

## Information technologies and multimedia Informacinės technologijos ir multimedija

### THE AIR VISCOSITY COEFFICIENT AND OTHER RELATED VALUES

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**Abstract.** Experimental measurements of the dynamic coefficient of air viscosity were done. Numerous other related values such as the kinematic coefficient of air viscosity, the mean free path and the mean thermal square velocity of air molecules were determined. The dependence of the air dynamic viscosity coefficient on temperature was also obtained. It allowed us to determine the Sutherland's corrections and to estimate the dimension of air molecules and the temperature gradients of the dynamic and kinematic coefficients of the air viscosity. The determined values were compared to the similar theoretical and experimental data obtained by other authors.

**Keywords:** air viscosity, mean thermal velocity, mean free path, air density, Sutherland's corrections, dimension of molecules.

#### Introduction

Viscosity, or the internal friction of liquid or gas molecules, belongs to the class of transport phenomena associated with the motion of molecules in fluid materials that create internal molecular resistance to the motion (Halliday et al., 2014). For this reason, the work done by the friction force leads to the transformation of the kinetic energy into the heat energy (Lide, 2006; Weast, 1984). In liquids and gases, freely moving molecules interact with each other, which leads to the equalization of their speeds. This alignment forms the internal friction which is characterized as the viscous force (Bogdanovičius, 2010).

The viscosity of the gas is also related to the diffusion and free path values, the mean thermal rate of molecular motion and the others quantities. Measurements of the gas viscosity coefficient will allow to determine the values of these values. Once the measurements were obtained, the coefficient of dynamic viscosity was determined as a function of temperature. These measurements make it possible to determine the size of the air molecules.

The aim of this work was to determine the dynamic viscosity coefficient ( $\eta$ ) of the air. Knowing the air dynamic viscosity coefficient and the air density ( $\rho$ ), we determine the mean free path of air molecules ( $\bar{\lambda}$ ), the mean thermal square velocity ( $\bar{v}$ ) and the kinematic viscosity coefficient ( $\nu$ ). After the measurements of the air viscosity coefficient dependence on temperature  $\eta(T)$  were ob-

tained, we have identified the temperature dependence on the following values:  $\rho(T)$ ,  $\lambda(T)$ ,  $\nu(T)$ ,  $\nu(T)$  and determined the dimensions ( $d$ ) of the air molecules.

#### 1. Measurement methodology and main formulas

In a laboratory conditions, there are several methods for determining the viscosity of air (see, for example, Halliday et al., 2014; Astrauskienė et al., 2009). Under our laboratory conditions, a stand of the connecting vessels shown in Figure 1 was used to perform measurements of the air dynamic viscosity coefficient ( $\eta$ ). The stand consists of vessels 2 and 4 connected by a capillary 3, the length ( $L$ ) and radius ( $r_0$ ) of which are known. The vessel 4 is connected to a water-filled vessel 5. The vessel 2 is connected to a vessel 1 filled with a moisture-absorbing material. The pressure difference at the ends of the capillary 3 is measured by the water manometer 6. A measuring beaker 7, a thermometer, a barometer, a chronometer and scales are also required for measuring the volume of water flowing from the vessel 5.

The viscosity appears when the air flows from the vessel 2 in which the atmospheric pressure is equal to  $p_0$ , the flow passes through the capillary 3 and reaches the vessel 4 in which the pressure is lower because of the water flowing from the vessel 5. The volume of the water that fills the beaker 7 is estimated, and the measured values the precisely adjusted with scales. The volume of water was

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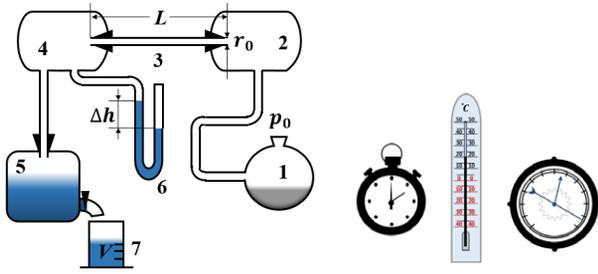


Figure 1. The measuring stand structural scheme and the devices for measurements: chronometer, thermometer, and barometer

determined by measuring the weight of the liquid with scales by applying the mass and volume ( $m/V$ ) relation.

