



## NEW AGGREGATION OPERATORS FOR DECISION-MAKING UNDER UNCERTAINTY: AN APPLICATIONS IN SELECTION OF ENTREPRENEURIAL OPPORTUNITIES

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**Abstract.** The main aim of this paper is to study how economic environment and logic reasoning guidance the decision-making process to start-up a new business by potential entrepreneurs. The study proposes a new method using the family of selection indices with OWA operator, which allows aggregating information according to the level of importance and their level of objectivity and subjectivity in the same formulation within the decision-making process. To develop case study, we have taken into account some industries of the sports sector and some critical environmental factors that influence the competitiveness and entrepreneurship in Colombia to start a new business. The results show in an orderly way all information aggregated, which can help potential investors and entrepreneurs to make a decision based on their preferences. Finally, the applicability of this method in real case can be given in aggregation different sources of information to help at dealing decision-making processes.

**Keywords:** decision-making process, fuzzy weighted, OWA and selection indices, entrepreneurial opportunities, new business.

**JEL Classification:** C44, C60, M13, L26.

### Introduction

Currently, entrepreneurship is a fundamental referent to the economic development for a region or a nation, because it is considered as an engine of change, innovation and economic growth (Audretsch, Peña-Legazkue 2012; Praag, Versloot 2007). Entrepreneurship, as fundamental factor within the economic development, has awake the interest of policy-

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makers and the academic community to study the factors that influence entrepreneur as an agent of change, and entrepreneurship as the process of the change (Audretsch, Keilbach 2004; Acs, Amorós 2008). Both elements are linked directly to competitiveness and its impact on the economy and are used to assessing the level of entrepreneurial activity (Carree, Thurik 2010). However, the impact of entrepreneurship on an economy depends on two elements, the stage of development of each country (van Stel *et al.* 2005) in accordance with their environmental conditions. Thus, it is noteworthy that entrepreneurship has a crucial role to play within economic and social environment. Environmental factors have relevant influence on the level of entrepreneurial activity because they cause different degrees of uncertainty that affect the determinants of entrepreneurship (Verheul *et al.* 2002). Potential entrepreneurs use their intuition and knowledge to perceive these factors, which condition their decision to be employed or not. Indeed, the information provided from the environment is used by the entrepreneur to form a positive or negative attitude, i.e. an attitude toward a degree of pessimism or optimism (Merigó 2009; Merigó *et al.* 2014; Merigó, Peris-Ortiz 2014).

The main aim of this paper is to study how economic environment and logic reasoning guidance the decision-making process to start-up a new business by potential entrepreneurs. Hence, we have proposed a new method using the family of selection indices with OWA operator (see Merigó 2009). This new method allows aggregating information according to the level of importance and their level of objectivity and subjectivity in the same formulation within the decision-making process. To develop mathematical application, we have used resulting data of competitiveness report and GEM report, which are focused on Colombia. Likewise, we have used as sector example six industries of the sports sector, which are considered as potential industries to star an entrepreneurial activity. According to Miller *et al.* (2014) Colombia is considered as one of the South American economies with better levels of macroeconomic stability for foreign investment. Furthermore, Colombia's competitiveness is on the *efficiency-drive stage*, which has a less positive impact on their entrepreneurial activity. In fact, Colombia produce goods and services with discreet value added and less innovative, which is characteristic of a managed economy, (Acs, Amorós 2008). On the other hand, sports sector, in Colombia, offers an opportunity to promote entrepreneurial activities, since the current sports environment generates new welfare needs, which stimulates the demand of sports goods and services. Thus, the dynamic of the sports sector promotes the creation of new sports businesses, which are able to offer new forms of consumption of services and sporting goods that fit the aspirations of its customers (Heinemann 1994).

This paper is structured as follows: firstly, we presents theoretical framework, which is composed by a briefly review of entrepreneurship from economy view and the main contribution of the OWA operators in the decision-making process on business. Secondly, we develop a case study based on the reports of competitiveness and entrepreneurship of Colombia, as well as a briefly description of the sports sector to carry out the application of the method designed. Thirdly, we define methodology, variables and mathematical model. Fourthly, we explain the application of the method and the main results. Finally, we present the conclusion and implications of the research.

## 1. Theoretical framework

### 1.1. Entrepreneurship for economic development

The economic crisis in development Western countries has made the competitiveness and entrepreneurship fundamental strategic axes that encourage and lead an economy. Competitiveness is defined by multiple factors, policies and institutions groups, which describe the economic environment the level of prosperity within a nation. Moreover, entrepreneurship contributes to economic development because it encourages job creation innovation, productivity and growth (Praag, Versloot 2007; Wennekers *et al.* 2005). In recent years, some researches have been directed to study the relationship between entrepreneurship and competitiveness (van Stel *et al.* 2005; Audretsch 2009; Carree, Thurik 2010; Audretsch, Peña-Legazkue 2012; Casero *et al.* 2013). Also, they have conducted studies that focus on the relationship between entrepreneurship and performance where the company is studied as a unit within a geographic region (Carree, Thurik 2010).

According to Carree and Thurik (2010) that entrepreneurship effects over economy are spread into four main strands across the country, such as, regional, industry, country and entrepreneurial activity level. At the regional level, these effects are concentrated on the size distribution in regions. At the industry level, they are focused on the share of market participation in an industry, aspect that can have an influence on the increase in the number of competitors. At the country level, effects are focused on the number of self-employment and entrepreneurial intention. These aspects are related to a higher level, intensive and extent of the entrepreneurial activity. Finally, the last approach assesses the level of national entrepreneurial activity, which is related to the rate of economic growth. Nevertheless, the impact of entrepreneurship on economy changes among countries and their different stages of economic development (van Stel *et al.* 2005). In fact, economic development has an indirect influence on determinants of entrepreneurship either on decision making process of individual, or on market specific or range of environmental factors (Verheul *et al.* 2002).

Globally, the competitiveness of countries is assessed through *the global competitiveness index* (GCI), which allows determining their stage of development (WEF 2002) and governments across the world agree this index widely. This index classifies the stage of development in three main factors (*Factor-Drive; Efficiency-Drive; Innovation-Drive*) and two-step transition. These factors are determined by the evaluation of the interrelation of the 12 pillars of competitiveness (WEF 2014). Competitiveness' analysis allows understanding the microeconomic and macroeconomic context within the social and the environmental dimension (Sala i Martín *et al.* 2014). According to Verheul *et al.* (2002) this analysis provides descriptive information about economic characteristics of the environment that could impact on the creation of entrepreneurial opportunities. Furthermore, competitiveness' analysis also allows comprehending the new view of the economic development, which has evolved from a notion of the economy of *accumulation* to a notion of the economy of *knowledge* (Wennekers *et al.* 2005). In fact, this aspect are linked to the evolution from the management economy to entrepreneurial economy (Audretsch, Thurik 2001). In this process, the change stage of development represents the evolutionary change of economic

development of a nation. These changes range from low cost production efficiency, efficient productive practices to exploit economies of scale to finally become a knowledge-based economy. Likewise, these change also influence the level of entrepreneurship because they can change push and pull factors within the labour market and product markets (Verheul *et al.* 2002). In fact, the best agent for this change is the entrepreneur (Acs, Amorós 2008) and the entrepreneurial activity is the process of the change (Audretsch, Keilbach 2004). These changes are highlighted because they reflect a more dynamic economic system.

