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# CALCULATION TECHNIQUE FOR STRESS-STRAIN ANALYSIS OF RC ELEMENTS SUBJECTED TO HIGH-CYCLE COMPRESSION

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**Abstract.** Theoretical models for the evaluation of fatigue on reinforced concrete structures most commonly fall into two major groups. The first and more widely used group of models is based on S-N curves and the static stress state. These models provide the final load cycle count until structural failure but do not consider redistribution of stresses within the structure or strain evolution over time. The second group accounts for deterioration of concrete over time. However, due to difficulties in application and high computational costs, these models are not fully evolved. A new simplified iterative method for reinforced concrete columns based on previous research by Zanuy *et al.* (2009) is presented in this paper. This model allows for estimation of stress redistribution and progressive degradation of concrete under high-cycle loading.

**Keywords: cyclic** loading, degradation, fatigue, reinforced concrete, S-N curves.

## Introduction

The effects of cyclic loading on reinforced concrete (RC) structures have become relevant in the last decades from the point of view of the adequate structural design and safety assessment.

The present tendency of optimization in structural design focuses on maximizing material efficiency, leading to higher stresses in the materials throughout their lifespan. In the case of RC members, structural optimization leads to slender elements becoming more widespread. It also explains a larger variation of stresses between the maximum and the minimum loads, which leads to increased fatigue problems. That is the case of approach slabs, road pavements or bridge deck slabs, where the dead load is only a fraction of the live load. An appropriate design of durable RC structures has to consider all the possible deterioration mechanisms. Therefore, fatigue performance should be given more attention as design criteria by structural designers working with structures susceptible to cyclic loading (wind power plants, offshore structures, roads, etc.).

This article introduces a new method to examine the behaviour of RC elements under cyclic actions. A time-dependent and appreciative of progressive degradation in concrete under compression model is applied (Zanuy *et al.* 2009). The application of this model allows for overcoming of problems associated with the usage of current design codes, which fail to account for the degradation mechanism of concrete. Such a method provides useful insights for assessing fatigue-carrying capacity of existing structures likely to be subjected to increased loads in the future. A new approach to investigate the stress and strain behaviour of reinforced concrete columns subjected to high-cycle compressive axial loading is proposed herein. The aims of this research paper are: (1) to develop a model for time-dependent stress-strain analysis of RC columns subjected to fatigue loading, also taking into account the degradation of concrete; (2) apply the proposed model to estimate the deformation response of mentioned RC columns; (3) to perform a parametric study for the stress-strain behaviour of these columns.

## Fatigue model for concrete in compression

A significant number of experimental studies have been performed to better appreciate the fatigue behaviour of concrete in compression. However, majority of them focused only on obtaining the number of load cycles to failure ($N_f$) under constant stress limits. A strong dependence of fatigue life on the upper stress level $\sigma_{max}$ has been observed (Holmen 1982). Other researchers have proposed different formulations to consider the effect of additional parameters, such as, load frequency $f$, minimum stress level $\sigma_{min}$ and stress ratio $R = \sigma_{min}/\sigma_{max}$ (Aas-Jakobsen 1970; Tepfers, Kutti 1979;

Stemland *et al.* 1990; Zhang *et al.* 1996). A model proposed by Hsu (1981) is the most applicable due to the inclusion of additional parameters that other authors neglect:

$$\log N_f = \frac{1 + 0{,}0294 \log f - S_{max}}{0{,}0662\left(1 - 0{,}556R\right)}, \quad N_f \geq 1000, \quad (1)$$

where $S_{max}$ is normalized maximum stress, $S_{max} = \sigma_{max} / f_{cm}$; $f_{cm}$ is the mean value of concrete compressive strength.

*Time-dependent material model*

Time-dependent material model was first introduced by Zanuy *et al.* (2009) and it focused on the uniaxial behaviour of concrete under cyclic loading. As stated before, the fatigue response of concrete in compression is largely dependent on the stress level. Therefore, a reliable stress-strain relationship for the short-term (static) behaviour of concrete is essential. The generally accepted nonlinear σ-ε curve for normal-strength concrete in compression of Model Code 2010 (CEB-FIP 2012) is taken to provide the initial stress-strain state of the fatigue process $\left(N = 1\right)$ and is also used as a failure envelope for the cyclic process (Fig. 1).

