



OPTIMAL PORTFOLIO SEARCH USING EFFICIENT SURFACE AND THREE-DIMENSIONAL UTILITY FUNCTION

Aleksandras Vytautas Rutkauskas¹, Viktorija Stasytytė²

Vilnius Gediminas Technical University, Saulėtekio al. 11, LT-10223 Vilnius, Lithuania

E-mails: ¹ar@vgtu.lt; ²viktorija.stasytyte@vgtu.lt (corresponding author)

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Abstract. The concepts of effectiveness, riskness and reliability are three cornerstones which together with utility of investor constitute the base for decisions perception and management logic in order to match the possibilities of investment space with investor's objectives. Risk, which is "a chance or possibility of danger, loss, injury, or other adverse consequences" (The Oxford Modern English Dictionary) or, specifically, in the area of investment management – "the chance that an investment (as a stock or commodity) will lose value" (Webster Dictionary) is the function of riskness of selected assets altogether with skills of a subject to manage the riskness of the analysed object, process, etc. Thus risk is analysed as an interaction of possibilities riskness and abilities of a subject (investor) to manage these possibilities. The paper will reveal a consistent way towards investment possibilities set description, when investment assets possibilities are under uncertainty, what is understood in this paper as under stochasticity. As a possible means of the above mentioned match the authors propose portfolio adequate for investment stochastic nature and present its formation and application principles. This model has broad application possibilities in investing in exchange and capital markets as well as in forming sustainable investment strategies. There are many figures and schemes in the text. This is caused by the consideration that where geometrical drawing can provide a non-false solution, this drawing becomes also a decision search visualization instrument.

Keywords: profitability, riskness, reliability, utility function, efficient surface, sustainable development.

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1. Introduction

The paper presents a framework to solve a new problem – an argumentation on how to commensurate the utility of amount of investment possibilities, its reliability and risk for investor

(subject). Along with that, the methods of sustainable investment return assurance continue to be developed. The novelty of the paper is expressed by its originality, which is stated by the fact that the paper addresses the problem not previously solved in such a manner and proposes the methods for its solving, developed by the authors.

The concept of risk developed by the authors and already presented in the abstract, from slightly to drastically differs from variability of process or event, which sometimes is used for quantitative risk measuring. Other concepts mentioned among the keywords will be explained in the text if they are used in a slightly different sense than commonly.

1.1. Development sustainability – a direction for the future

The broad concept of sustainable development gained prominence after the publication of the so-called Brundtland Report ‘Our Common Future’ (WCED 1987). Sustainable development was defined there as: *development that meets the needs of the present without compromising the ability of future generations to meet their own needs.*

Sustainable development has become an important part of international and national approaches to integrating economic, environmental, social and ethical considerations so that a good quality of life can be enjoyed by current and future generations for as long as possible (Moffatt, Hanley 2001).

Although sustainable development is difficult to define using mathematical terms, many researchers recognize that it is a function of two major components, ecological and human. Therefore, sustainable decision-making should have two simultaneous goals:

- achievement of human development to secure high standards of living;
- protection and improvement of the environment now and for the generations to come (Andriantiatsaholiniaina *et al.* 2004).

Research has produced numerous indicators of sustainable development so that it is possible to gain some insight into whether or not an area, region or nation is on a trajectory of sustainable development (Moffatt, Hanley 2001).

The concepts of sustainable evolution or development, which have become the categories of analysis and management of countries, regions and other multiaspect and complex systems (Wallner 1999; Čiegis *et al.* 2009; Hall *et al.* 2010; Todorov, Marinova 2010), probably inherited their constructive philosophy and methodology about these systems’ present and future necessity of harmonic interaction from the research of populations’ (microorganisms, flora, fauna, etc.) sustainable development possibilities. However, such a conversion has certain inadmissible losses. One of the main attributes describing the sustainable development of populations – guarantee, that the current state or embraced trend of development will remain unchanged during a long period of time with high enough probability – is not cherished. The essence of this concept of persistence or survival is visualized by the provision that probability (guarantee) P of the changing state (for example, the probability that the quantity ξ of a certain population during a long enough period of time $t \in (0, T)$, will not drop lower than a certain value K_g , critical for the population) should remain at a certain level g (Rutkauskas, Stasytytė 2010):

$$P\left\{\xi_{t \in (0, T)} \geq K_g\right\} = g. \quad (1)$$

There is no doubt that analysis and management of the country and region sustainable development, related with research on such multidimensional processes when separate aspects are linked with each other by the complex interdependencies, is a complicated problem, hardly conforming to operational management decisions. However, in many cases a provision about preservation of quantitatively measured guaranty for certain economic, demographic and financial proportions can become a fundamental framework of the entire sustainable development nurturance.

The point already mentioned is very important in the projection of decisions for the large investment subjects, such as investment banks, mutual and pension funds, etc. An attempt of quantitative evaluation of the possible reliability or guarantee of activity results should mobilize the makers of such strategies to reveal the problems, solutions for which strongly influence company success, and which still do not have decisions giving satisfactory results.

One of the authors' areas of intensive research is the search for sustainable return in exchange markets (Rutkauskas *et al.* 2006, 2008; Rutkauskas, Stasytytė 2006).

1.2. Investment – a constructive dialog with the future

It is necessary for selected methods in future possibilities discussion, as well as for core concepts clearly to associate with analysed objects, processes and decisions made. In order to illustrate investment results possibilities description and estimation of the best possibility a one-step portfolio investment is selected, and on its basis the decision making and implementation will be discussed. In the given case the portfolio P is composed of 3 assets, which are described by certain probability distributions of their possibilities if a unit capital is invested:

$$D_1(a_1, \sigma_1), D_2(a_2, \sigma_2), D_3(a_3, \sigma_3).$$

$$a_1 = 1.1; a_2 = 1.4; a_3 = 1.7.$$

$$\sigma_1 = 0.11; \sigma_2 = 0.15; \sigma_3 = 0.19.$$

Thus $P = p(N_1, N_2, N_3)$ is a random variable with probability distribution of its possibilities and survival function (Fig. 1). Here function p is assumed to be simply the sum of the assets $p = A_1 + A_2 + A_3$, and probability distributions D_i are represented by Normal probability distributions N_i .

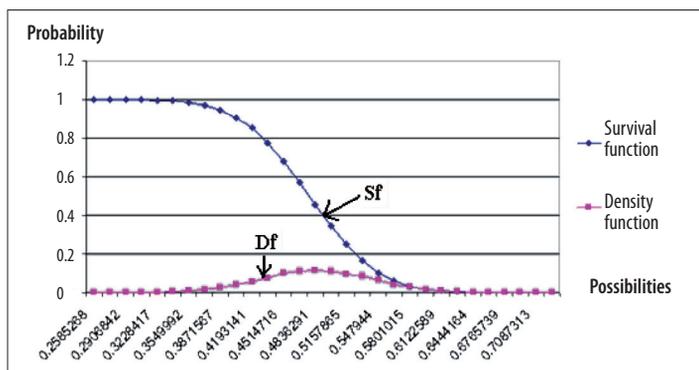


Fig. 1. One-step effect (value increase or decrease) portfolio possibilities' probability distribution (density function – Df) and survival function (Sf)

In Fig. 1 the process is presented as a discrete case. Thus, the sum of positive probabilities of the values (possibilities) described by the density function compose 1.