Another important aspect is the economic performance of entrepreneurial activity because it is a key factor that promotes the introduction of innovation, enhances rivalry, creates competition and takes advantage of the new gaps within the market (Wong *et al.* 2005). Indeed, large firms have been thoroughly developed these activities in the last few decades. Similarly, large industries and incumbent firms have conducted accumulation processes, in which have concentrated on key activities (production, distribution and R&D) that have enabled them to increase their productivity. These processes of industrial development are related to the period of *scale and scope* (Chandler, Hikino 1996). The relationship between entrepreneurship and large firms can be explained from creative destruction concept and creative accumulation as an economic system dynamics (Backhaus 2003). Creative destruction highlights the importance of the role of the entrepreneur because it is considered as the prime cause of the economic development where one of this characteristic is the proliferation of small firms (Carree, Thurik 2010). On the other hand, creative accumulation emphasises its attention on the innovative activities, which are developed by large industries and incumbent firms forming a concentrated market structure (Carree, Thurik 2010). The development of some of these types of industries is given by the stage of development of the country in which they are located (Verheul *et al.* 2002). In fact, high levels of research and innovation are more intensive in big industries while the development of innovative advantage comes from gaps of market that small and new business has detected enabling the development of a small-scale production within the service economy. Indeed, countries with high levels of research and innovation are more likely to develop and foster small and new business that provide an innovative advantage. Thus, depending on the state of development of a country, this can create specific conditions for entrepreneurship.

Hence, the market and the industrial structure determine the creation of opportunities to entrepreneurship and technological development, globalization and economic development influence it (Verheul *et al.* 2002). The demand for entrepreneurship is configured by these sets of factors where environmental factors create conditions for the entrepreneurial activity. Thus, the selection of business opportunity and makes a decision for potential entrepreneurs have a degree of uncertainty because it considers multiple variables related to risk and reward. Indeed, the entrepreneur decision making take into account personal preferences, environmental factors, individual characteristics, which depends on weighing risk-rewards between self-employment and salary employment (Wennekers, Thurik 1999; Verheul *et al.* 2002). Furthermore, there are factors as the changing markets conditions and a possible increase of the opportunity costs, which cause more uncertainty in the individual, hindering the process of entrepreneurial decision-making. Hence, the treatment of personal characteristics and intuition of the entrepreneur can contribute to fostering new

entrepreneurial initiatives (Cuervo 2005; Merigó, Peris-Ortiz 2014). Currently, in soft computing science are being developing techniques that allow processing personal preferences and the attitudinal character. Thus, with the use of these techniques, the decision maker can take a decision to invest and to create a new firm in the most appropriate market within a complex environment (Merigó, Peris-Ortiz 2014; Merigó *et al.* 2014).

## 1.2. Decision making with the ordered weighted averaging (OWA) operator

The ordered weighted averaging (OWA) operator is an instrument that allows aggregating information obtaining a single representative value of the information. The OWA operator meets the parameters of considered optimism and pessimism in an aggregated value. The OWA operator unifies the criteria of Optimist, Pessimist, Hurwicz and de Laplace in a single formulation (Yager 1988). Likewise, the OWA operator can aggregate information differently depending on its degree of pessimism or optimism (Gil-Lafuente, Merigó 2009), which are defined by limits the minimum and maximum. Based on OWA operator, new aggregation operators have developed new aggregation operators providing a parameterized family (Emrouznejad, Marra 2014; Merigó, Gil-Lafuente 2012a). In this sense, Yager (1993) proposed OWA operator families; Yager and Kelman (1999) suggested analytical hierarchy process, Yager and Kacprzyk (1997) and Yager *et al.* (2011) developed applications of OWA operator, Herrera (1995), Herrera and Martinez (2000) and Xu (2004) developed works focused on linguistic OWA operator. Furthermore, it has been developed aggregation operators in combination with OWA operator and other instruments. Merigó (2009) Gil-Lafuente and Merigó (2007) Merigó and Gil-Lafuente (2006, 2008a, 2008b, 2008c, 2013) suggested distance and index selection for OWA operators, Wei (2009) developed an induced linguistic hybrid geometric aggregation of OWA operators, Zhao *et al.* (2010) proposed OWA operators for intuitionistic fuzzy sets, Zhou and Chen (2010) worked in logarithm aggregation in OWA operators, Peng and Ye (2014) introduced the induced uncertain pure linguistic hybrid harmonic averaging operator, Xu *et al.* (2014) proposed intuitionistic fuzzy Einstein Choquet integral operators, Zhou *et al.* (2014) suggested intuitionistic fuzzy weighted cosine similarity measure, He *et al.* (2014) introduced an intuitionistic fuzzy geometric, Wei *et al.* (2014) worked on multiple attribute decision making, Peng *et al.* (2014) suggested non-homogenous preferences information, He *et al.* (2015a) proposed intuitionistic fuzzy information, He *et al.* (2015b) introduced fuzzy interaction Bonferroni, He *et al.* (2015c) suggested hesitant fuzzy power Bonferroni. All these operators offer multiple techniques that can be used to solve decision-making problems. Furthermore, some of these applications are used in the process of decision-making over uncertainty in several fields.

In business and economic field, some techniques are used to dealing with uncertainty and opinion of the decision-maker, specially, when there are not shortage of information and high level of subjectivity. Firstly, we have Hamming distance (HD) (Hamming 1950), which allow aggregating information and being used when a strictly established figure is considered as the ideal, i.e., it is not a threshold with valid results (Merigó 2010). Secondly, we have adequacy coefficient (AC) (Kaufmann, Gil-Aluja 1986, 1987), which is an

extension of HD and allows analysing the information when the results are higher than the ideal by using a t-norm. Thirdly, we have the index of maximum and minimum level (IMAM) (Gil-Lafuente 2001, 2002), which combines the characteristics of HD and AC in the same formulation (Merigó, Gil-Lafuente 2010; Merigó *et al.* 2011, 2013; Merigó, Gil-Lafuente 2012a, 2012b) and allows taking into account the situation in which it is as good to achieve the ideal level as to overcome it. Thus, it is noteworthy that these techniques let the decision-maker can take a decision according to its interests. Besides, decision-maker can also focus on the results obtained in different situations, based on its optimism degree (Merigó 2010). Recently, using these techniques with OWA, it has been developed new aggregation operators, which are called *selection indices* (SI): Gil-Lafuente and Merigó (2007), Merigó (2009) and Merigó and Gil-Lafuente (2010) have proposed the ordered weighted averaging distance (OWAD) and Merigó (2010), Merigó and Gil-Lafuente (2012a; 2012b) have suggested the ordered weighted averaging adequacy coefficient (OWAAC) operator and ordered weighted averaging index of maximum and minimum level (OWAIMAM) operator. These new extensions are used in business and economics decision-making problems as entrepreneurship, human resources, investment, marketing, strategy, production, policy-making, regional development and public economics. Furthermore, it is noteworthy that these new operators reflecting the attitudinal character of the decision-maker at the time of decision-making.