According to the Poiseuille's formula (e.g., Halliday et al., 2014), where the radius ( $r_0$ ) and the length ( $L$ ) of the capillary tube, the pressure difference ( $\Delta p$ ) at the ends and the time of the measurements ( $t$ ) are known, the volume of the water ( $V$ ) is expressed by the equation

$$V = \frac{\pi r_0^4 t}{8\eta L} \Delta p. \quad (1)$$

The difference of the pressure at the ends of the capillary could be measured by a water manometer (Bogdanovičius, 2010):

$$\Delta p = \rho g \Delta h, \quad (2)$$

where  $\Delta h$  is the level height difference of the liquid manometer,  $\rho$  – the density of manometer liquid,  $g$  – the free fall acceleration.

From equations (1) and (2) the air dynamic viscosity coefficient ( $\eta$ ) is:

$$\eta = \frac{\pi g r_0^2 \rho t \Delta h}{8VL}. \quad (3)$$

Knowing the air viscosity coefficient ( $\eta$ ), the other related parameters of air could be determined. According to the molecular kinetic theory (e.g., Halliday et al., 2014; Lide, 2006; Weast, 1984; Bogdanovičius, 2010), the coefficient of viscosity ( $\eta$ ) is related with the density of gas  $\rho_0$ , the mean thermal square velocity ( $\bar{v}$ ) of molecules, and the mean free path ( $\bar{\lambda}$ ) of molecules.

It is important to clarify that in the case of “air molecules”, air is not considered to be a mixture of different gases, but a mono-constituent diatomic gas of the same molar mass  $M = 29.0 \cdot 10^{-3}$  kg/mol. Considering the chemical composition of air gas (e.g., Zhang, 2004), we see that nitrogen molecules  $N_2$  make up to 78.084% and oxygen molecules up to  $O_2$  20.946% of the air volume. Accordingly, 99.03% of the air volume consist of diatomic molecules and only 0.97% consist of polyatomic molecules. The characteristics of air are indeed similar to those of diatomic gas (Tiwary et al., 2019). The degrees of freedom, thermal velocity, air density, and other measurements can be made, which rather accurately indicate that the air can be considered a monoconstituent diatomic gas (Halliday et al., 2014; Lide, 2006; Weast, 1984; Bogdanovičius, 2010).

## 2. Dimension clarification of the air molecules

The simplest model to determine size of molecules is the model of solid spherical approximation. Knowing the air pressure ( $p_0$ ), the temperature ( $T$ ) and the mean free path of air molecules ( $\bar{\lambda}$ ), the diameter ( $d$ ) of the air molecules can be determined as follows (Weast, 1984):

$$d = \left( \frac{kT}{\sqrt{2}\pi\bar{\lambda}p_0} \right)^{1/2}. \quad (4)$$

When the temperature ( $T$ ), the pressure ( $p$ ) and the mean free path of molecules ( $\bar{\lambda}$ ) were inserted into the formula from Table 2, the diameter value of  $d = 272$  pm of the air molecules was calculated.

The model of solid spheres does not take into account the interaction between the molecules and the peculiarities of the external electronic states. These molecular phenomena are considered by taking into account the Sutherland's corrections (Chapman & Cowling, 1991; Smits & Dussauge, 2006), which make the equation of viscosity more complex (Smits & Dussauge, 2006; White, 2005):

$$\eta = \frac{5}{16} \sqrt{\frac{MkT}{\pi N_A}} \cdot \frac{1}{1+C/T} \cdot \frac{1}{d_0^2}. \quad (5)$$

To determine the Sutherland's constant ( $C$ ), the measurement of the air viscosity coefficient at different temperatures was performed (Table 1 and Figure 2). Knowing the air density ( $\rho_0$ ), not only the dynamic ( $\eta$ ), but also the kinematic viscosity  $\nu = \eta/\rho$  dependence on temperature  $\nu = \nu(t)$  can be determined. The dynamic ( $\eta$ ) and kinematic ( $\nu$ ) viscosity coefficients dependent on temperature are presented in Figure 2. Due to the influence of air density ( $\rho$ ) on the kinematic viscosity coefficient ( $\nu$ ), this dependence is not parallel.

The Sutherland's constant ( $C$ ) could be determined by the following formula:

$$C = \frac{\eta_2 T_1^{\frac{3}{2}} T_2^{\frac{3}{2}} - \eta_1 T_2^{\frac{3}{2}} T_1^{\frac{3}{2}}}{\eta_1 T_2^{\frac{3}{2}} - \eta_2 T_1^{\frac{3}{2}}}. \quad (6)$$

By inserting the temperature and viscosity values into equation (6), the Sutherland's constant for air was estimated:  $C = 39.6$  K.