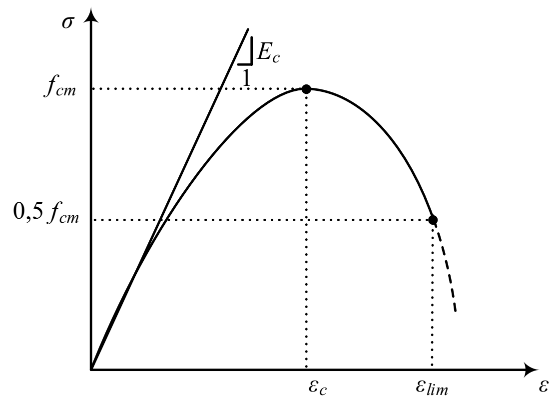


Fig. 1. Static stress-strain relationship for concrete in compression

The expression is as follows:

– for ascending branch of the σ-ε curve:

$$\sigma = \frac{E_c \dfrac{\varepsilon}{f_{cm}} - \left(\dfrac{\varepsilon}{\varepsilon_c}\right)^2}{1 + \left(E_c \dfrac{\varepsilon_c}{f_{cm}} - 2\right)\dfrac{\varepsilon}{\varepsilon_c}} f_{cm}, \quad (2)$$

– for descending branch of the σ-ε curve:

$$\sigma = \left[\begin{array}{c} \left(\dfrac{1}{\varepsilon_{lim}/\varepsilon_c}\xi - \dfrac{2}{\left(\varepsilon_{lim}/\varepsilon_c\right)^2}\right)\left(\dfrac{\varepsilon}{\varepsilon_c}\right)^2 \\ + \left(\dfrac{4}{\varepsilon_{lim}/\varepsilon_c} - \xi\right)\left(\dfrac{\varepsilon}{\varepsilon_c}\right) \end{array}\right]^{-1} f_{cm}, \quad (3)$$

$$\xi = \frac{4\left[\left(\dfrac{\varepsilon_{lim}}{\varepsilon_c}\right)^2\left(E_c\dfrac{\varepsilon_c}{f_{cm}} - 2\right) + 2\dfrac{\varepsilon_{lim}}{\varepsilon_c} - E_c\dfrac{\varepsilon_c}{f_{cm}}\right]}{\left[\left(E_c\dfrac{\varepsilon_c}{f_{cm}} - 2\right)\left(\dfrac{\varepsilon_{lim}}{\varepsilon_c}\right) + 1\right]^2}; \quad (4)$$

where $E_c$ is the initial tangent modulus of deformation, $\varepsilon_{lim}$ is the concrete strain at $0{,}5 f_{cm}$ in the softening part of the curve, and $\varepsilon_c$ indicates the concrete strain that corresponds to a peak stress $f_{cm}$.

To examine the fatigue effect of concrete, a time-dependent material model based on maximum concrete strain $\varepsilon_{max}$ is suggested. Selected variable is able to define the material state at a determined instance in fatigue life ($N/N_f$) (see Fig. 2).

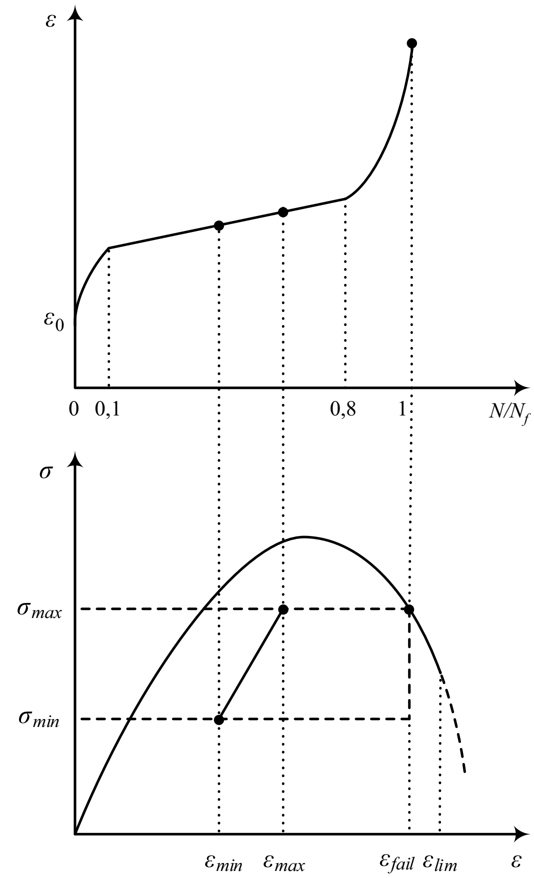


Fig. 2. Theoretical model (according to Zanuy *et al.* 2009)

