THE UNCERTAINTY EVALUATION METHOD OF SUPPLY CHAIN RELIABILITY

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Abstract. The objective of the article is to scientifically evaluate supply chain reliability (SCR). We argue that this problem relates to two aspects - the cognition and expression of SCR. The paper considers SCR as a unification of a fuzzy and random meaning in a dynamic environment. Furthermore, intrinsic relationship between the theoretical foundation of SCR evaluation and the cloud theory is discovered, accordingly to which, the cloud theory is applied to study the evaluation of SCR from a holistic perspective. According to the comprehensive invalidation degree of a supply chain, SCR is differentiated as six grades and the influencing factors of SCR are classified taking into account five aspects. A comprehensive performance model is developed to measure five aspects of influencing factors and to evaluate the exact class SCR belongs to. As we know, the cognition of SCR depends on human mind while the natural language is an appropriate medium to express human mind. Therefore, linguistic terms are adopted to express uncertain transformation between qualitative concepts and their corresponding quantitative values. This method is further demonstrated using a numerical example.

Keywords: supply chains, reliability, evaluation, uncertainty, cloud theory, random variables, fuzzy rules.

1. Introduction

Supply chain design models have traditionally treated the world as if we knew everything about it with certainty. In reality, however, parameter estimates may be inaccurate due to poor forecasts, measurement errors, changing demand patterns or some other factors. Moreover, even if all of the parameters of the supply chain are known with certainty, the system may face disruptions from time to time, for example, due to inclement weather, labour actions or sabotage (Snyder 2003). A supply chain needs high reliability to insure its effectiveness and efficiency (Burkovskis 2008).

The increasing reliability-related researches in engineering and management fields are carried out; however, they seldom refer to concepts in the literature of the academic supply chain. Up to now, there has not been a generally acknowledged definition of supply chain reliability (SCR). Thomas (2002) was the first who explicitly presented the concept of SCR defined as ‘the probability of the chain meeting mission requirements to provide the required supplies to the critical transfer points within the system’. Some sources presented the concept of SCR from a specific perspective, for example, arrival time (Van Nieuwenhuyse and Vandaele 2006) or potential failure (Quigley and Walls 2007). In general, a supply chain is reliable in case it performs well when the parts of the chain fail.

2. Problem Analysis

The concept of SCR belongs to a fuzzy linguistic representation in essence. Under the influence of various dynamic influencing factors (Miao and Xi 2008), the evaluation of SCR is the comprehensive consideration of random variables and fuzzy rules. Human thinking is fuzzy in essence. Managers care much about whether a supply chain is reliable or not (Vasilis Vasiliauskas and Jakubauskas 2007), and a precise number cannot suit for answering the question expressed by fuzzy linguistics. If SCR is divided into different grades according to probability, the boundary between abutting states is also fuzzy. One cannot clearly identify to which state that supply chain belongs, whether being reliable or not the chain belongs to fuzzy rules and various numerical expressions about SCR belonging to random variables. Therefore, the concept of SCR can be seen as the unification of random variables and fuzzy rules.

While the fuzzy theory and the random theory form the foundation of the cloud theory (Li et al. 1998)
that is a mathematical tool specializing in dealing with uncertain and fuzzy knowledge and managing to realize the transformation between concepts and corresponding data. The cloud theory can obtain its explanation from the classical random theory and the fuzzy theory, reflects the relationship between randomness and fuzziness and forms mapping between qualitative concepts and quantitative data. Therefore, there is subtle coincidence between reliability measure and the cloud theory. Generally speaking, the uncertainty of the concept can be represented by multiple eigenvalues. Expectation and variance in the probability theory can be seen as this kind of eigenvalues, however, they have no relation to fuzziness; membership is a kind of the simplified precise method of the fuzzy theory, but it does not give consideration to randomness; precise-knowledge-based sets are used in the rough set theory to measure uncertainty but the uncertainty of the background knowledge is overlooked. The cloud theory uses expectation ($Ex$), entropy ($En$), hyper-entropy ($He$) and higher order entropy to depict uncertainty by integrating the random theory and the fuzzy theory.

