kappa$ diagrams are shown by dashed lines in Figs 2 and 3 for the first and third series respectively. For the purposes of analysis, the experimental moment-curvature diagrams were smoothed by MATLAB. Previous analysis [24] has shown that due to some extent irregular distribution of experimental points, the smoothed $M - \kappa$ diagrams have slightly wavy form leading to a similar shape of the computed $\sigma - \varepsilon$ curve. In order to obtain smoothed shapes for the material $\sigma - \varepsilon$ diagrams, the experimental $M - \kappa$ curves have to

### Table 1. Main characteristics of specimens

<table>
<thead>
<tr>
<th>Beam No</th>
<th>Name</th>
<th>Depth [mm]</th>
<th>Width [mm]</th>
<th>Effective depth [mm]</th>
<th>100 mm cube strength [MPa]</th>
<th>Tensile steel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Diameter [mm]</td>
<td>Section area $\times 10^{-4}$[m$^2$]</td>
<td>Yield strength [MPa]</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Reinforcement ratio [%]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>Series I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>P3-1Kk</td>
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<td>180</td>
<td>225</td>
<td>20.0</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>P3-2Kd</td>
<td>249</td>
<td>181</td>
<td>224</td>
<td>30.5</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>P2-2Kd</td>
<td>250</td>
<td>180</td>
<td>225</td>
<td>21.0</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>P2-1Kk</td>
<td>249</td>
<td>179</td>
<td>225</td>
<td>20.0</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>P1-1Kk</td>
<td>251</td>
<td>179</td>
<td>228</td>
<td>28.5</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
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<td>180</td>
<td>227</td>
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<tr>
<td>Series III</td>
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<td></td>
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<td>7</td>
<td>P3-2Pd</td>
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<td>180</td>
<td>230</td>
<td>31.5</td>
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<tr>
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<td>P3-1Pd</td>
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<td>180</td>
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<td>12</td>
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<td>P2-2Pb</td>
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<tr>
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<td>180</td>
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<tr>
<td>13</td>
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<td>248</td>
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<td>7</td>
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<tr>
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<td>180</td>
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</tr>
<tr>
<td>16</td>
<td>P0-1Pb</td>
<td>250</td>
<td>180</td>
<td>227</td>
<td>34.0</td>
<td>7</td>
</tr>
</tbody>
</table>
be averaged. The averaged curves were simply constructed from several characteristic experimental points. For most of the cases analysed, 5-8 experimental points were sufficient to obtain a numerically averaged curve which adequately represented all experimental points.

Averaged and smoothed $M - \kappa$ curves for the beams of the first and third series are shown by solid lines in Figs 2 and 3 respectively. The computed $\sigma_f - \epsilon_f$ curves for these diagrams are shown in Figs 4 and 5.
Although the curves shown in Figs 4 and 5 have quite a similar shape, the most striking is the difference in their extension length, which is characterised by strain $\varepsilon_{t,o}$ corresponding to zero stress. The present results in general support the previous findings [24] that strain $\varepsilon_{t,o}$ increases with decrease of reinforcement ratio $p$. Graphical presentation of this dependence taking $\varepsilon_{t,o}$ values in terms of parameter $\beta$ (Fig 1) is given in Fig 6, where signs “x” and “+” correspond to data points of beams of the first series (plain bars) and data points of the third series (deformed bars) respectively. It must be said that some computed $\sigma_t - \varepsilon_t$ curves did not have the actual $\varepsilon_{t,o}$ strain corresponding to zero stress. Most often this was due to early termination of the test. For such beams imaginary strains $\varepsilon_{t,o}$ were assumed, however for beam P3-1Kk of the first series and beam P1-2Pk of the third series $\varepsilon_{t,o}$ were not defined. Fig 6 also contains data points (shown by circles) of the previous analysis [24] of 16 beams tested by Clark and Speirs [5]. These beams were reinforced with deformed bars and had different reinforcement ratios and bar diameters. Curve fitting performed by MATLAB for points in Fig 6 corresponding to points (shown by circles) of the previous analysis [24] of series were reinforced with deformed bars and had different imaginary strains was due to early termination of the test. For such beams the first series (plain bars) and data points of the third series (deformed bars) respectively.

Therefore, the ascending branch of the calculated $\sigma_t - \varepsilon_t$ relationship had a missing part of initial stresses corresponding to the shrinkage stresses. With the shorter ascending branch, the zero point moves up and the descending branch enters the negative stress zone; c) assumption of the constant $\sigma_t - \varepsilon_t$ diagram for all tensile concrete fibres is inaccurate. As it was described earlier, the $\sigma_t - \varepsilon_t$ diagram is computed for the extreme fibres assuming that other fibres follow the same law. At the initial cracking stages, the computed $\sigma_t - \varepsilon_t$ curve actually represents average stresses of the fibres close to the extreme surface and reinforcement. An extreme fibre at an average strain $\bar{\varepsilon}_t > \varepsilon_{cr}$ carries some average stress $\bar{\sigma}_t$. It is assumed that a tensile fibre distant from the reinforcement also carries $\bar{\sigma}_t$ when the strain in that fibre reaches $\bar{\varepsilon}_t$. However, this can not be true, because distant fibres are less affected by bond with reinforcement and, therefore, carry lesser stresses. When equilibrium equations are solved, in order to compensate for these increased stresses, stresses in the extreme fibre are reduced what in some cases might lead to negative stresses.

Analysis has shown that $\sigma_t - \varepsilon_t$ curves for members with higher reinforcement ratios have little effect on the curvature and deflection calculation. Therefore, the nega-

\[ \beta = 32.8 - 27.6p + 7.12p^2 \] (8)
tive stress portions of $\sigma_t - \varepsilon_t$ diagrams can be simply excluded. Besides, the negative stress parts can be reduced or even eliminated if shrinkage effects are assessed.