According to Zanuy *et al.* (2009), the model assumes that the replication of the fatigue process cycle by cycle is numerically ineffective, so an analytical expression for the maximum strain with respect to the number of cycles is required. The equation of $\varepsilon_{max}$ is expressed as a function of the number of cycles and the loading conditions:

$$\varepsilon_{max} = f\left(\frac{N}{N_f}, \frac{\sigma_{max}}{f_{cm}}, \frac{\sigma_{min}}{f_{cm}}\right). \quad (5)$$

In order to identify the relative lifetime instant, the resistant number of cycles ($N_f$) is necessary. To obtain a value of $N_f$, the S-N curves proposed by Hsu (refer to Eq. (1)) are selected due to the most extensive approach on the influence of several parameters. As mentioned before, cyclic loading causes a redistribution of stresses in concrete components. This means that a process of variable stress limits is developed in each material fibre. Therefore, a direct application of equation (5) is not possible and the process is evaluated by a number of shorter processes for which constant stress limits can be presumed. To relate them to each other, an accumulation criterion is required, thus the concept of the equivalent number of cycles ($N_{eq}$) is now introduced as a new accumulation rule (Zanuy *et al.* 2009). The parameter $N_{eq}$ is the number of load cycles that is necessary to be applied in a fatigue process with constant limits ($\sigma_{max}, \sigma_{min}$) until a total strain of $\varepsilon_{max}$ is reached. It may be simply calculated using the complete analytical expression (refer to Eqs. (5) and (7) to (16)).

The new concept can be easily explained by imagining a concrete specimen, which is first subjected to $N_n$ load cycles with constant limits ($\sigma_{max,n}$, $\sigma_{min,n}$) and afterwards to $N_{(n+1)}$ additional cycles varying between ($\sigma_{max,(n+1)}$, $\sigma_{min,(n+1)}$) (see Fig. 3(a)).

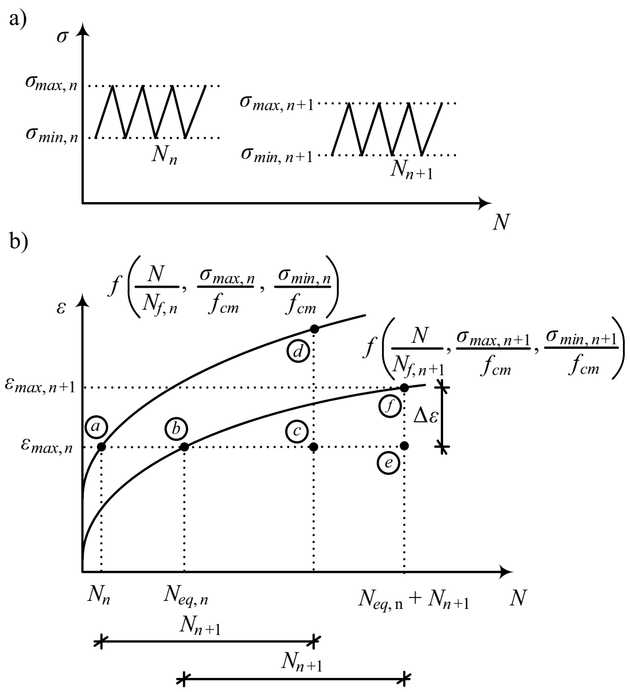


Fig. 3. (a) Load history with two different stress level steps; and (b) strain increment employing equivalent number of cycles (Zanuy *et al.* 2009)