3. The Basic Theory

If $X$ is a quantitative domain expressed with an accurate number and $C$ is a qualitative concept in $X$, if a quantitative value $x \in X$ and $x$ is a random realization of the qualitative concept $C$, $\mu(x)$ is the membership of $x$ to $C$, $\mu(x) \in [0, 1]$, it is the random number that has the steady tendency (He et al. 2007; Hui et al. 2009):

$$
\mu(x): X \rightarrow [0, 1], \ \forall x \in X, \ x \rightarrow \mu(x).
$$

The membership of a certain point in the quantitative domain varies subtly and brings little effect to the holistic character of the cloud. For a fuzzy set, the holistic character of the cloud is important, i.e. the shape and variation law of the cloud. The following two laws have been proved as the truth (Li et al. 1998):

1. For substantive fuzzy concepts in natural and social sciences, the expectation curves of the cloud approximately obey normal or half normal distribution.
2. The membership distribution of certain points in the quantitative domain has the shape of normal distribution.

It is meaningless to discuss the membership of a certain single point if no consideration is given to the holistic shape and agglomeration character of the cloud.

A normal cloud has universal applicability and reveals the basic laws of substantive fuzzy concepts in natural and social sciences. There are two kinds of uncertainty: one is randomness which is uncertainty in the event having a clear definition but does not necessarily emerging, whereas the other is fuzziness that is uncertainty in the event that has emerged but does not have a clear definition. Through the cloud theory, the problems of a fuzzy and random character can obtain an integrated expression.

The normal cloud is approximately determined by three eigenvalues, including expectation ($Ex$), entropy ($En$) and hyperentropy ($He$) (Li et al. 2000):

- **Expectation ($Ex$)** lies in $X$ corresponding to the center of the gravity of the cloud. In other words, the element $Ex$ in the quantitative domain is fully compatible with the linguistic term. In Figure 1, the center of the area under the cover of the membership cloud can be denoted as $G(Ex, \sqrt{2}/4)$ reflecting the central information value of the corresponding fuzzy concepts.
- **Entropy ($En$)** is a measure of the coverage of the concept within the quantitative domain. It can be also considered as a measure of the fuzziness of the concept. $En$ is defined by the bandwidth of the Mathematical Expected Curve (MEC) of the normal cloud and shows how many elements in the quantitative domain can be included in the linguistic term. The MEC of the normal cloud corresponding to a linguistic term may be considered as its membership function.
- **Hyperentropy ($He$)** is the entropy of $En$. It is a measure of the dispersion of the cloud drops. Fig. 1 shows, that $He$ denotes the stochastic distribution variance of the membership degree corresponding to the point $M(Ex + \sqrt{\ln 8 \cdot En}, \sqrt{2}/4)$ in the expectation curve of the membership cloud. $He$ can be used to express the thickness of the cloud. The higher is the value of $He$, the larger is the degree of cloud droplets dispersion and the greater is the thickness of the cloud. The variances of two points $A(Ex, 1)$ and $B (Ex + 3En, 0.0111)$ in the membership cloud curve can be taken as 0. The variances of other points in the curve present half-normal distribution along the degressive direction, from $He$ to 0.

As can be seen from Fig. 1, three eigenvalues of the cloud model unify fuzziness and randomness into the whole, form mapping from qualitative concepts to quantitative data and reflect a quantitative character of qualitative knowledge.

For indexes with bilateral constraint $[C_{min}, C_{max}]$, the following formula can be used to approximate three eigenvalues (Li 1997).

![Fig. 1. Membership cloud and its eigenvalues](image-url)


\[ Ex = \frac{C_{\text{min}} + C_{\text{max}}}{2}, \quad (1) \]

\[ En = \frac{C_{\text{max}} - C_{\text{min}}}{6}, \quad (2) \]

\[ He = k, \quad (3) \]

where: \( k \) is a constant and can be adjusted according to concrete indexes.

The generation algorithm of the cloud is shown as follows (Li et al. 2000):
- \( x_i = G(Ex, En) \). Generate random data satisfying normal distribution having expectation \( Ex \) and standard deviation \( En \);
- \( En_i = G(En, He) \). Generate random data satisfying normal distribution having expectation \( En \) and standard deviation \( He \);
- Calculate \( \mu_i = \exp[-(x – Ex)^2/2En_i^2] \) and let \( (x_i, L_i) \) be the cloud drops. Assign numerical values to \( Ex, En \) and \( He \) respectively.