Further in the text as possibilities of effect (or effectiveness) determining rule of portfolio or asset a survival function will be used, each point of which denotes a numerical value of effect possibility and reliability or guarantee $P\{\xi \geq x\} = g$ of this value.

1.3. Past versus future

Often speaking about determination and uncertainty past is accepted as a bastion of determination and that things happened in the past can be estimated (measured, accepted) unambiguously, while future cannot be treated unambiguously and forces us to learn the particular language for analysing certain events in the future.

Difference between past and future becoming possibilities simply is accepted as a dogma. We never think that if the time would stop the events for several moments or transfer them forward, then, first of all, no one of existing persons or animals would live. There would have been another people, instead of which now we are here, and in the former case the world would be completely different. Accepting the reality in such a perception, deliberating what would be now if time would have shortened for a while or would have lengthened is no less complex than attempting to answer what will be in the future. There is no doubt that concentrating attention on stochasticity of the real processes in the past we could more reliably answer the question of “what...if” for the past. And this could serve for developing of “what...if” analysis methods for the future. Even theoretically more argumented and detailed analysis of past possibilities realization should be the strong methodological argument and source of information for analysing “what...if” in the future. Thus, consideration of “what...if” could become not only increasingly interesting story for science fiction novel, but also the reasoning of future possibilities evaluation. Further the main concepts and thought logics will be discussed, related with future analysis considering possibilities of a person to influence the future realization of events with the help of his today’s decisions. Of course, the mankind is not omnipotent while choosing the future, but there exist such situations when the set of future events fully depends on today’s individual steps. Investment will be selected as an exceptional personal activity, because here today’s actions are projected in order to convert our decisions and intentions into attained objectives, implementing our expectations.

In order to associate the core concepts of the selected language style with the analysed actions, processes and decisions made, in the paper, as it was mentioned before, one portfolio investment management step is selected, in the beginning of which we invest a unit of capital, and possible result at the end of this step is described by investment effect possibilities’ probability distribution. As it was already mentioned, in the analysed case portfolio is composed of three assets, for which the perspective of unit capital investment one step period is described respectively by certain probability distributions of possibilities.

2. Adequate portfolio as natural result of modern investment portfolio development, intended for integrated profitability, risk and reliability application regarding investor’s utility function

Function of fundamental modern (Markowitz) portfolio and its further amplifications (Fabozzi, Markowitz 2002; Reilly, Brown 2003) is an intention to commensurate investment

profitability and risk objectively and to give an opportunity to select a portfolio taking into consideration investor's indifference curve. Efficiency line of portfolio values is fundamental mean of such choice and optimization (Sharpe 1964). However, evaluation of the aimed profitability's reliability and along with general commensuration of profitability, risk, and reliability levels, the essence of which discloses analytically through designing an effective surface in three-dimensional – profitability, risk, reliability – space is of premium and natural importance for today's investor. Effective surface, which is formed as an intersection of survival functions of portfolio possibilities values and iso-guaranties, not only contributes for such a commensuration, but also becomes a set of constraints searching for the possibility of the highest profitability for an investor, in other words a criteria invoking his utility function, that depends on profitability, *risk*, and reliability. Here the word risk is distinguished in order to stress the principal difference between the riskness of investment possibilities' and investor's risk, which depends also on individual features of an investor.

In order to reveal in details the contents and mechanism of portfolio investment decisions' reliability concept, we will briefly take a look over adequate for investment decisions reliability assessment portfolio anatomy.

Fig. 2 presents adequate portfolio for investment decisions reliability assessment, formed for a case of statistically independent assets. "Mean – standard deviation" portfolio (modern, or Markowitz portfolio (Markowitz 1952)) is shown in Fig. 2 section *a*. Next, a bunch of the possible values of all possible "quintiles – standard deviation" portfolios (Fig. 2, section *b*) is formed. More precisely speaking, not all the quintiles were used for this bunch here, but all percentiles. Thus, the bunch consists of 100 layers of possibilities' set, each representing different reliability level. On the basis of the set of formed portfolio values and on the analogy of the efficiency line of modern portfolio, a set of all efficiency lines is composed, which is the framework for efficient surface formation. The efficient surface is presented in Fig. 2, section *c* for the three selected assets, the possibilities of which here are defined by the Normal probability distributions:

$$N_1(a_1 = 0.12, \sigma_1 = 0.03), N_2(a_2 = 0.07, \sigma_2 = 0.02), N_3(a_3 = 0.06, \sigma_3 = 0.02).$$

There is no doubt that investor is interested not only in quantitative indicators of investment profitability possibilities, but also in the guarantee of each possibility – i.e. the probability that investment profitability (return) will not drop below the certain level. In case of modern stock portfolio, the guarantees of investment profitability possibilities are usually not discussed, although in case when portfolio return possibilities' probability distribution is a Normal one, there is a direct possibility to evaluate these guarantees, if mean value and standard deviation are known (Rutkauskas 2000). Moreover, if the probability distribution of return possibilities is not Normal and possesses a substantial enough amount of skewness, then guarantee as a third indicator for possible value description gains even bigger importance as only in such a manner the expected value of return can be fully perceived and thoroughly quantitatively described. Raising every efficiency line from the Fig. 2 section *b* by the level of its guarantee or reliability, the three-dimensional view (profitability, riskness, reliability) of the investment portfolio is formed (Fig. 2 section *c*). The precise description of the process of adequate portfolio formation and its geometrical view development can be found in (Rutkauskas 2000, 2006; Rutkauskas, Stasytytė 2010). In Fig. 2 section *d* we have the three-dimensional view of utility function, but it will be discussed in details in next chapter.