The concept of the equivalent number of cycles allows obtaining the increase in strain due to the last $N_{(n+1)}$ cycles by means of the following expression:

$$\Delta\varepsilon = \varepsilon_{max,(n+1)} - \varepsilon_{max,n} = f\left(\frac{N_{eq,n}+N_{(n+1)}}{N_f}, \frac{\sigma_{max,(n+1)}}{f_{cm}}, \frac{\sigma_{min(n+1)}}{f_{cm}}\right) - \varepsilon_{max,n}. \quad (6)$$

The calculation of the strain increment is explained in Fig. 3(b). There $N_{eq,n}$ is the number of load cycles needed to develop a total strain $\varepsilon_{max,n}$ in the fatigue process under stress levels ($\sigma_{max,(n+1)}$, $\sigma_{min,(n+1)}$). The additional number of cycles $N_{(n+1)}$ must be introduced from point B, leading to the strain increase EF. It is apparent from Fig. 3(b) that this value is smaller than the one obtained without introducing the equivalent number of cycles (segment CD), which is based on generally accepted damage accumulation model presented by Palmgren-Miner (Miner 1945).

*Evolution law for estimating strain*

According to Zanuy *et al.* (2009), the evolution law for the total strain is determined from the data available in the literature. The proposed curve of strain evolution (see Fig. 4) reproduces three typical stages of the fatigue process of concrete. Second-order parabolic equations (Eqs. (7) and (9)) are used for the first and third stages, whereas a linear expression (Eq. (8)) is employed for the second stage, so that the experimental S-shaped evolution (see Fig. 4) is attained. According to Holmen (1982), the transition points between stages 1–2 and 2–3 are supposed to occur at 10% and 80% of the fatigue live, respectively.

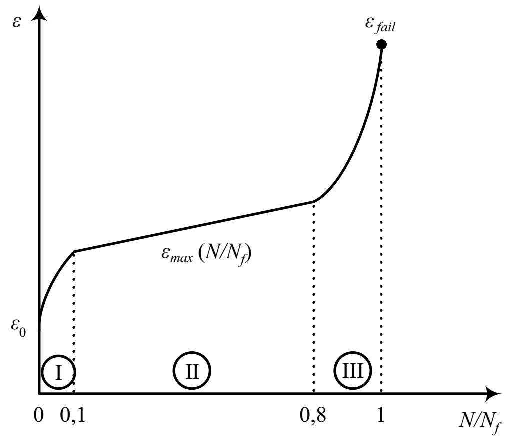


Fig. 4. Variation of concrete deformations with number of cycles

The strain evolution curve is defined as follows:

$$\frac{\varepsilon_{max}}{\varepsilon_0} = 1 + A\frac{N}{N_f} + B\left(\frac{N}{N_f}\right)^2, \quad 0{,}0 \le \frac{N}{N_f} < 0{,}1, \quad (7)$$

$$\frac{\varepsilon_{max}}{\varepsilon_0} = \varepsilon_{1-2} + \varepsilon_2\left(\frac{N}{N_f} - 0{,}1\right), \quad 0{,}1 \le \frac{N}{N_f} < 0{,}8, \quad (8)$$

$$\frac{\varepsilon_{max}}{\varepsilon_0} = \varepsilon_{1-2} + \varepsilon_2\left(\frac{N}{N_f} - 0{,}1\right) + C\left(\frac{N}{N_f} - 0{,}8\right)^2,$$

$$0{,}8 \le \frac{N}{N_f} < 1; \qquad (9)$$

where

$$A = 20(\varepsilon_{1-2} - 1) - \varepsilon_2, \qquad (10)$$

$$B = 100(1 - \varepsilon_{1-2}) + 10\varepsilon_2, \qquad (11)$$

$$C = 25\left(\frac{\varepsilon_{fail}}{\varepsilon_0} - \varepsilon_{1-2} - 0{,}9\varepsilon_2\right). \qquad (12)$$

Equations (7) to (9) give the maximum strain with respect to its initial value $\varepsilon_0$ at the first load cycle ($N = 1$). The ratio $\varepsilon_{1-2}$ defines the relative strain at $0{,}1N_f$, which is the transition between stages 1 and 2, and $\varepsilon_2$ provides the constant strain rate of the second domain. Both are defined according to Holmen's approach (Holmen 1982):

$$\varepsilon_{1-2} = \frac{1{,}184}{S_{max}}, \qquad (13)$$

$$\varepsilon_2 = \frac{0{,}74037}{S_{max}}, \qquad (14)$$

$$\varepsilon_2 \le \frac{1}{0{,}9}\left(\frac{\varepsilon_{fail}}{\varepsilon_0} - \varepsilon_{1-2}\right). \qquad (15)$$

## Evaluation of stress redistribution in concrete

Due to an extensive and complicated disclosure of the proposed algorithm a further sectional analysis is not presented in this paper. A detailed description is provided in the author's thesis (Tamulenas 2014).