## 2. Case study

Case study is focused on entrepreneurial conditions, stage of economy development and problematic condition for doing business in Colombia. Mathematical experiment is oriented to start new business in the sports sector. Since the 1990s, Colombia has adopted policies toward enhancing competitiveness in order to improve the macroeconomic conditions (CEC 2012). However, in the last competitiveness report, Colombia has evidenced the existence of weak institutions and a considerable corruption, transport infrastructure insufficiency, failures of the education system and lack of diversification in the economic base as well (WEF 2014). Currently, Colombia is a stage of *efficiency-drive* economy, which is characterized by its efficiency production process and increasing product quality (WEF 2014). This stage of the economy develops special environment characteristics for doing business and starting new businesses. Structural conditions that affect the business dynamic, business opportunities, personal business preferences and decision-making processes of individuals are composed by these characteristics, which can be defined by key entrepreneurial framework conditions (EFC's). Colombia's EFC's, compared to other economies, show negative differences in financing, legal, trade and physical infrastructure, R&D transfer, dynamics and opening of domestic market and social norms (GEM Colombia 2014). However, Colombia has shown a considerable enhancement on macroeconomic environment, specially in trade, investment and fiscal freedom, which has aided to become a "mostly free" economy and increasing investor confidence (Miller *et al.* 2014).

Sports have become a dynamic and attractive economic sector (Heinemann 1994), where both public and private sector take part in its development. Nowadays, sport has

generated a huge global interest, which has become an economically attractive sector, creating a great deal of opportunities for the development of new sports business (Ratten, V., Ratten, H. 2011). According to Heinemann (1994) these opportunities arise when demand is stimulated by new welfare needs. In order to supply these new needs, suppliers offer new services, goods, locations and forms of consumption in line with the aspiration of the customers. Likewise, these are complemented with activities related to leisure and recreation. Furthermore, it is noteworthy that professional and spectacle sports support the potential of sports business because they receive financial resources both for public and private sectors. Besides, mass media promotes its local and international consumption, improving the revenues. Finally, sports business initiatives are able to affect different areas of the economy through its social impact (Ratten, V., Ratten, H. 2011). Thus, the development of the sports sector in Colombia can have a chance of growing, since the current environmental conditions, the existing structure market and the sports movement can stimulate the creation of new businesses.

### 3. Methodology

In this paper, we have proposed a new algorithm for decision-making process under uncertainty. Likewise, we have also developed an application that allows the comparison and the selection between new several business alternatives. To develop case study, we have taken into account some industries of the sports sector and some critical environmental factors that influence the competitiveness and entrepreneurship in Colombia to start a new business. In order to carry out mathematical application, we use the Colombia's resulting data contents within global competitiveness report (GCR) and global entrepreneurship monitor (GEM) in the period 2013–2014.

#### 3.1. Global competitiveness index (GCI)

The global competitiveness report (GCR) provides the global competitiveness index (GCI), which allows assesses the national competitiveness worldwide from drivers of their productivity and prosperity (WEF 2014). GCI is supported by 12 different pillars of competitiveness, which determine 3 stages of development and 2 stage of transition. The first stage is called *factor-driven* and is related to pillars from 1 to 4. It is noteworthy that countries within this stage develop economies based on natural resources endowments and primarily unskilled labour. The second stage is called *efficiency-driven* and is related to pillars from 5 to 10. Countries within this stage develop more efficient production processes and increase product quality. Thus, for a country to move from stage 1 to stage 2, it must improve industrialization, raise wages and become more capital intensive in order to become more competitive. The third stage is called *innovation-driven* and is related to pillars 11 and 12. Countries within this stage are characterized by the use of the most sophisticated production processes and innovation in order to produce new and different goods and services. Thus, for a country to move from stage 2 to stage 3, their companies must be at the global technological frontier (WEF 2014). It is highlighted that pillars tend to reinforce

each other, and a weakness in one area often has a negative impact in the others (WEF 2014). In addition, the GCR's highlights and assesses *the most problematic factors for doing business* (Table 1). These resulting data are significant for the study. This table summarizes those factors seen by business executives as the most problematic ones when doing business in their economy. From a list of 16 factors, respondents were asked to select the five most problematic and rank them from 1 (most problematic) to 5. The results were then tabulated and weighted according to the ranking assigned by respondents.

Table 1. The most problematic factors for doing business

Vector $w_{PF}$	Problematic factors	Vector $w_{PF}$	Problematic factors
$w_{PF} 1$	access to financing	$w_{PF} 9$	inflation
$w_{PF} 2$	crime and theft	$w_{PF} 10$	insufficient capacity to innovate
$w_{PF} 3$	corruption	$w_{PF} 11$	policy instability
$w_{PF} 4$	foreign currency regulations	$w_{PF} 12$	poor public health
$w_{PF} 5$	government instability/coups	$w_{PF} 13$	poor work ethic/labor force
$w_{PF} 6$	inadequately educated workforce	$w_{PF} 14$	restrictive labor regulations
$w_{PF} 7$	inadequate supply of infrastructure	$w_{PF} 15$	tax rates
$w_{PF} 8$	inefficient government bureaucracy	$w_{PF} 16$	tax regulations

Source: Global Competitiveness Report, 2013–2014 (WEF 2014).

### 3.2. Global entrepreneurship monitor (GEM)

Global entrepreneurship monitor (GEM) provides significant information of the level of entrepreneurship within a country. One of its key goals is helping establishing the way entrepreneurship relates to economic growth and, in a longer-term perspective, economic development. The report also shows the entrepreneurial behaviour and attitudes of the individuals (GEM 2014). These factors mentioned above, we allow understudying of the entrepreneurship environment, which are represented on entrepreneurial framework conditions (EFC's) (see Table 2) (GEM 2008).