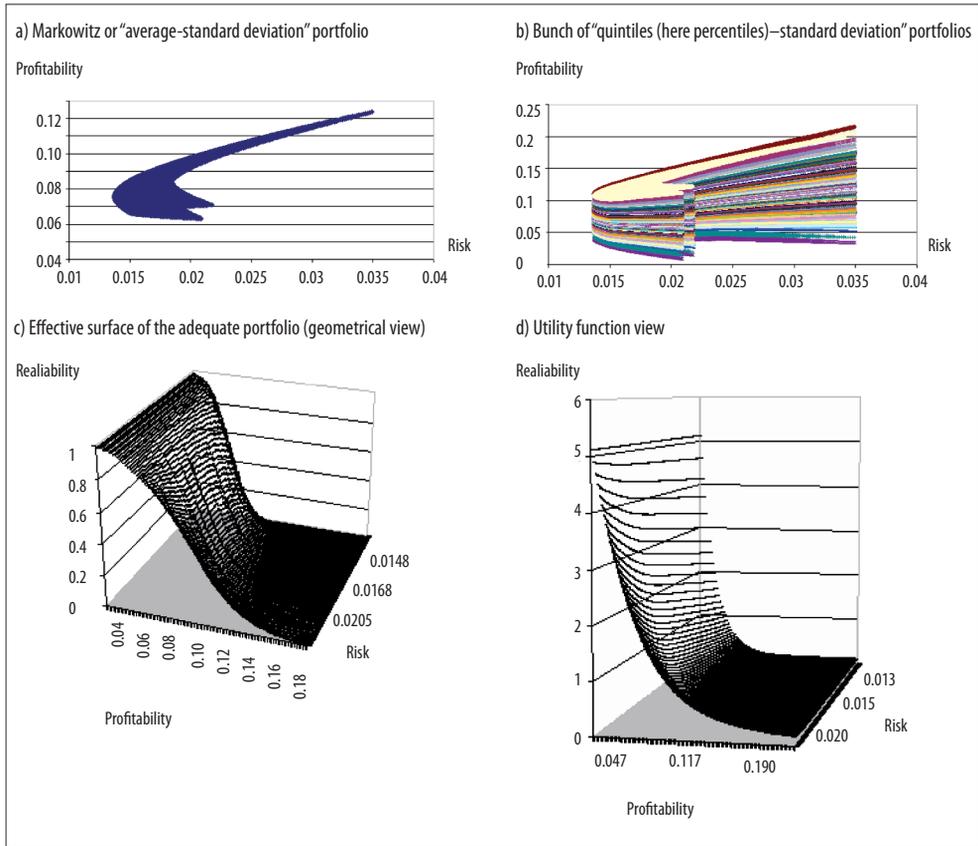


Fig. 2. Succession of adequate portfolio formation

3. The problems of efficiency, reliability and risk commensuration in selection of investor's utility function

Selection of multicriteria functions and assessment of possibilities of practical application are highly important problems which receive much attention of mathematics, mechanics and other representatives of "quantitative" science. None the less attention should be given to these problems in social sciences, where great part of factors is described only qualitatively, and its direct application in multicriteria analysis provoke a lot of questions (Steuer, Na 2003; Bivainis, Drejeris 2009; Ginevičius, Podvezko 2008; Ginevičius, Zubrecovas 2009; Turskis *et al.* 2009; Zavadskas *et al.* 2010).

This paper intensively analyses a problem of factors commensuration problem – detection of possibilities which determine how much the change of one factor amount (unit) varies in terms of other factor certain amount (number of units), when it is attempted to retain the same weight of multicriteria function. Utility function and izoguarantees are examples of exceptional importance in this paper.

3.1. Let us begin from efficiency and reliability commensuration

The common utility function, depending on the two parameters – p (the size of possibility) and g (reliability (guarantee) of possibility) – looks as follows:

$$U = u(p, g) = \exp\{p\} \cdot g. \quad (2)$$

Even though in risk management literature the mentioned problem of profitability, reliability and risk commensuration constantly receives a high attention, but in job situation analysing even such classical problem as selection of the best solution with regard to profitability and reliability together in a situation represented in Fig. 1, A. Riabacke (2006) makes a conclusion that “risk and probability estimation made by managers are often based on inadequate information and intention, that not formal analysis is carried out, that no computer based decision tools are used in the decision making process, and therefore most decisions are based on intuition and gut feeling”.

Thus even in situation presented in Fig. 3, which is so simple and natural that it seems to be dictated by the high intelligence, there are some doubts regarding which X value is most suitable for an investor. Moreover, it is not advised to make any recommendations for a particular investor without a clear explanation of future impact of every decision made when uncertainty is present. Probably utility function as a decision making instrument (Fig. 2 section *e*) should incorporate evaluation of possibilities of individual loss, or risk.

It is worth repeating that full understanding of risk should be interpreted as subject's (investor's) probability distribution of loss possibilities. Along with that it is clear that investor's risk always depends on investment riskness. Utility function, or simply indifference curve of an investor whose loss in case of failure is big demands a non-diminishing guarantee even for a highly profitable and promising investment.

3.2. Evaluation of possibilities reliability – the core problem in investment decisions management

Evaluation of solutions reliability or guarantee is the urgent decision management (decisions selection and implementation) problem, that differentiates and at the same time associates logics and methods of solutions, gained under the terms of determinated relation, and solutions, gained under the terms of uncertainty and risk. Reliability of solutions, under the terms of determinated relation, associates with the accuracy of relation measurement and the propriety of decision methods, while evaluation of reliability, under the terms of uncertainty and risk, assumes to be entirely distinctive problem. In the paper we will use the portfolio, adequate to the evaluation of investment possibilities reliability, or simply adequate portfolio, which appears to be not only an innovative approach to investment decisions management, but also an effective mean to analyze possibilities and project sustainable development of sophisticated systems (Rutkauskas, Stasytytė 2010). Analyzing stochastic (probabilistic) values or processes we will measure reliability of possibility as reliability or survival function $S(x) = 1 - F(x)$, here $F(x) = P\{\xi < x\}$ is accumulated distribution function of investment possibilities.

It is important to notice that in literature for decisions reasoning often profitability and risk concepts are proposed to use, and even more precisely – the interaction of processes

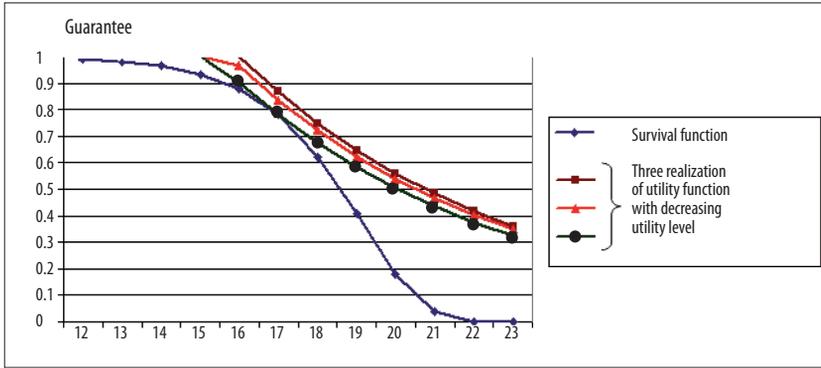


Fig. 3. Tangent point of sub-utility 1 and survival function as an instrument for decision search (Rutkauskas, Stasytytė 2010)

described by these concepts. However, according our proposed logics, risk is intended for reflection of not only investor’s possible losses due to investment possibilities riskness, but also investor’s ability (or inability) to manage the consequences of this riskness.

3.3. The concept of iso-guarantee

In order to approach the evaluation of investment utility for investor according effectiveness of possibility, possibilities’ riskness and reliability of every possibility, a concept of iso-guarantee is used and applied, which was proposed by the authors (Rutkauskas 2003). Here and further in the paper investment portfolio and every asset effectiveness will be perceived as portfolio profitability.

Q-level isoguarantee of investment portfolio is an efficiency line of „q-level quintile – risk“ portfolio possibilities, which is raised to q level on z axis (applicate axis) and connecting possibility set values of the same guarantee under changing risk conditions.

In probability theory and mathematical statistics terminology, iso-guarantee should indicate a line, connecting q-level quintiles $\xi_q^s : \left(P\left\{ \xi^s \geq \xi_q^s \right\} = q \right)$ in a set of portfolio profitability possibilities when portfolio riskness (s – standard deviation) changes (grows).