A flow-chart of the proposed algorithm based on a time-dependent material model developed by Zanuy *et al.* (2009) is presented here (see Fig. 5). The algorithm considers the redistribution of stresses in reinforced concrete columns subjected to high-cycle fatigue loading. A step-by-step procedure is introduced in order to obtain a full evolution of the degradation mechanism of concrete until instant when a failure occurs. The limit state of RC column is governed either by the yielding of reinforcement (e.g. 400 MPa, referring to S400) or by concrete reaching the maximum number of load cycles ($N_f$).

## Numerical study of reinforced concrete columns

A numerical study is carried out by applying an algorithm presented above. Four columns subjected to cyclic loading have been analysed (see Table 1 and Fig. 6). Results of numerical analysis are summarized by graphical illustrations in Figure 7 to Figure 9, and in Table 2.

Table 1. Main characteristics of analysed columns

| No. | ρ, % | $f_{cm}$, MPa | $E_c$, GPa | $E_s$, GPa | $f$, Hz | $S_{0,max}$ | $S_{0,min}$ |
|---|---|---|---|---|---|---|---|
| A | 0,503 | 38 | 33,5 | 205 | 5 | 0,80 | 0,40 |
| B | 1,131 | | | | | | |
| C | 1,689 | | | | | | |
| D | 2,182 | | | | | | |