Both reports give information about environmental conditions within economic space. On the one hand, we have considered factors that have considerably influenced at doing business. On the other hand, we have considered factors that define the necessary conditions to entrepreneur (EFC's). In this sense, we assume that a group entrepreneur potentials desire to start a new business within an industry of sports sector and they want to choose the best option among a group of existing opportunities. Thus, in order to aggregate and order economic information and entrepreneur expectations, we use the OWA operator. Since our main aim is to make a selection, we use selection indices, OWAD operator, OWAAC operator, OWAIMAM operator. These operators allow achieving an accurate representation of the decision-making problem, because they take into account the situations where it is as good to get a particular ideal level as to surpass it. Right after now, aggregator operators and its extensions will be defined.

Table 2. Key entrepreneurial framework conditions (EFC's)

EFC's		EFC's	
C <sub>1</sub>	Finances: financial resources-equity and debt-for small and medium enterprises (SMEs)	C <sub>7</sub>	R&D transfer: national research and development will lead to new commercial opportunities
C <sub>2</sub>	National policy-general policy: public policies give support to entrepreneurship	C <sub>8</sub>	Commercial infrastructure: property rights, commercial, accounting and other legal and assessment services and institutions
C <sub>3</sub>	National policy-regulation: taxes and regulations	C <sub>9</sub>	Internal market-dynamics: the level of change in markets from year to year
C <sub>4</sub>	Government programs: the presence and quality of programs directly assisting SMEs	C <sub>10</sub>	Internal market-openness: the extent to which new firms are free to enter existing markets
C <sub>5</sub>	Entrepreneurship education-primary and second	C <sub>11</sub>	Physical infrastructure: access to physical resources-communication, utilities, transportation, land or space
C <sub>6</sub>	Entrepreneurship education-post-school	C <sub>12</sub>	Culture and social norms: norms encourage or allow actions leading to new business methods

Source: Based on Global Entrepreneurship Monitor 2013 Global Report (Amorós, Bosma 2014: 45).

### 3.3. HD, AC, IMAM, OWAD, OWAAC AND OWAIMAM definitions

In this section, we briefly review some basic concepts about the Hamming distance, the adequacy coefficient (AC), the index of maximum and minimum level (IMAM) and their extensions with the OWA operator, which are used throughout the paper.

#### 3.3.1. The Hamming distance

The Hamming distance (Hamming 1950) is a useful technique to calculate the differences between two elements, two sets, etc. In fuzzy set theory, it can be useful, for example, for the calculation of distances between fuzzy sets, interval-valued fuzzy sets, intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets. For two sets A and B, the weighted Hamming distance can be defined as the following:

**Definition 1.** A weighted Hamming distance of dimension n is a mapping  $d_{WH}:R^n \times R^n \rightarrow R$  that has an associated weighting vector W of dimension n with the sum of the weights being 1 and  $w_j \in [0,1]$  such that:

$$d_{WH}(\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle) = \sum_{j=1}^n w_j |x_j - y_j| \tag{1}$$

where  $x_i$  and  $y_i$  are the *i*th arguments of the sets X and Y.

#### 3.3.2. The adequacy coefficient

The adequacy coefficient (Kaufmann, Gil-Aluja 1986, 1987) is an index that is used for calculating the differences between two elements or two sets. It is very similar to the Hamming distance, but the difference is that it neutralizes the result when the comparison shows that

the real element is higher than the ideal one. For two sets A and B, the weighted adequacy coefficient can be defined as follows:

**Definition 2.** A weighted adequacy coefficient of dimension  $n$  is a mapping  $K:[0,1]^n \times [0,1]^n \rightarrow [0,1]$  that has an associated weighting vector  $W$  of dimension  $n$  with the sum of the weights 1 and  $w_j \in [0,1]$  such that:

$$K(\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle) = \sum_{j=1}^n w_j [1 \wedge (1 - x_1 + y_i)], \tag{2}$$

where  $x_i$  and  $y_i$  are the  $i$ th arguments of the sets  $X$  and  $Y$ .

### 3.3.3. The index of maximum and minimum level

The index of maximum and minimum level is an index that unifies the Hamming distance and the adequacy coefficient in the same formulation (Gil-Lafuente 2001, 2002). For two sets A and B, the weighted index of maximum and minimum level can be defined as follows:

**Definition 3.** An AWIMAM of dimension  $n$  is a mapping  $K:[0,1]^n \times [0,1]^n \rightarrow [0,1]$  that has an associated weighting vector  $W$  of dimension  $n$  with the sum of the weights 1 and  $w_j \in [0,1]$  such that:

$$\eta(\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle) = \sum_u Z_i(u) * |x_i(u) - y_i(u)| + \sum_v Z_i(v) * [0 \vee x_i(v) - y_i(v)], \tag{3}$$

where  $x_i$  and  $y_i$  are the  $i$ th arguments of the sets  $X$  and  $Y$ .

### 3.3.4. The OWA operator

The OWA operator (Yager 1988) provides a parameterized class of mean type of aggregation operators. It can be defined as follows:

**Definition 4.** An OWA operator of dimension  $n$  is a mapping  $OWA:R^n \rightarrow R$  that has an associated weighting vector  $W$  of dimension  $n$  with  $w_j \in [0,1]$  and  $\sum_{j=1}^n w_j = 1$ , such that:

$$OWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^n w_j b_j, \tag{4}$$

where  $b_j$  is the  $j$ th largest of the  $a_i$ .

The OWAAC operator (Merigó, Gil-Lafuente 2008b, 2010; Gil-Lafuente, Merigó 2009) is an aggregation operator that uses the adequacy coefficient and the OWA operator in the same formulation. It can be defined as follows for two sets X and Y.

**Definition 5.** An OWAAC operator of dimension  $n$  is a mapping  $OWAAC:[0,1]^n \times [0,1]^n \rightarrow [0,1]$  that has an associated weighting vector  $W$ , with  $w_j \in [0,1]$  and  $\sum_{j=1}^n w_j = 1$ , such that:

$$OWAAC(\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle) = \sum_{j=1}^n w_j K_j, \tag{5}$$

where  $K_j$  represents the  $j$ th largest of  $[1 \wedge (1 - x_1 + y_i)]$ ,  $x_i, y_i \in [0,1]$ .

The OWAD operator (Gil-Lafuente, Merigó 2007; Merigó 2009; Merigó, Gil-Lafuente 2010) is an aggregation operator that uses OWA operators and distance measures in the same formulation. It can be defined for two sets  $X$  and  $Y$ .

**Definition 6.** An OWAD operator of dimension  $n$  is a mapping  $OWAD:R^n \times R^n \rightarrow R$  that has an associated weighting vector  $W$ ,  $\sum_{j=1}^n w_j = 1$  and  $w_j \in [0,1]$  such that:

$$OWAD(\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle) = \sum_{j=1}^n w_j D_j, \tag{6}$$

where  $D_j$  represents the  $j$ th largest of the  $|x_i - y_i|$ .