In general (Markowitz) case, efficiency line of modern portfolio is not iso-guarantee. If portfolio profitability possibilities mean value equals median for each risk level, then efficiency line becomes the iso-guarantee of 0,5 level.

According efficient frontier generation logics, if all the possible quintiles (here – percentiles) become profitability resultant, then structural formation of the set of iso-guarantees is presented in Figs 4 and 5. Here, as it was assumed earlier, our assets selected in 2.1 subchapter are independent random values $N(0.12; 0.03)$, $N(0.07; 0.02)$, $N(0.06; 0.02)$.

After plotting all efficiency lines of selected step between neighbouring quintiles, a set of efficiency lines will be formed (Fig. 5), constructing the base for efficient surface formation. If all iso-guarantees from “risk-profitability” plane could be lifted into “risk-profitability-reliability (guarantee)” space according to their reliability level, the surface of all possibilities

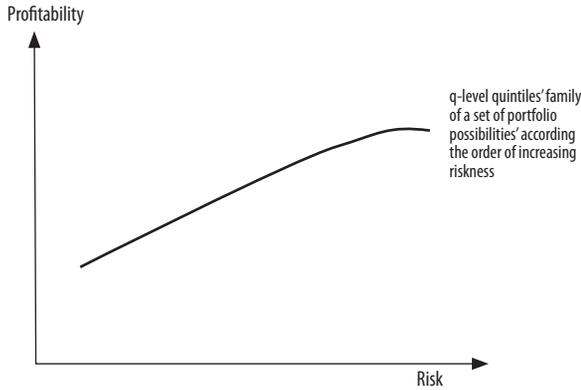


Fig. 4. The origin of iso-guarantee

of investment portfolio would be depicted as in Fig. 6. This surface in the paper, according the analogy of modern portfolio efficiency line title, is named the efficient surface.

Fig. 6 presents an adequate portfolio efficiency zone (surface). Efficient surface is a network of intersecting survival functions and iso-guarantees.

While analysing investment portfolio efficiency lines in two-dimensional plane, we determine possible values of optimal portfolio, and, in turn, propose the selection of optimal portfolio, when its utility is measured according utility function depending on profitability and riskness in “risk-profitability plane”. This line further will be called sub-utility 2 function, which is in fact indifference curve (Fig. 7).

On the efficient surface, i.e. in three-dimensional space, the role of efficiency lines is assigned to iso-guarantees. Here it is possible to analyse the selection of utility possibilities measured in three parameters: profitability, reliability of profitability and risk with the help of three-dimensional utility function.

3.4. Sub-utility 2: commensuration of profitability and risk

In paragraph 3.1 we have already discussed the so-called sub-utility 1 function, which allows selecting optimal profitability and reliability ratio for particular investor, and the efficiency line itself becomes a source of information estimating how much reliability (guarantee) one should dispose in order to achieve profitability increase of one unit. However, classical portfolio management conception states that every investor selects the portfolio that satisfies the investor’s desired risk level and provides the maximum profitability under accepted risk level (Hirt, Block 1993; Lumby 1994). Such selection is made with the help of indifference curve, which will be named as sub-utility 2 here. Thus sub-utility 2 function helps the investor selecting optimal portfolio with regard to profitability – riskness ratio. Sub-utility 2 function

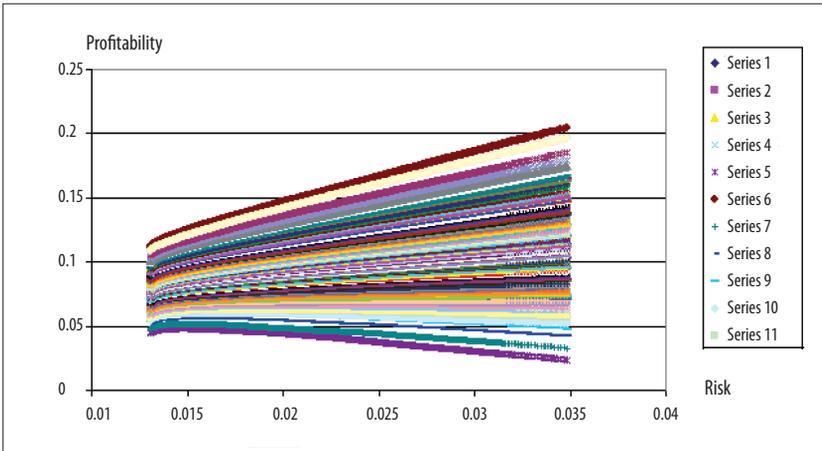


Fig. 5. Projection of efficiency lines and iso-guarantees in risk-profitability plane

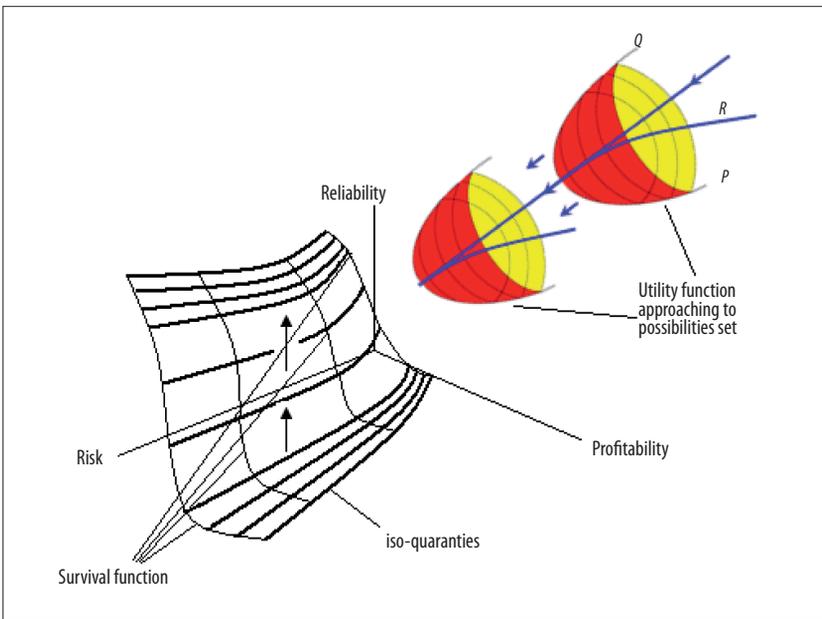


Fig. 6. A scheme of efficient surface (Rutkauskas, Stasytytė 2010)

is viewed as the family of indifference curves, approaching the efficient frontier as the utility level decreases (Rutkauskas 2006). Graphically it is shown in Fig. 7.

It is worth noticing that Fig. 7 presents only the family of indifference curves of an investor accepting moderate risk level.