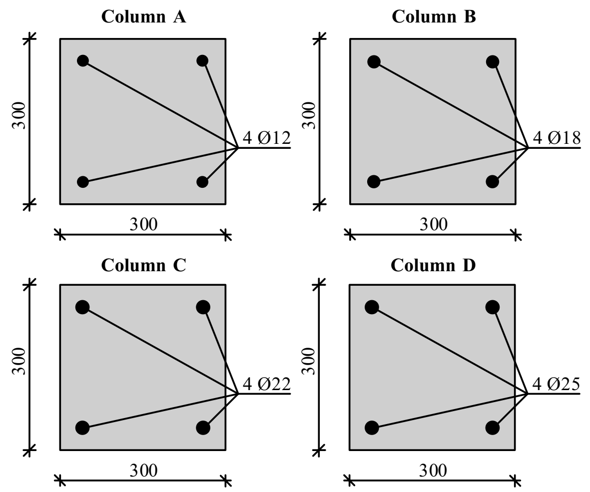


Fig. 6. Cross-sections of the studied columns

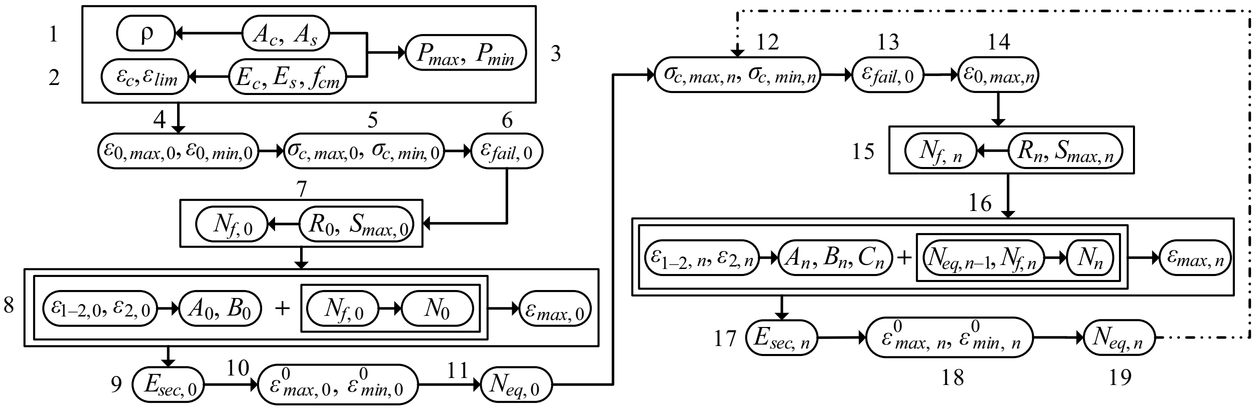


Fig. 5. A flow-chart of the proposed algorithm

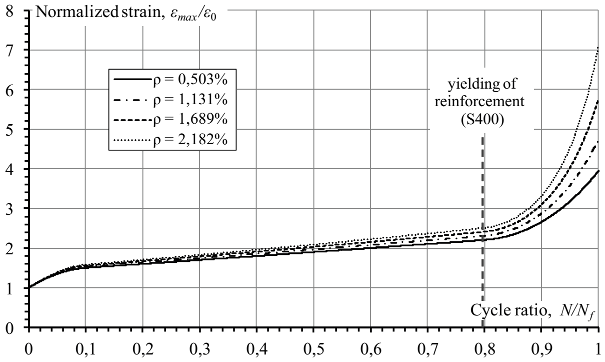


Fig. 7. Evolution of longitudinal strain with number of cycles regarding to reinforcement ratio

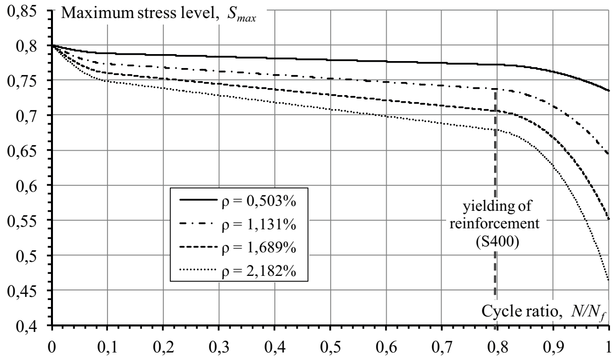


Fig. 9. Evolution of maximum stress level with number of cycles regarding to reinforcement ratio

As can be seen from Figure 7, total strain evolution has three distinct stages. Reinforcement ratio has no significant influence on the total strain in the initial two stages. While the third stage, particularly at the failure instant, exhibits a rather crucial dependency of total strain on reinforcement ratio. Thus, the total strain rises more rapidly with increase of reinforcement ratio.

Figure 8 and Figure 9 demonstrate the evolution of maximum stress and stress level with regard to number of load cycles. The most substantial stress drop in comparison with its initial value can be observed in a member with the largest reinforcement ratio. Therefore, the redistribution process of stresses in concrete becomes more apparent as the reinforcement ratio increases.

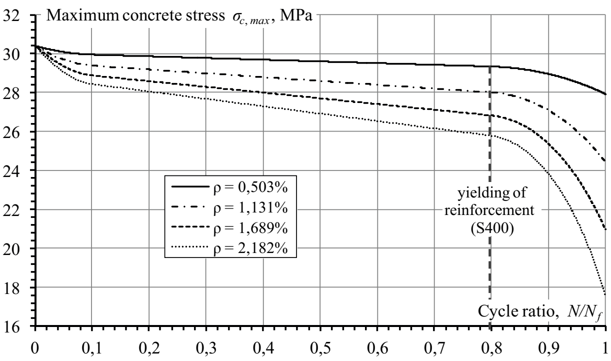


Fig. 8. Evolution of maximum stress with number of cycles regarding to reinforcement ratio

In order to evaluate the adequacy of the proposed technique, obtained results have been compared with two other generally prevailing fatigue assessment methods. The first one is taken from Model Code 2010 and is based on research done by Stemland *et al.* (1990). The second model proposed by Hsu (1981) is applied since it takes into account the influence of several key parameters.