The OWAIMAM operator (Merigó 2009; Merigó, Gil-Lafuente 2011, 2012b; Merigó *et al.* 2011) is an aggregation operator that uses the Hamming distance, the adequacy coefficient and the OWA operator in the same formulation. It can be defined as it follows:

**Definition 7.** An OWAIMAM operator of dimension  $n$ , is a mapping  $OWAIMAM:[0,1]^n \times [0,1]^n \rightarrow [0,1]$  that has an associated weighting vector  $W$ , with  $w_j \in [0,1]$  and the sum of the weights is equal to 1, such that:

$$OWAIMAM(\langle x_1, y_1 \rangle, \langle x_2, y_2 \rangle, \dots, \langle x_n, y_n \rangle) = \sum_{j=1}^n w_j K_j, \tag{7}$$

where  $K_j$  represents the  $j$ th largest of all the  $|x_i - y_i|$  and the  $[0 \vee (x_i, y_i)]$ .

#### 4. A new method to deal with the weight of the OWA operator

Following (Gil-Aluja 1999) information can be aggregated according to the significance of its characteristics. Thus, this proposition allows aggregating information according to a certain level all characteristics because it takes into account the importance of the relationship of each characteristic, which is considered to make a decision. In order to deal with a decision-making problem, the following proposition is realized.

**Proposition 1.** An OWAAC operator with fuzzy-significance can have a weighting vector  $W$ , with  $w_j \in [0,1]$  and  $\sum_{j=1}^n \frac{w_j}{\max(w_j)}$ , such that:

$$FS-OWAAC(\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle) = \sum_{j=1}^n \frac{w_j}{\max(w_j)} K_j, \tag{8}$$

where  $K_j$  represents the  $j$ th largest of  $[1 \wedge (1 - x_i + y_i)]$ ,  $x_i, y_i \in [0,1]$ .

**Proposition 2.** An OWAD operator with fuzzy-significance can have a weighting vector  $W$ ,  $\sum_{j=1}^n \frac{w_j}{\max(w_j)}$ , and  $w_j \in [0,1]$  such that:

$$FS-OWAD(\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle) = \sum_{j=1}^n \frac{w_j}{\max(w_j)} D_j, \tag{9}$$

where  $D_j$  represents the  $j$ th largest of the  $|x_i - y_i|$ .

**Proposition 3.** An OWAIMAM with fuzzy-significance can have a weighting vector  $W$ , with  $w_j \in [0,1]$  and  $\sum_{j=1}^n \frac{w_j}{\max(w_j)}$ , such that:

$$FS-OWAIMAM(\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle) = \sum_{j=1}^n \frac{w_j}{\max(w_j)} K_j, \tag{10}$$

where  $K_j$  represents the  $j$ th largest of all the  $|x_i - y_i|$  and the  $[0 \vee (x_i, y_i)]$ .

Note that it is easy to obtain a result higher than the maximum; hence, the boundary and idempotency condition is not accomplished in the operator. This occurs because the sum of the weights can be higher than one. Thus, we carry out a normalization process (Merigó, Gil-Lafuente 2009) in order to accomplish the boundary condition. A practical method for doing so is by dividing the weights by the sum of all of them. Therefore, this operator is very useful as a method for decision-making but it does not represent the information from a classical point of view.

These OWA operators are commutative, monotonic, non-negative and reflexive. They are commutative from the OWA perspective because  $f(\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle) = f(\langle c_1, d_1 \rangle, \dots, \langle c_n, d_n \rangle)$  where  $(\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle)$  is any permutation of the arguments  $(\langle c_1, d_1 \rangle, \dots, \langle c_n, d_n \rangle)$ . They are also commutative from the distance measure perspective because  $f(\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle) = f(\langle y_1, x_1 \rangle, \dots, \langle y_n, x_n \rangle)$ . They are monotonic because if  $|x_i - y_i| \geq |c_i - d_i|$ , for all  $i$ , then  $f(\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle) \geq f(\langle c_1, d_1 \rangle, \dots, \langle c_n, d_n \rangle)$ . Non-negativity is also accomplished always, that is,  $f(\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle) \geq 0$ . Finally, they are also reflexive because  $f(\langle x_1, x_1 \rangle, \dots, \langle x_n, x_n \rangle) = 0$ .

Another important issue is to consider the formulation for the characterization of the weighting vector under this framework: the degree orness/andness (degree of optimism or pessimism) (Yager 1988) is defined in the following way with this new approach:

$$\alpha(w) = \sum \frac{n-j}{n-1} * \frac{w_j}{\max(w_j)}, \tag{11}$$

where  $\min \rightarrow 0$ , even though the maximum can be higher than 1. By the introduction of significance degree in more than one variable is where the result is not contained in  $[0,1]$ .

The entropy of dispersion (Shannon 1948; Yager 1988) can be defined as follows:

$$H(w) = - \sum_{j=1}^n \left( \frac{w_j}{\max(w_j)} \right) \ln \left( \frac{w_j}{\max(w_j)} \right). \tag{12}$$

Finally, let us briefly study some key particular cases of these operators. If one of the sets is empty, we get the FS-OWA operator in the FS-OWAD. In the FS-OWAAC, if X is empty all the individual comparisons are 1, so the result is 1 or higher. The FS-OWAIMAM is a combination of the previous ones.

If  $w_1 = 1$  and  $w_j = 0$  for all  $j \neq 1$ , we get the maximum distance,  $FS-OWAD = \text{Max}\{D_j\}$  and if  $w_n = 1$  and  $w_j = 0$  for all  $j \neq n$ , the minimum distance,  $FS-OWAD = \text{Min}\{D_j\}$ . Note that similar results are also found with the FS-OWAAC and the FS-OWAIMAM operator. If  $w_j = 1/n$ , for all  $j$ , the result is the total operator, which is also the sum of all the individual distances (Merigó et al. 2014). This occurs because in this situation all the weights are  $(1/n)/(1/n) = 1$  so the sum is  $n$ . The result obtained is the absolute distance.

The step-FS-OWA aggregation (Yager 1993) provides the same results than the classical OWA approach. The median-FS-OWA aggregation (Merigó, Gil-Lafuente 2009) gives the same results only when it is odd. If it is even, the results of the FS-OWAD are twice as high as they should in the classical framework. The Olympic-FS-OWAD (Merigó, Gil-Lafuente 2009; Yager 1993) is used when  $w_1 = w_n = 0$ , and for all others  $w_{j^*} = 1/(n - 2)$ . In this case, the result is  $n - 2$  which is close to the total and has close connections with the arithmetic distance.

## 5. Application

Now, we present an application of the new operator proposed. The main advantage on using OWA operators is that they can overestimate information according to attitudinal character of the decision-maker. The application is centred on a sample selection of business opportunities between six industries in the sports sector.