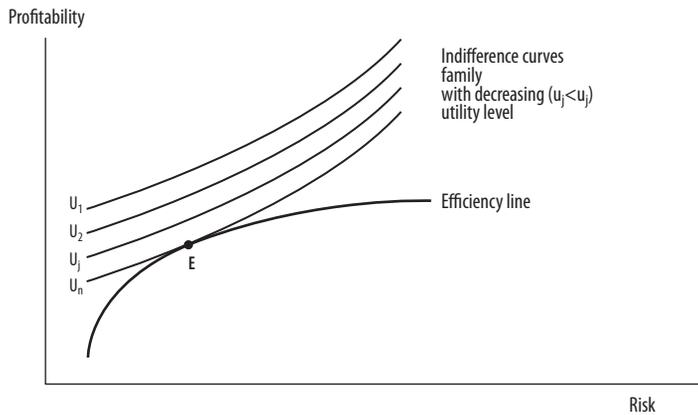


Fig. 7. The intersection (point E) of indifference curve family and efficiency line – the optimal portfolio for the investor in “profitability-risk” plane (Rutkauskas, Stasytytė 2010)

Now, returning to Fig. 6, we can recall that if adequate portfolio possibilities’ set is an intersection network of survival functions and iso-guarantees, then in space utility function is an intersection network of sub-utility 1 and sub-utility 2 functions.

3.5. Multifactor stochastic optimization applying efficient surfaces

The mean-variance methodology (Markowitz 1952) for portfolio selection problem has been central to research activity and has served as a basis for the development of modern financial theory. However, the expected return of a portfolio can be used only as an approximation, because returns are random. Markowitz model had been extended including additional parameters (for example, skewness) into portfolio selection problem (Lai 1991; Konno *et al.* 1993; Prakash *et al.* 2003; Konno, Yamamoto 2005; Steuer *et al.* 2005; Joro, Na 2006). Portfolio selection problem can be then viewed as a multiobjective mathematical problem.

Multiple criteria decision making (MCDM) has been widely applied in finance (Zouponidis, Doumpos 2002; Costa, Soares 2004; Zavadskas, Turskis 2010). Steuer and Na (2003) presented a categorized bibliography on the application of multiple criteria decision making. Classifying papers according the area of application of MCDM, they found out that 29% of the analysed scientific researches dealt with portfolio selection problem.

Along with that, the perception of various objectives, and mainly portfolio return, as stochastic values in portfolio selection problems became obvious (Steuer *et al.* 2005; Huang 2007; Buckley *et al.* 2008; Ehrgott *et al.* 2009). Stochastic programming (SP) and particularly multi-objective stochastic programming models can be used to deal with such difficulties. Among the applications of multi-objective stochastic programming in portfolio selection the development of a mean-variance approach having several scenarios with known probabilities, a formulation of stochastic goal programming based on utility function and “mean-variance”

model, a multi-objective stochastic linear programming formulation of portfolio selection problem under uncertainty, a proposition of a Compromise Chance Constrained Programming model for portfolio selection are worth mentioning (Ben Abdelaziz *et al.* 2007). All these models assume that the objectives are random and normally distributed. However, the huge amount of research has been dedicated to the development of arguments that one of the main objectives in portfolio selection – portfolio return – is not normally distributed (Elton, Gruber 1974; Eberlein, Keller 1995; Constantinou *et al.* 2006; Yang 2006; Ben Abdelaziz, Masri 2010).

In such a case multi-objective stochastic programming problems and models became even more difficult to apply. However, such problems can be possibly solved to the desired degree of precision under certain limitations with the help of imitative technologies. Such technologies allow to design a certain computational framework (Rutkauskas 2006) and visualize the decision search with the help of efficient surface formation.

Fig. 6 shows geometrical view of efficient surface, the structure of which is revealed here in details. As it was already mentioned, efficient surface is formed by raising efficiency lines of „risk-quintiles“ sets of portfolios into the level α ($0 \leq \alpha \leq 1$) of a certain quintile on applicate axis Z above the risk-profitability plane (X, Y). In every risk point r (on intercepted ordinate Y) the plane which is perpendicular to Y axis, the intersection point to the made surface results in a survival function of a certain random value. The value N is a function of the selected assets $N_1(a_1, s_1)$, $N_2(a_2, s_2)$, ..., $N_k(a_k, s_k)$ and certain portfolio structure (w_1, w_2, \dots, w_k) :

$$N = w_1 N_1 + w_2 N_2 + \dots + w_k N_k. \quad (3)$$

In our analysed situation three Normal random values $N(m_1 = 0.12, s_1 = 0.03)$, $N(m_2 = 0.07, s_2 = 0.02)$, $N_3(m_3 = 0.06, s_3 = 0.02)$ are chosen as portfolio assets.

Decision search with the help of efficient surface and dimensional utility function allows us to make one more step aside from the multicriteria methods and multi-objective mathematical problems, which are so much popular in investment science nowadays. The selection of particular portfolio structure in the certain intersection point of two surfaces does not limit itself to the traditional optimization problem: to find the best solution according several criteria, selecting one criterion for maximization (minimization) and others treating as constraints. The proposed geometrical decision-making method allows commensurating all the three criteria and solve the tasks of slightly different nature – for example, to answer the question: how much risk one should incur in order to get profitability higher by one unit, the general utility of the decision being the same? Such problems are called multifactor problems.

3.6. Practical application of utility function and the random factor

The configurations of possibilities' set (efficiency zone) and utility (objective) function and their inter-position, as well as analytical expression of our applied utility function points out that the magnitude of the possibility, as well as the increase of reliability of possibility both positively influence the growth of utility. However, the analytical expression of the utility function being used provides that the increase of risk negatively influences the growth of utility.

Unfortunately, analytical possibilities of comparing different and distinct enough possible portfolio values according their profitability, riskness and reliability, if they were generated

from probability distributions of different forms of assets, remain limited enough. However, there is no doubt that as the network of izoguarantees and survival functions becomes more solid, the continuous surface of the set of possibilities (efficiency zone) is being formed, and along with that the network of utility values, which is composed on the basis of functions subutility 1 and subutility 2 and which generates continuous spatial utility function, is getting more solid as well.

Thus universal analytical possibilities of commensurating profitability, reliability and risk on the surface of adequate portfolio possibilities' appear with regard to the surface of utility values. Also, these two surfaces are convex surfaces with regard to each other. Considering the continuity and convexity of these two surfaces, in order to find a solution of the desired precision or simply their intersection point, one can use their discrete adequate values. This simplifies finding the solution.

The more delicate problem is the treatment of uncertainty inherency. It is important to admit that for several centuries in scientific field it is being discussed whether it is the truth that uncertainty appears because of inability to precisely measure and unanimously evaluate the possibility being analysed with the help of analytical computations, or that in the framework of present and future cognition there will be plenty of such situations when under uniform conditions the obtained results will be different. In other words, we observe the phenomena of stochasticity, which influences the realization of the events or processes being analysed (Knight 1921; Kahneman, Tversky 1979; Tversky, Kahneman 1992; Nakamura 1999; Bo, Sterken 2007; Anderson *et al.* 2009; Gayer 2010).

The paper admits the second assumption stating that selection of equal investment proportions into the same assets can provide different set of investment possibilities, even if they are described with the same probability distribution.

But discussing the decision-making for perspective it is worth recalling that our constructed surface of possibilities is obtained with the help of determinate characteristics of assets as stochastic values: quintiles, survival functions, etc. Thus the obtained surface of possibilities is merely theoretical, even though very informative scheme of the real situation.