Previous analysis [24] has shown that the phenomenon of the negative stress portions is more common for beams having higher reinforcement ratios. However, a $\sigma_t - \varepsilon_t$ curve has a reduced effect on curvatures for members with higher reinforcement ratios. Therefore, the negative portions of the curves can be simply excluded.

4. A new constitutive relationship for cracked tensile concrete in flexure

The stress-strain relationships for tensile concrete obtained from beam tests of Clark and Speirs [5] and Figarovskij [38], see Figs 4 and 5, have been used for derivation a new constitutive relationship. From a number of fitting curves considered, as a compromise between accuracy and simplicity, the following shape for the descending part of the $\sigma_t - \varepsilon_t$ relationship shown in Fig 7 has been proposed:

$$\sigma_t = a\sigma_{cr}\left(1 - \frac{\varepsilon_t}{\varepsilon_{cr}}\right)^{1 + \beta(1 - a)/a}$$

(9)

where

$$\varepsilon_t = \varepsilon_{cr} - \varepsilon_{cr}(1 - a)$$

(10)

Parameter $\beta$ characterising the length of the descending branch of the $\sigma_t - \varepsilon_t$ curve (see Fig 7) is equal to such $\varepsilon_t$ which corresponds to zero stress. Parameter $\beta$ is taken from Eq (8) and $\sigma_{cr}$ and $E_c$, if absent can be assessed from Eqs (6) and (7) respectively.

Concluding remarks

The present research is dedicated to investigation of tension stiffening effect in lightly reinforced concrete beams using short-term experimental data reported in literature. Applying the method developed by the author and his co-workers, average stress-strain relationships for cracked tensile concrete have been derived for beams reinforced with plain and deformed bars. In general, results of this analysis fitted within the trends set by the earlier work [24]. It has been shown that the shape of the relationships mostly depend on reinforcement ratio and surface of reinforcement bars. The length of the descending branch of the curves reflecting the tension stiffening effect was considerably more pronounced for beams with smaller reinforcement ratios. Use of deformed bars in the tensile zone also secured greater tension stiffening. Based on these and previously obtained stress-strain curves, a new constitutive relationship for tensile concrete in flexure has been proposed. The relationship in a simplified integrated manner takes into account complex effects of cracking, bond and shrinkage. This constitutive model can be applied not only in a finite element analysis, but also in a simple iterative technique based on classical principles of strength of materials extended to layered approach. The latter technique as a universal, simple and accurate tool can serve as an alternative to the code methods.

References


Išteikta 1999 10 27

NAUJA SUPLEIŠĖJUSIO TEMPIAMO BETONO ĮTEMPIŲ-DEFORMACIJŲ PRIKLASOMYBĖ LENKIAIEMS ELEMENTAMS

G. Kaklauskas

Santrauka

Straipsnyje pasiūlyta nauja supleišėjusio tempiamo betono įtempių-deformacijų priklausomybė lenkiamų gelžbetoninių elementų deformacijoms skaiciauti. Taikant šią priklausomybę, lenkiamų elementų deformacijos gali būti apskaičiuojamos tiek baigninių elementų metodu, tiek klasiškėmis medžiagės atsparumo formulėmis, skerspjūvi sudalijus į betono ir armatūros sluoksnius. Įvade pateikta plati supleišėjusio tempiamo betono įtempių-deformacijų priklausomybų [4-35] apžvalga.

Kuriant medžiagos priklausomybę, buvo taikomas autoriaus ir jo kolegų pasiūlytas metodas [24, 37], kuriuo iš eksperimentinių lenkiamų gelžbetoninių sijų momentų-kreivų ir (arba) momentų-deformacijų diagramų nustatoma visa tempiamo betono vidutinės įtempių-deformacijų diagrama, išskaitant ir jos krintančią dalį.

Šiame darbe tempiamo betono vidutinių įtempių-deformacijų kreives buvo nustatytos 16 mažai ir vidutiniškai armuotų stačiakampio skerspjūvio gelžbetoninių sijų, kurias trumpalaikė atkrova (dviejų koncentruotomis įegomis) išbandė Figarovskis [38]. Darbe panaudoti pirmosios ir trečiosios serijos sijų (1 lentelė), armuotų atitinkamai lygiai ir rumbuota armaitūra, duomenys. Iš eksperimentinių momentų-kreivių diagramų (2 ir 3 pav.) nustatytos tempiamo betono vidutinės įtempių-deformacijų priklausomybės pateiktos 4 ir 5 pav. Šių kreivių krintančių dalis charakterizuojama supleišėjusio betono darbą. Nors kreivių forma skirtingoms sijoms yra pakankamai panaši, kreives labiausiai skirtis krintančiosios dalies ilgiu. Krintančių kreives dalis charakterizuojama deformacija ε1,0, atitinkančia įtempių, lygius nuliu. Šios deformacijos, išreikštos santykiniai dydžiai $\beta = \varepsilon_{1,0} / \varepsilon_\text{cr}$ (kur $\varepsilon_\text{cr}$ yra betono supleišėjimo deformacija, žr. 1 pav.), priklausomybė nuo armavimo procento yra pateikta 6 pav. Šiame paveiksle kartu pateikti ankstesni tyrimų rezultatai, gauti Clarko ir Speirso eksperimentinės sijoms [5]. Aktualu $\beta$ priklausomybė nuo armavimo procento $p$ (6 pav.) aprašoma (8) priklausomybe. Apdorojus šiame darbe bei ankščiau gautus tempiamo betono įtempių-deformacijų diagramas, buvo pasiūlyta minėtoji medžiagos priklausomybė, aprašyta (9) arba (11) lygtimi.

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