### 5.1. Mathematical approach for decision making

The new operator allows comparing between each alternative in order to select option according environmental information and attitude of entrepreneur. The procedure is designed in four steps, which are described as follows:

Step 1. We assumed that the entrepreneurs want to select an industry to start a business within the sports sector, and there are six possible options  $NB_1$ ,  $NB_2$ ,  $NB_3$ ,  $NB_4$ ,  $NB_5$  and  $NB_6$ , which have different characteristics (see Table 3). In this case, each characteristic of the sector is considered a property.

Table 3. Sports industries business opportunity

$NB_1$	Textile and manufacturing industry
$NB_2$	Sports services
$NB_3$	Sports and leisure events
$NB_4$	Sports management consultancy services
$NB_5$	Media
$NB_6$	Marketing, selling and distributions

Source: Own elaboration.

Step 2. It has fixed real level and the ideal conditions for each significant characteristic. The real level is current conditions available under the environment to start a business in Colombia (see Table 4). The ideal conditions define the necessary conditions for entrepreneur an economic activity (see Table 5).

Table 4. Real level: Colombia's EFC's rates

variable X	EFC's	Rates
C <sub>1</sub>	finances	2.3
C <sub>2</sub>	national policy-general policy	2.8
C <sub>3</sub>	national policy-regulation	2.6
C <sub>4</sub>	government programs	3
C <sub>5</sub>	education-primary and second	2.3
C <sub>6</sub>	education-post-school	3.2
C <sub>7</sub>	R&D transfer	2.4
C <sub>8</sub>	commercial infrastructure	2.8
C <sub>9</sub>	internal market-dynamics	2.9
C <sub>10</sub>	internal market-openness	2.8
C <sub>11</sub>	physical infrastructure	3.3
C <sub>12</sub>	culture and social norms	3.1

Source: Own elaboration based on Global Entrepreneurship Monitor 2013–2014 (GEM 2014).

Table 5. Ideal conditions for entrepreneur

X	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>	C <sub>6</sub>	C <sub>7</sub>	C <sub>8</sub>	C <sub>9</sub>	C <sub>10</sub>	C <sub>11</sub>	C <sub>12</sub>
Colombia	0.46	0.56	0.52	0.6	0.46	0.64	0.48	0.56	0.58	0.56	0.66	0.62

Source: Own elaboration based on EFC's rates Table 1.

Step 3. It has fixed the necessary conditions of each characteristic to start a different business opportunity in a sports industry. Here, each characteristic can consider objective or subjective information, which is provided by expert specifications and its suggestions. They recommend the most necessary conditions to start a business opportunity highlighting its degree of importance. In this case, hypothetical degrees of level in each characteristic for each of the options have been proposed (see Table 6).

Table 6. Necessary conditions to start a new business in a sports industry

Y	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>	C <sub>6</sub>	C <sub>7</sub>	C <sub>8</sub>	C <sub>9</sub>	C <sub>10</sub>	C <sub>11</sub>	C <sub>12</sub>
NB <sub>1</sub>	0.1	0.6	0.4	0.8	0.4	0.7	0.7	0.8	0.7	0.7	0.9	0.9
NB <sub>2</sub>	0.4	0.6	0.4	0.2	0.7	0.6	0.3	0.6	0.6	0.6	0.7	0.6
NB <sub>3</sub>	0.6	0.6	0.5	0.5	0.7	0.6	0.3	0.6	0.1	0.3	0.9	0.9
NB <sub>4</sub>	0.1	0.6	0.5	0.3	0.7	0.9	0.2	0.2	0.5	0.7	0.5	0.3
NB <sub>5</sub>	0.1	0.6	0.7	0.2	0.3	0.6	0.3	0.4	0.1	0.1	0.7	0.6
NB <sub>6</sub>	0.1	0.6	0.2	0.2	0.6	0.6	0.5	0.8	0.6	0.7	0.7	0.9

Source: Own elaboration.

Step 4. We have used the FS-OWA to make a comparison between the EFC's in Colombia and the different options. It will consider the FS-OWAAC, the FS-OWAIMAM and the FS-OWAD operator. In this application, we assume that the experts decide that the most problematic factors for doing business in Colombia are considered as the weighting

vectors  $W_{PF}$  (see Table 7). Each of the factors is selected from the (WEF 2014). The main idea is to aggregate and compare how environmental conditions in combination with attitudinal character of the decision-maker can influence and guide the decision to start a new business.

Table 7. Colombia’s PF for doing business rates

vector $w_{PF}$	rates %	N	IN	vector $w_{PF}$	rates %	N	IN
$w_{PF}$ 1	20.2	1	0	$w_{PF}$ 9	3.3	0.163	0.837
$w_{PF}$ 2	14.6	0.723	0.277	$w_{PF}$ 10	2.8	0.139	0.861
$w_{PF}$ 3	12.2	0.604	0.396	$w_{PF}$ 11	2.4	0.119	0.881
$w_{PF}$ 4	8.1	0.401	0.599	$w_{PF}$ 12	2.3	0.114	0.886
$w_{PF}$ 5	8.1	0.401	0.599	$w_{PF}$ 13	1.8	0.089	0.911
$w_{PF}$ 6	7.7	0.381	0.619	$w_{PF}$ 14	1.7	0.084	0.916
$w_{PF}$ 7	7.4	0.366	0.634	$w_{PF}$ 15	0.9	0.045	0.955
$w_{PF}$ 8	6.3	0.312	0.688	$w_{PF}$ 16	0.2	0.01	0.99

Source: Own elaboration based on Global Competitiveness Report, 2013–2014 (WEF 2014). Note that resulting data has been Normalized (N) to establish the weight of each factor and Inverse Normalized (IN) for showing positive factor effects.

### 5.2. Results

We were proposed and implemented the new weighting vector  $W_{PF}$ , which are called FS-OWAAC operator, the FS-OWAIMAM operator, and the FS-OWAD operator. We elaborated two different applications of the FS-OWAIMAM operator, since this operator uses the Hamming distance and the adequacy coefficient simultaneously. Thus, it is shown the versatility of this operator to aggregate preferences of the decision-makers. Furthermore, FS-OWAAC provides more widespread results, while FS-OWAD and FS-OWAIMAM provide more specific results. Finally, we have ordered and charted the results obtained from the comparison between each single considered industry. It is noteworthy that these operators allowed aggregating objective – resulting data – and subjective – necessary condition – information to obtain a characteristic value for each of the alternative  $NB_n$  suggested within an uncertain environment. We have calculated the vector  $W_{PF}$  with 16 cases (see Tables 8 to 11) and its influence at doing business according to new business opportunity. After that, each alternative  $NB_n$  was ordered according to average vectors (Ave) in each table. Results obtained from the FS-OWAAC are sorted in descending order because in this case the highest average is the most favourable alternative. Results obtained from FS-OWA operator are sorted in ascending order because the lowest average is the most favourable alternative. It is noteworthy that each alternative reflects a different degree of optimism or pessimism concerning the weighting vector. There is an interesting issue regarding the degree of optimism for the vector  $W_{PF}$  1, since it is the lowest vector and has a negative influence in an individual decision-making. Likewise, vectors allow observing differences between each of the alternatives, which appear to be small. However, it is important to highlight that although the differences are small, into a macroeconomic context, they have a different connotation, since each point of difference shows different levels of development.