The essence of the mentioned situation is in that every of the selected assets gains values out of its set of possibilities with the participation of the phenomena of stochasticity, and that process recalls the formation of portfolio as a set of possibilities of the stochastic value:

$$P: w_1^i D_1(a_1, s_1) + w_2^i D_2(a_2, s_2) + w_3^i D_3(a_3, s_3), \quad (4)$$

where $w_j^i, i = \overline{1, n}, j = 1, 2, 3$ are proportions of the invested capital $\sum_{j=1}^3 w_j^i = 1$ distribution to every i . Thus the forms of probability distributions of assets in turn construct the shape of probability distribution of portfolio possibilities' set. The knowledge of probability theory allows us evaluating it or even recognizing it as one of standard probability distributions.

However, efficient surface can be seen as stochastic process, or simply a family of stochastic values, in which the risk value r is a critical parameter, depicted on ordinate axis Y , and possibilities of the respective random values (efficient portfolios) are described by the survival function of the random variable corresponding to the mentioned risk value.

As deciles, percentiles or more small parts of a percent can be selected as quintiles, selecting infinitely small step between neighboring structures $(w_1^i, w_2^i, \dots, w_k^i)$ and $(w_1^j, w_2^j, \dots, w_k^j)$, the surface network can seem converging to continuous surface. On this surface the interaction of the core parameters, describing the possibilities of uncertainty occurrence can be analysed – i.e. performing differential valuations of different type, including the search of parameters' mutual equivalency to various functions, depending on the following three parameters: p – profitability, g – reliability (guarantee) and r – risk.

One of important and interesting solutions of the problems is visualization of process and quantitative relations between profitability, reliability and risk, which leads to the development of cardinal method of stochastic optimization. Efficient surface is composed of possibilities available for investor from portfolio investment into selected assets, the investment possibilities being described by profitability, reliability and risk indicators. These are the indicators which fully describe the utility of a possibility for an investor. Thus, investor's utility function can become an objective function in stochastic possibilities' set.

3.7. Through sub-utility 1 and sub-utility 2 towards the selection of utility function

We have already exploited the concept of utility function through the so-called sub-utility 1 and sub-utility 2 cases. Let's further explore how the combination of these functions into integrated network could result in a three-dimensional utility function, described by portfolio possibility's profitability, reliability of this profitability and risk, which is faced by the investor if the mentioned possibility occurs.

Also, the development of this network is a logically comprehensive procedure. It must necessarily be a solid network, allowing to compare all possibilities of the set of adequate portfolio values with regard to three attributes of a possibility: profitability, reliability and risk. To implement this objective one should select the logics and technique of utility function.

Fig. 1 presents a schematic view of two-parametrical utility function in “profitability-reliability” plane, where it is named as sub-utility1, while in “quintiles-risk” plane (Fig. 5) it is named as sub-utility 2. However, distinct application of separate two-dimensional utility functions is not fully informative, because cannot directly dispose the information about the interaction of possibilities' profitability, reliability and risk in possibilities' utility formation. For this reason (Fig. 6) three-parametrical utility function is composed in “profitability-risk-reliability” space (right side), which is approaching the set of adequate portfolio values and thus indicates the best portfolio value, and, in turn, the portfolio itself.

Graphical view of such utility function is composed using the following analytical function:

$$U = \exp\left(\frac{p}{r}\right) \cdot g, \quad (5)$$

where U is the utility level of possibility, p denotes profitability, r is the risk and g – the guarantee.

Such specification of utility function and decision-making procedure is analytically meaningful, because it allows to solve a complex stochastic programming task with the help of imitative technologies and graphical decision-making methods.

Fig. 8 exhibits the mutual position of possibilities' set, being analysed in Fig. 2, efficiency zone and utility function under certain (according selected utility function) utility level.

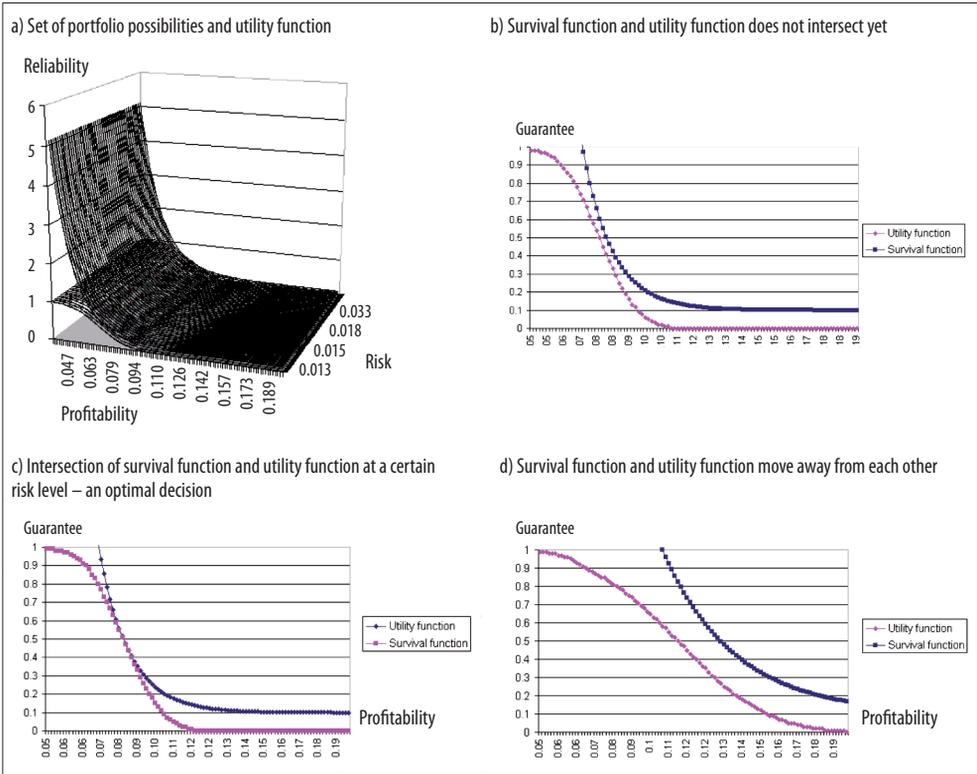


Fig. 8. The process of optimal decision formation

Fig. 8a discloses that this is the position of intersection of two almost continuous and convex with regard to each other surfaces. After dividing both three-dimensional surfaces into separate sections according certain risk level, one can watch the process of optimal decision formation. In Fig. 8b survival function and utility function does not intersect yet. Fig. 8c shows the moment of decision fixing. And in Fig. 8d both surfaces again move away from each other again.

Indeed, when utility degree of utility function is decreasing, one of the sections of efficient surface, perpendicular to the abscissa axis OX and, passing through certain survival function, is first to touch the utility function itself, along with that touching one of iso-guarantees, while the respective sections of the higher or lower risk levels do not reach their survival functions.

The resulting tangency point denotes the possibility having the highest utility, which at the moment possesses the following characteristics: $p = 0.086227$; $g = 0.44$ and $r = 0.01439$. The structure of optimal portfolio is: $w_1 = 0.265$; $w_2 = 0.34$; $w_3 = 0.395$.