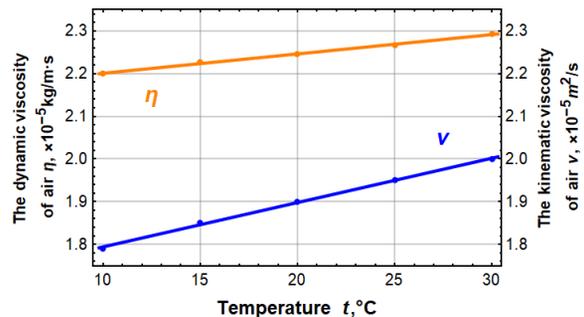


Figure 2. The dynamic ( $\eta$ ) and kinematic ( $\nu$ ) viscosity coefficient dependence on temperature

Knowing the value of the Sutherland's constant ( $C$ ), the diameter ( $d_0$ ) of the air molecule according to formula (5) can be determined as follows:

$$d_0 = \left[ \frac{25MkT}{256\pi N_A \eta^2 (1 + C/T)^2} \right]^{\frac{1}{4}}. \quad (7)$$

Using the value of viscosity determined during the experiment and the Sutherland's constant, from formula (7) we obtain:

$$d_0 = 346.7 \text{ pm}. \quad (8)$$

This dimension is equal to the effective diameter of molecules, when  $T \rightarrow +\infty$ . To determine the effective diameter of air molecules at a finite temperature, the diameter dependence on temperature should be used (White, 2005):

$$D = d_0 \sqrt{1 + \frac{C}{T}}. \quad (9)$$

The effective diameter ( $D$ ) of air molecules was determined in RT ( $T = 273 \text{ K}$ ):

$$D = 370.9 \text{ pm}. \quad (10)$$

Remind here that air is considered to be a one-component diatomic gas of the same molar mass  $M = 29.0 \cdot 10^{-3} \text{ kg/mol}$  (see section 1).

It is well known that in the large temperature change range the dependences of  $\eta = \eta(T)$  and  $\nu = \nu(T)$  are quite complex (Weast, 1984). Nevertheless, in the range of the temperature  $T = 283 \div 303 \text{ K}$  these temperature dependences are almost linear (Table 1 and Figure 2).

Table 1. The air density ( $\rho$ ), dynamic ( $\eta$ ) and kinematic ( $\nu$ ) coefficients of viscosity, the mean square velocity ( $\bar{\nu}$ ) and the mean free path ( $\bar{\lambda}$ ) dependence on temperature ( $T$ )

$T, \text{ K}$	283	288	293	298	303
$\eta, 10^{-5} \text{ kg/(m}\cdot\text{s)}$	2.20	2.23	2.25	2.27	2.29
$\rho, 10^{-3} \text{ kg/m}^3$	1.22	1.20	1.18	1.16	1.14
$\nu, 10^{-5} \text{ m}^2/\text{s}$	1.79	1.85	1.90	1.95	2.00
$\bar{\nu}, \text{ m/s}$	454	459	463	466	470
$(\bar{\lambda}), 10^{-7} \text{ m}$	1.19	1.22	1.24	1.26	1.28

This allowed us to determine the gradients of dynamic ( $\eta$ ) and kinematic ( $\nu$ ) coefficients of the air viscosity, which respectively are  $\Delta\eta / \Delta T = 4.91 \cdot 10^{-8} \text{ kg/m}\cdot\text{s}\cdot\text{K}$  and  $\Delta\nu / \Delta T = 5.17 \cdot 10^{-8} \text{ m}^2/\text{s}\cdot\text{K}$ . In Figure 2 these values correspond to the angle of the inclination of the measured dependencies to the temperature axis  $T$ .

### 3. The average weighted mean of the molecule dimension and others results

Knowing the volumetric and mass chemical composition of the dry air (e.g., Zhang, 2004; Tiwary et al., 2019) as well as the values of the covalent radii of the main air-forming gases (Lide, 2006; Weast, 1984), the dimensions of

air molecules could be estimated as the average weighted mean of the molecule dimensions of the air-forming gases. The average mass and volume diameter of air molecules is correspondingly  $\bar{D}_m = 335.2 \text{ pm}$  and  $\bar{D}_v = 338.9 \text{ pm}$ . However, the weighted average value of the air molecule dimension is not satisfactory. Historically, the first to evaluate the dimensions of the air molecules was Loschmidt (1995). The value he obtained was inaccurate and 2.6 times over the true value. The values of other authors ( $D = 355.7 \text{ pm}$  (Nave & Nave, 1985),  $D = 360.5 \text{ pm}$ ) in our opinion are slightly reduced, because the actual value is  $D = 370 \div 372 \text{ pm}$  (Davisson et al., 2012; Zhang, 2004; Chapman & Cowling, 1991) (Table 2).