Table 8. Results of vector  $W_{PF}$  with FS-OWAAC operator

$w_{PF}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Ave
NB <sub>1</sub>	0.00	3.18	4.55	6.88	6.88	7.11	7.28	7.90	9.61	9.61	10.1	10.2	10.5	10.5	11.0	11.4	7.91
NB <sub>2</sub>	0.00	3.12	4.46	6.74	6.74	6.96	7.13	7.74	9.41	9.41	9.92	9.97	10.3	10.3	10.8	11.1	7.75
NB <sub>3</sub>	0.00	3.06	4.38	6.62	6.62	6.84	7.00	7.61	9.25	9.25	9.74	9.79	10.1	10.1	10.6	10.9	7.62
NB <sub>4</sub>	0.00	2.83	4.04	6.11	6.11	6.31	6.46	7.02	8.53	8.53	8.99	9.04	9.29	9.34	9.75	10.1	7.03
NB <sub>5</sub>	0.00	2.73	3.90	5.91	5.91	6.10	6.25	6.78	8.25	8.25	8.69	8.74	8.98	9.03	9.42	9.76	6.79
NB <sub>6</sub>	0.00	2.99	4.27	6.46	6.46	6.67	6.83	7.42	9.02	9.02	9.50	9.56	9.82	9.88	10.3	10.6	7.43

Table 9. Results of vector  $W_{PF}$  with FS-OWAD operator

$w_{PF}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Ave
NB <sub>1</sub>	0.00	0.85	1.21	1.83	1.83	1.89	1.94	2.10	2.56	2.56	2.69	2.71	2.78	2.80	2.92	3.02	2.10
NB <sub>2</sub>	0.00	1.09	1.55	2.35	2.35	2.42	2.48	2.70	3.28	3.28	3.45	3.47	3.57	3.59	3.74	3.88	2.70
NB <sub>3</sub>	0.00	0.98	1.41	2.13	2.13	2.20	2.25	2.44	2.97	2.97	3.13	3.15	3.23	3.25	3.39	3.51	2.45
NB <sub>4</sub>	0.00	1.28	1.82	2.76	2.76	2.85	2.92	3.17	3.85	3.85	4.06	4.08	4.19	4.21	4.40	4.56	3.17
NB <sub>5</sub>	0.00	1.49	2.13	3.23	3.23	3.33	3.41	3.70	4.50	4.50	4.74	4.77	4.90	4.93	5.14	5.33	3.71
NB <sub>6</sub>	0.00	1.11	1.59	2.41	2.41	2.49	2.55	2.77	3.36	3.36	3.54	3.56	3.66	3.68	3.84	3.98	2.77

Table 10. Results of vector  $W_{PF}$  with FS-OWAIMAM I operator

$w_{PF}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Ave
NB <sub>1</sub>	0.00	0.22	0.32	0.49	0.49	0.50	0.51	0.56	0.68	0.68	0.71	0.72	0.74	0.74	0.77	0.80	0.56
NB <sub>2</sub>	0.00	0.27	0.39	0.59	0.59	0.61	0.62	0.67	0.82	0.82	0.86	0.87	0.89	0.90	0.93	0.97	0.67
NB <sub>3</sub>	0.00	0.33	0.47	0.71	0.71	0.73	0.75	0.81	0.99	0.99	1.04	1.04	1.07	1.08	1.13	1.17	0.81
NB <sub>4</sub>	0.00	0.66	0.95	1.44	1.44	1.48	1.52	1.65	2.01	2.01	2.11	2.12	2.18	2.20	2.29	2.37	1.65
NB <sub>5</sub>	0.00	0.64	0.91	1.38	1.38	1.42	1.45	1.58	1.92	1.92	2.02	2.03	2.09	2.10	2.19	2.27	1.58
NB <sub>6</sub>	0.00	0.41	0.58	0.88	0.88	0.91	0.94	1.02	1.23	1.23	1.30	1.31	1.34	1.35	1.41	1.46	1.02

Table 11. Results of vector  $W_{PF}$  with FS-OWAIMAM II operator

$w_{PF}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Ave
NB <sub>1</sub>	0.00	0.46	0.66	1.00	1.00	1.04	1.06	1.15	1.40	1.40	1.48	1.48	1.52	1.53	1.60	1.66	1.15
NB <sub>2</sub>	0.00	0.21	0.30	0.45	0.45	0.46	0.47	0.51	0.63	0.63	0.66	0.66	0.68	0.69	0.71	0.74	0.52
NB <sub>3</sub>	0.00	0.47	0.67	1.02	1.02	1.05	1.08	1.17	1.42	1.42	1.50	1.51	1.55	1.56	1.62	1.68	1.17
NB <sub>4</sub>	0.00	0.50	0.71	1.08	1.08	1.11	1.14	1.24	1.51	1.51	1.59	1.60	1.64	1.65	1.72	1.78	1.24
NB <sub>5</sub>	0.00	0.59	0.85	1.28	1.28	1.32	1.36	1.47	1.79	1.79	1.89	1.90	1.95	1.96	2.04	2.12	1.47
NB <sub>6</sub>	0.00	0.47	0.67	1.01	1.01	1.04	1.07	1.16	1.41	1.41	1.49	1.50	1.54	1.55	1.61	1.67	1.16

We have depicted results obtained in order to understand the behaviour of the information aggregated to specific industry (see Fig. 1). Illustration A shows the results of the FS-OWAAC operator, which are nearest of upper limit. They do not display significant differences between each alternative, even though alternatives NB<sub>1</sub>, NB<sub>2</sub> and NB<sub>3</sub> have more potential to be developed. Illustration B shows the results of the FS-OWAD, which are nearest of lower limit. They display significant differences between each alternative;