4. About the shape of assets possibilities probability distributions and influence of statistical dependencies of these possibilities on decision-making

Conceptually, there is no doubt that solution should depend on every assets' possibilities probability distributions' shape, as well as on statistical interdependence degree of these possibilities.

In fact, we use the so-called principle of *all-round optimization*. All-round optimization, or, simply, financial optimization, means a certain optimization scenario, when the whole of the separate objectives is presented as a composition of these objectives. Also, this composition is being expressed as mathematical or logical function of criteria (indicators) of these objectives, while factors themselves, their relations and constraints, as well as separate aspects are evaluated with the help of value equivalents of these factors and constraints.

In essence, solving such a problem is a means of reaching the primary economic objective – research on the usage of possessed primary resources, which are measured in value expression creating the highest value-providing supply for the user.

Fig. 9 exhibits the same mean values and standard deviations having efficient surfaces of assets as in Figs 2 and 8, but having the shape of Lognormal probability distributions (Fig. 9, section *a*). Thus retaining the same utility function the best possibility appears to be as follows: ($p = 0.078809$, $g = 0.66$, $r = 0.015118$). The presented possibility is guaranteed by the portfolio ($w_1 = 0.055$, $w_2 = 0.755$, $w_3 = 0.19$). Thus it is obvious that not only externally visible attributes, but also the result of the decision changes substantially if the shapes of assets' possibilities probability distributions are modified.

The potential of the system proposed by the authors allow to analyse practically every probability distribution of possibilities, including those presented empirically.

Fig. 10 exhibits the case when probability distributions of assets' possibilities have Lognormal distribution shapes and also are statistically dependent according the following matrix:

$$C_{ij;ij=\overline{1,4}} = \begin{pmatrix} 1 & 0.8 & 0.3 \\ & 1 & 0.7 \\ & & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0.9 & 0.7 \\ & 1 & 0.8 \\ & & 1 \end{pmatrix}$$

Comparing Figs 10 and 9, as well as comparing Fig. 9 with Figs 2 and 8 one can see that the variations in external attributes must inform about optimal solution change. Indeed, in such a case the obtained solution is a possibility with ($p = 0.0978$, $g = 0.4$, $r = 0.025268$), the utility level of which and portfolio generating this possibility is ($w_1 = 0.455$, $w_2 = 0.105$, $w_3 = 0.44$). Thus, comparing the numerical results and graphical representation of the set of portfolios, one can see the correlation impact on decisions' result.

Thus, Fig. 10 clearly exhibits the impact of correlation of assets' probability distributions on the final result – the point where optimal portfolio is found.

In case of complex probability distributions of investments assets' profitability possibilities, their interrelations, and complex form of utility function, formation of optimal portfolio is complicated and has no universal methods for decision making. Thus, in order to have an operative mean of such decision, that is necessary for the application of adequate portfolio for decision making while investing in exchange and capital markets, one should apply imitative technologies, which are almost the only mean of information supply for the quick decision management process.

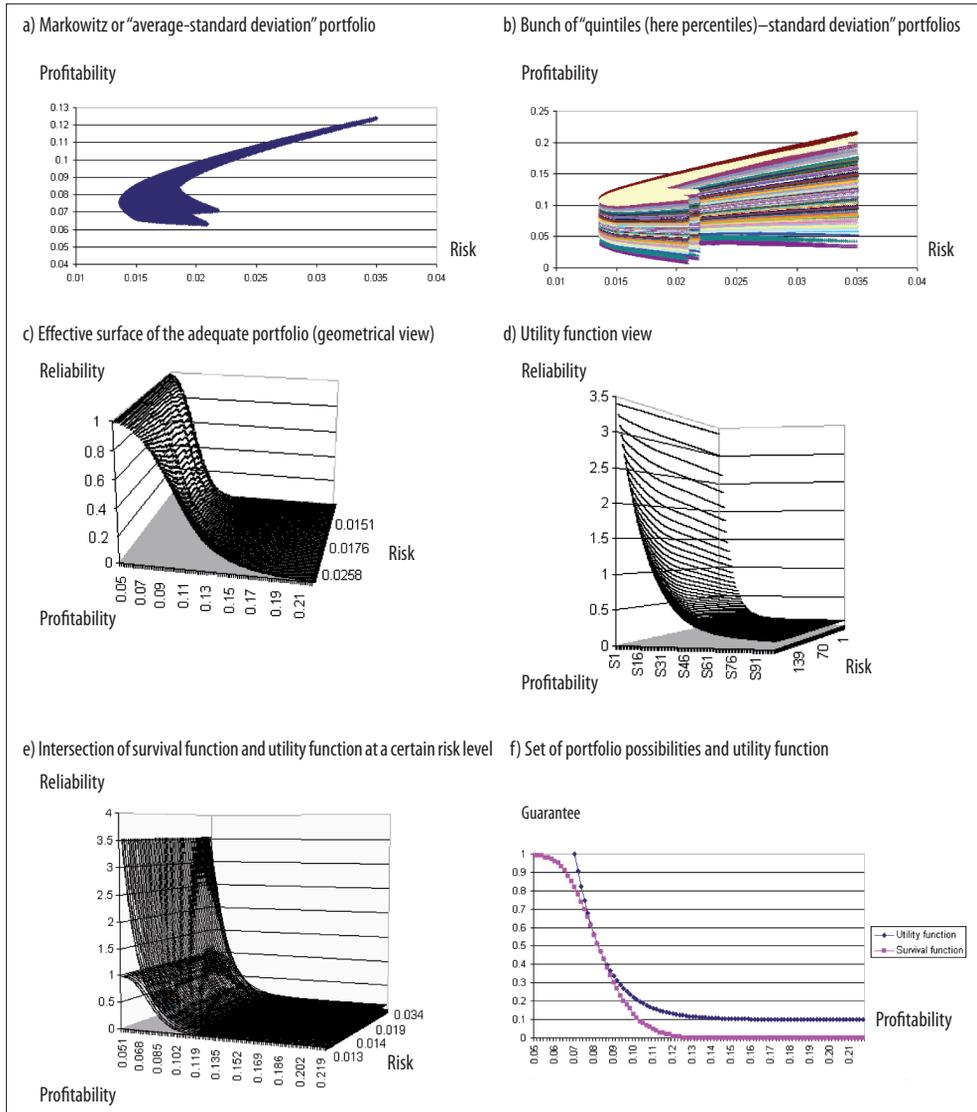


Fig. 9. Succession of adequate portfolio formation (Lognormal probability distributions)

5. Conclusions

1. Profitability, riskness and reliability are three main attributes that ought to be used integrally when investment decision is under preparation.
2. Probability distribution of profitability possibilities, and precisely – the survival function developed on its basis, – allows evaluating the interaction of profitability and reliability levels. Risk, in turn, reveals the dependence between the riskness of the analysed objects (processes) and possibilities of subjects to cope with consequences of this riskness.