Table 2. The sizes of air molecules established by different authors

Reference	(Loschmidt, 1995)	(Tiwary et al., 2019)	(Shakhashiri, 1986)	(Nave & Nave, 1985)	(Zhang, 2004; Davisson et al., 2012)	Measured value	(Lide, 2006)
$D, \text{ pm}$	969	355.7	360	360.5	370	370.9	372

As one can see from Table 3, the value of 370.9 pm obtained by this research is close to the values of 370÷372 pm given in manuals and monographs.

## The results and conclusions

Finally, after experimental measurements and theoretical calculations we can formulate the following results and conclusions:

1. Determined by us the air viscosity coefficient  $\eta = (2.245 \pm 0.101) \cdot 10^{-5} \text{ kg/m}\cdot\text{s}$  differs from the theoretical value  $\eta = 1.85 \cdot 10^{-5} \text{ kg/m}\cdot\text{s}$  (Halliday et al., 2014). However, it must be borne in mind that the theoretical value of the air viscosity coefficient describes the viscosity of dry air, while in this research the measurements were made in humid air which obviously has a higher viscosity coefficient. The humidity of the air in our experimental measurements was the cause of the difference between theoretical and experimental values of the air viscosity.
2. Determining the mean free path ( $\bar{\lambda}$ ) and the mean thermal square velocity ( $\bar{\nu}$ ) of air molecules, the following data were obtained:  $\bar{\lambda} = (1.237 \pm 0.061) \cdot 10^{-7} \text{ m}$  and  $\bar{\nu} = 462.5 \pm 3.6 \text{ m/s}$  which corresponds to the known values of the free path and the mean thermal square velocity of oxygen and nitrogen molecules under normal conditions (e.g. Lide, 2006; Weast, 1984; Tiwary et al., 2019).
3. The performed measurements of the viscosity coefficient dependence on temperature allowed us to determine the gradients of dynamic ( $\eta$ ) and kinematic ( $\nu$ ) coefficients of the air viscosity, which

respectively are  $\Delta\eta/\Delta T = 4.91 \cdot 10^{-8} \text{ kg/m} \cdot \text{s} \cdot \text{K}$  and  $\Delta\nu/\Delta T = 5.17 \cdot 10^{-8} \text{ m}^2/\text{s} \cdot \text{K}$ . If in the larger temperature change range the dependences of  $\eta = \eta(T)$  and  $\nu = \nu(T)$  are quite complex (Weast, 1984), then in the temperature range  $T = 283 \div 303 \text{ K}$  these temperature dependences are almost linear (Table 1 and Figure 2).

4. For the estimation of the air molecule dimensions we obtained two values. In the approximation of solid spheres, according to equation (4) the value of the molecular diameter  $d = 272 \text{ } \mu\text{m}$  was obtained. However, after temperature measurements of the coefficient of viscosity  $\eta$  (Chapter 2), we obtained a more accurate air molecule diameter  $d = 370.9 \text{ pm}$ . This value is close to the values of  $370 \div 372 \text{ pm}$  given in manuals and monographs (Table 2).

By placing the stand in an airtight transparent shell, the method can be applied to homogeneous gases of a known chemical composition. For example, by placing a stand in an airtight transparent container and filling it with pure nitrogen, we can determine all of the above values for nitrogen: its dynamic ( $\eta$ ) and kinematic ( $\nu$ ) viscosity coefficients and their dependences on temperature  $\eta(T)$  and  $\nu(T)$ , the mean free path of nitrogen molecules ( $\lambda$ ), the mean square velocity ( $\bar{v}$ ), the nitrogen density ( $\rho$ ) and the diameter ( $D$ ) of the nitrogen molecule.

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## ORO KLAMPUMO KOEFICIENTAS IR KITI SUSIETI DYDŽIAI

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Santrauka

Buvo atlikti eksperimentiniai oro klampumo koeficiento matavimai. Matuojant buvo nustatyti kitų susijusių dydžių eksperimentinės vertės: oro klampumo kinematinis koeficientas, vidutinis laisvasis kelias ir vidutinis oro molekulių šiluminis kvadratinis greitis. Taip pat gauta oro dinaminio klampumo koeficiento priklausomybė nuo temperatūros. Gautosios vertės leido nustatyti Sazerlendo pataisą, įvertinti oro molekulių matmenis, taip pat oro klampumo dinaminių ir kinematinų koeficientų temperatūros gradientus. Vertės palygintos su kitų autorių gautais teoriniais ir eksperimentiniais rezultatais.

**Reikšminiai žodžiai:** oro klampa, vidutinis šiluminis greitis, vidutinis laisvasis kelias, oro tankis, Sazerlendo pataisos, molekulių matmenys.