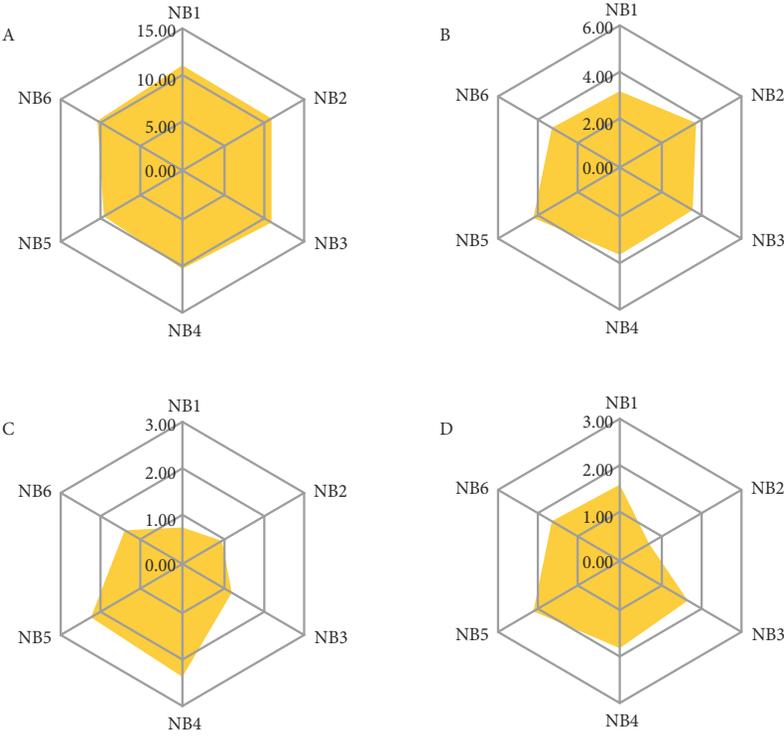


Fig. 1. Illustrated representation of each alternative

it is noteworthy that alternatives  $NB_1$  and  $NB_3$  can be developed with significance difference between other ones. Illustrations C and D show the results of the FS-OWAIMAM I and II, which are nearest of lower and upper limit simultaneously according to preferences of decision-maker. On the one hand, illustration C is noteworthy that the  $NB_1$  alternative is the most probable to be developed than other ones. On the other hand, illustration D is noteworthy that  $NB_2$  alternative is the most probable to develop than other ones.

Likewise, we can rank each alternative using the results obtained, which allows establishing order for all vectors (see Table 12). Since, we have used resulting data of current economic structure in Colombia, it is possible to analyse the individual decision-making and possibilities of development new business within the sports sector. It is to highlight that this methods are able to capture and deal with the environmental uncertainty and the subjectivity of the decision-maker. In this table, we show the order given for each method applied, the most favourable opportunities to be developed and their relationship with stage of development. Firstly, order has shown that the most favourable alternative to be carried out is  $NB_1$  followed by  $NB_2$ . Likewise, order has shown also that alternatives  $NB_3$  and  $NB_6$  have great potential to develop an entrepreneurial initiative. Notwithstanding, environmental factors do not offer some necessary conditions to raise the possibility of development. Finally, in the last part of the order, we have found that alternatives  $NB_4$  and  $NB_5$  have limited opportunities to be developed because environmental factors are more

hostile and less favourable. Secondly, we have classified each alternative according to the stage of development in order to determine the necessary stage to carry out some entrepreneurial alternatives proposed. Thus, the alternatives NB<sub>1</sub> and NB<sub>2</sub> are classified within the efficiency-stage and transition 2 of development. This stage is focused on promoting and helping to develop small businesses within the service economy. The alternatives NB<sub>3</sub> and NB<sub>6</sub> are classified in the transition 1 and efficient-stage of development. These kinds of industries are characterized by the development of scale economies, which are developed easily by large industries and incumbent firms since they have a dominant position on the market. Hence, current market conditions can affect the entrepreneurial activities positively or negatively, since they are focused on developing small-scale of manufactures and trade with the highest levels of self-employment. Finally, alternatives NB<sub>4</sub> and NB<sub>5</sub> need to develop high market sophistication. Currently, Colombia does not reach this level of development and sophistication, which impedes to encourage entrepreneurial activities with higher added valued within these kinds of industries. Thus, government’s intervention becomes necessary to help reducing entrance barriers, fostering sports business with finance programs and promoting technological development and innovation within the sports sector.

Table 12. Matrix ordering of alternatives

Order	1	2	3	4	5	6
FS-OWAAC	NB <sub>1</sub>	NB <sub>2</sub>	NB <sub>3</sub>	NB <sub>6</sub>	NB <sub>4</sub>	NB <sub>5</sub>
FS-OWAD	NB <sub>1</sub>	NB <sub>3</sub>	NB <sub>2</sub>	NB <sub>6</sub>	NB <sub>4</sub>	NB <sub>5</sub>
FS-OWAIMAM I	NB <sub>1</sub>	NB <sub>2</sub>	NB <sub>3</sub>	NB <sub>6</sub>	NB <sub>5</sub>	NB <sub>4</sub>
FS-OWAIMAM II	NB <sub>2</sub>	NB <sub>1</sub>	NB <sub>6</sub>	NB <sub>3</sub>	NB <sub>4</sub>	NB <sub>5</sub>
<i>decision-making possibilities</i>	<i>the most favourable</i>		<i>some possibility</i>		<i>more difficult</i>	
<i>stage of development</i>	<i>Efficient-stage and transition 2</i>		<i>Efficient-stage and transition 1</i>		<i>Innovation stage and transition 2</i>	

Source: own elaboration.

### Conclusions

In this paper, we have analysed three main items, the economic environment factors, individual decision-making processes and degree of uncertainty and subjectivity in order to determine the influence on decision-maker to start new businesses. Likewise, we have studied entrepreneurship from economic view, the factors that take into account the entrepreneur to make the decision to carry out an entrepreneurial activity or not, and its importance on the process of change towards knowledge-based economy. We have proposed a mathematical operator, which allow aggregating resulting data of the environmental factors and attitudinal character of decision-maker that represents the degree of subjectivity and the degree of uncertainty. Specifically, this new proposal allows aggregating information of an uncertain environment from the significance of its characteristics and developing a practical tool to select of the entrepreneurial opportunities according to a) the market and economic conditions within a specific country, b) attitude of potential entrepreneur and c)

uncertainty generated by politic, economy and social changes. We have considered resulting data of the economic environment of Colombia and the sports sector as case study to carry out the application of the method proposed. The results have shown the possibility of development of each alternative according to variables studied. These results have shown also in an orderly way all information aggregated, which can help potential investors and entrepreneurs to make a decision based on their preferences. It is noteworthy that the results allow analysing each alternative according to the stage of development, which provided overview about the possibilities to start new business within the current market structure. Furthermore, this new extension of the OWA operator has several implications. Firstly, the results of the economy and social conditions obtained from local and global reports can be better interpreted using this new method. Secondly, objective and subjective information from different sources can be aggregated in the same formulation and obtained a representative value, which provide a holistic view of the specific environment. Thirdly, this new method allows dealing with uncertainty beyond the importance of the characteristics that has the information. Finally, the applicability of this method in real case can be given in aggregation different sources of information to help at dealing decision-making processes, either entrepreneurial activity context or formulation of new policies and programs for the industrial and social development.

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