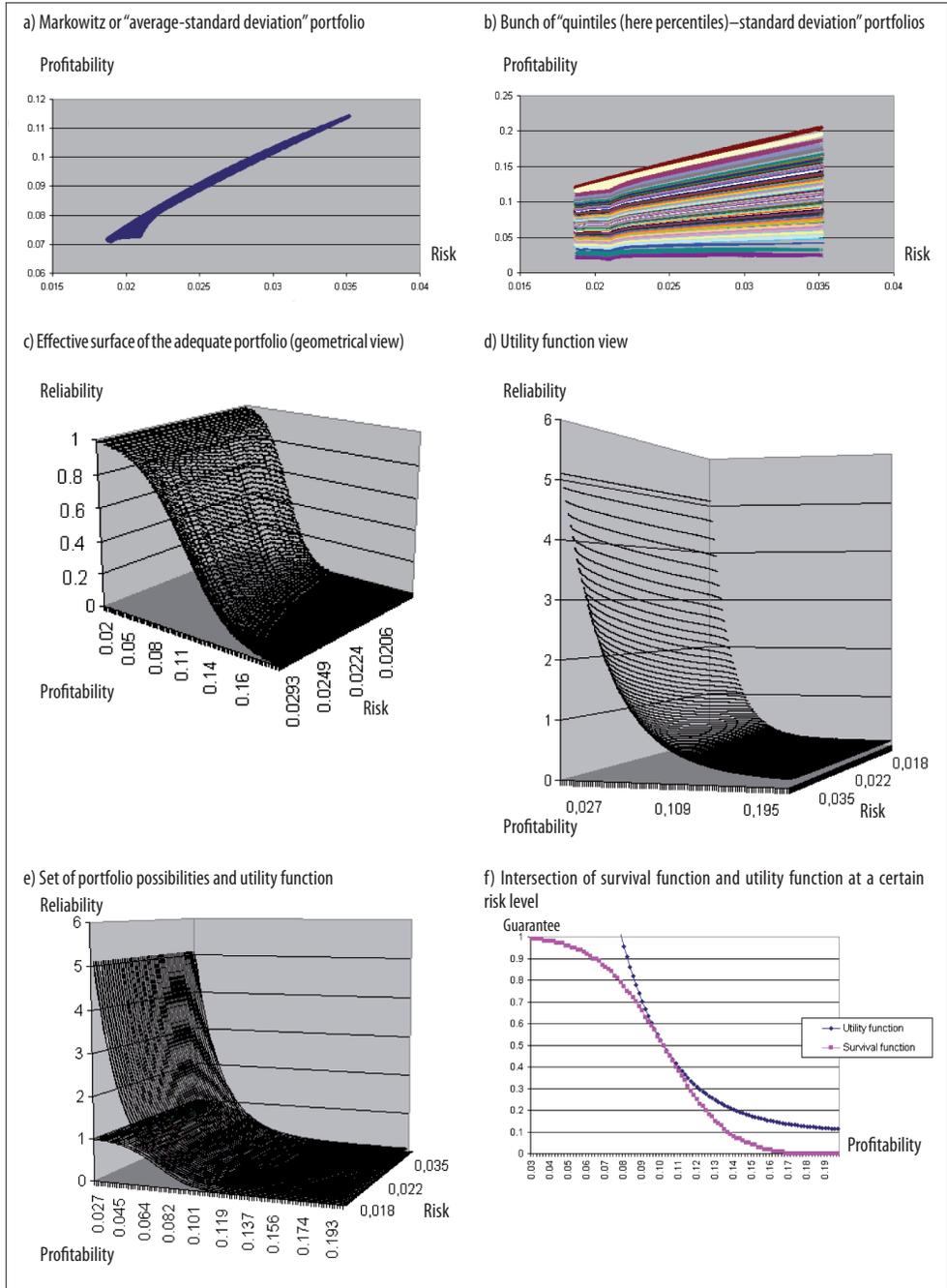


Fig. 10. Succession of adequate portfolio formation (mutually correlated probability distributions)

3. The possibilities' set, interconnected into the unique network, forms a convex surface, which in its intersection point with utility function provides an unambiguous investment portfolio structure, which, in turn, generates a random variable having attained profitability, its guarantee and risk.
4. Adequate investment portfolio, on the basis of which the efficient surface is being formed, seems to be theoretically sound and practically effective instrument for investment decision making in global capital and exchange markets.
5. Graphical visualization of decision search or utility maximization explicitly unfolds the convexity of efficient surface and multiplicative utility function with regard to each other.
6. The geometry of the efficient surface reacts very sensitively to the contents of the assets composing the investment portfolio, the forms of possibilities' probability distributions and to the degree of interdependence between the assets.
7. Applying proper forecasting system of stock prices and exchange rates changes, the adequate portfolio becomes an effective instrument of investment decision management in capital and exchange markets.

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OPTIMALAUS PORTFELIO PARINKIMAS PANAUDOJANT EFEKTYVIUOSIUS PAVIRŠIUS IR TRIMATĘ NAUDINGUMO FUNKCIJĄ

A. V. Rutkauskas, V. Stasytė

Santrauka. Efektyvumo, rizikingumo ir patikimumo sąvokos – tai trys kertiniai akmenys, ant kurių pasitelkiant investuotojo naudingumą laikosi sprendimų suvokimo ir valdymo logika, siekiant suderinti investicijų erdvės teikiamas galimybes ir investuotojo siekius. Rizika, kuri apibrėžiama kaip „pavojaus, praradimo, sužeidimo arba kitų neigiamų pasekmių šansas arba galimybė“ (The Oxford Modern English Dictionary), arba, konkrečiai investicijų valdymo srityje – „galimybė kad investicija (akcija arba prekė) praras savo vertę“ (Webster Dictionary), yra visų pasirinktų aktyvų rizikingumų funkcija pamatuota su subjekto įgūdžiais valdyti nagrinėjamo objekto, proceso ar pan. rizikingumus. Taigi, rizika nagrinėjama kaip galimybių rizikingumo ir subjekto (investuotojo) gebėjimų juos valdyti sąveika. Straipsnyje bus atskleistas nuoseklus kelias į investavimo galimybių aibės aprašymą, kuomet investicinių aktyvų galimybės yra neapibrėžtos, kas šiame straipsnyje suprantama kaip stochastinės. Kaip galimą aukščiau paminėto suderinimo priemonę autoriai siūlo adekvatųjį investicijų stochastinei prigimčiai portfelių, pateikdami jo sudarymo ir panaudojimo principus. Šis modelis turi plačias pritaikymo galimybes tiek investuojant valiutų ir kapitalo rinkose, tiek formuojant tvarios plėtros strategijas. Tekste pateikta labai daug paveikslų ir schemų. Tai sąlygota tų aplinkybių, kad ten, kur geometrinis brėžinys gali teikti neklaidingą atsakymą, tas brėžinys tampa ir sprendimo paieškos vizualizavimo priemone.

Reikšminiai žodžiai: pelningumas, rizikingumas, patikimumas, naudingumo funkcija, efektyvusis paviršius, tvari plėtra.

Aleksandras Vytautas RUTKAUSKAS. Doctor Habil, Professor, the Head of the Faculty of Business Management, Vilnius Gediminas Technical University. Research interests: capital and exchange markets, sustainable investment strategies development, regional development.

Viktorija STASYTYTĖ. PhD student at the Department of Finance Engineering, Vilnius Gediminas Technical University. Research interests: capital markets, stock prices forecasting, investment portfolio management.