Palmgren-Miner damage accumulation hypothesis has been adopted to assess the stress redistribution of concrete in models developed by Stemland, and Hsu. While the concept of equivalent number of cycles ($N_{eq}$) has been applied for the model proposed by Zanuy *et al.* (2009). A comparison of results is presented in Table 2. According to Stemland. and Hsu, the maximum number of load cycles to failure ($N_f$) considering the stress redistribution of concrete is 1.8 to 31.9 and 2.1 to 93.3 times, respectively, greater than under constant stress limits. Although Stemland, Hsu and Zanuy models do take into account stress redistribution, the estimated maximum number of load cycles differs significantly. Larger reinforcement ratio in RC columns shows the Palmgren-Miner hypothesis to be inaccurate while giving a smaller resistant number of cycles, thus overestimating failure state of concrete.

Table 2. Comparison of results of RC columns A, B, C and D under different models

| No. | Reinforcement ratio | Stemland *et al.* (constant stress limits) | Stemland *et al.* (stress redistribution) | Hsu (constant stress limits) | Hsu (stress redistribution) | Zanuy *et al.* S400 |
|---|---|---|---|---|---|---|
| | ρ,% | $N_f$ | $N_f$ | $N_f$ | $N_f$ | $N_f$ |
| A | 0,503 | 8629 | 15621 | 41149 | 88391 | 127616 |
| B | 1,131 | 8629 | 45035 | 41149 | 336943 | 691906 |
| C | 1,689 | 8629 | 117687 | 41149 | 1197729 | 3108498 |
| D | 2,182 | 8629 | 275388 | 41149 | 3841468 | 11683885 |

## Conclusions

The aim of this paper is to contribute to a better understanding of fatigue influence on the stress and strain behaviour of reinforced concrete columns subjected to high-cycle loading. After performing numerical studies, the following conclusions can be drawn:

1. An adequate time-dependent model evaluating concrete under fatigue effects is necessary to accurately replicate the behaviour of reinforced concrete under cyclic loading.
2. A simplified and transparent iterative analysis method for reinforced concrete columns subjected to high-cycle fatigue loading has been proposed based on previously developed time-dependent material model by Zanuy.
3. The proposed method enables the evaluation of the progressive time-dependent degradation and redistribution of stresses in concrete.
4. A numerical analysis of reinforced concrete columns subjected to high-cycle fatigue loading has shown that the resistant number of cycles, next to well established parameters such as maximum stress level, stress ratio and loading frequency, is greatly affected by reinforcement ratio. The members with higher ratio of reinforcement have larger redistribution capacity and are able to reach a higher number of load cycles until failure.
5. The performed comparative analysis of prevailing fatigue assessment methods has shown that RC columns with larger reinforcement ratio do not abide by the generally accepted Palmgren-Miner damage accumulation rule. Which gives conservative results, overestimating the failure state of concrete by giving a smaller resistant number of cycles.


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## DAUGIACIKLE GNIUŽDYMO APKROVA VEIKIAMŲ GELŽBETONINIŲ ELEMENTŲ ĮTEMPIŲ IR DEFORMACIJŲ SKAIČIAVIMO METODIKA


**V. Tamulenas, V. Gelazius, R. Ramanauskas**



Santrauka

Teoriniai modeliai, taikomi gelžbetoninių konstrukcijų nuovargio poveikiui vertinti, paprastai yra skirstomi į dvi pagrindines grupes. Pirmoji modelių grupė, kuri ir yra labiausiai paplitusi, remiasi S-N kreivėmis ir statiniu konstrukcijos įtempių būviu. Šie modeliai pateikia tik galutinį apkrovos ciklų skaičių iki konstrukcijos suirimo ir juos taikant neatsižvelgiama į betono įtempių persiskirstymą ar deformacijų kitimo raidą. Antroji grupė apima irimo teorijos modelius, kuriuose atsižvelgiama į laipsnišką betono silpnėjimą. Tačiau dėl pernelyg sudėtingo realaus konstrukcinio pritaikymo ir dėl per didelių skaičiavimo programų resursų poreikio praktinis šių modelių taikymas nėra išvystytas. Todėl, remiantis Zanuy *et al.* (2009) pasiūlytu modeliu, kuris leidžia įvertinti laipsnišką betono įtempių degradaciją ir jų perskirstymą, buvo sukurtas ir išplėtotas supaprastintas ir aiškus gelžbetoninių kolonų, veikiamų daugiacikle apkrova, iteracinis analizės metodas.



**Reikšminiai žodžiai**: ciklinė apkrova, degradacija, gelžbetonis, nuovargis, S-N kreivės.