The above algorithm can generate a normal cloud with an arbitrary number of the cloud drops. Three eigenvalues are enough to depict the configuration of the whole cloud.

4. The Evaluation Model of SCR

We can construct a comprehensive performance model of a supply chain to evaluate SCR. The following method provides an approach for the uncertainty evaluation of SCR and is helpful to discover the relation between qualitative fuzzy linguistic terms and quantitative numerical values about SCR.

When a supply chain is considered as the whole, one cannot easily describe its reliability. In this paper, SCR is divided into six grades:
1. Ideality: highest reliability and perfect operational ability;
2. Superior: high reliability keeping the main operational ability well;
3. Satisfaction: some drawbacks that emerge in the supply chain while the basic operational ability is unaffected;
4. Inferior: the main operational ability is hampered;
5. Crisis: the major drawbacks emerge in the supply chain;
6. Disruption: a supply chain has lost its operational ability.

The influencing factors of SCR depend on many indexes for expression (Jaržemskis 2007) and these indexes are hard to compare one by one. To tackle this problem, we propose the following classification of factors influencing SCR:
- logistics;
- capital;
- information;
- behaviour;
- environment.

The above classification covers various influences including temporal and spatial, inherent and external and provides a relatively comprehensive summarization of the factors influencing SCR.

Since the influencing factors depend on many indexes for expression and these indexes are hard to compare, therefore, the normalization of the concrete indexes of the above introduced five aspects should be done.

Then, the comprehensive performance model can be constructed as follows. First, introduce a circle. Second, divide the circle into five equal parts and obtain five points on the circle. Third, draw five radii from the centre of the circle to five points and let the length of the radii represent the ideal value of each influencing aspect when SCR is in the ideal state. The area \( S_0 \) of the regular pentagon constituted by the links between the above points represents a comprehensive performance when the supply chain is in the ideal state. An actual value of the comprehensive performance of the supply chain each time form a smaller polygon (broken line segments represent the sides of the polygon) as shown in Fig. 2 and the polygonal area \( S \) represents the actual comprehensive performance of the supply chain each time. When SCR decreases, the polygonal area \( S \) shrinks correspondingly.

![Fig. 2. Comprehensive performance model of a supply chain](image)

The polygonal area \( S \) can be applied to evaluate the comprehensive performance of the supply chain in the dynamic environment. The ratio \( S/S_0 \) can represent SCR each time. Next, we need a concrete approach to determine to which grade SCR belongs. Considering the cloud theory, intrinsic relationship with the character of SCR and the idea of the cloud theory can be applied and the following numerical example will demonstrate the created situation.

5. Numerical Example

For a supply chain, the regular pentagonal area \( S_0 \) and a smaller polygonal area \( S \) should be calculated. Let the radius of the circle equal to 1 and assume the above five aspects of the indexes influencing SCR respectively equal to 0.50, 0.60, 0.75, 0.75 and 0.60 which are relative values obtained by unitary processing. Consequently, we can calculate and obtain \( S = 1.03125 \sin(2\pi/5) \) and \( S_0 = 2.5 \sin(2\pi/5) \), and therefore \( S/S_0 = 0.4125 \).

Then, the scope of each grade of SCR needs to be determined. Generate bilateral constraint \([C_{\text{min}}, C_{\text{max}}]\)
for each of the above six grades of SCR. The numerical eigenvalue \((E_x, E_n, H_e)\) for each linguistic term can be obtained from the above formula (1), (2) and (3). The results are shown in Table 1.

### Table 1. Numerical eigenvalues of the membership cloud of SCR evaluation

<table>
<thead>
<tr>
<th>Quantitative linguistic term</th>
<th>(C_{\min})</th>
<th>(C_{\max})</th>
<th>(E_x)</th>
<th>(E_n)</th>
<th>(H_e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideality</td>
<td>0.93</td>
<td>0.99</td>
<td>0.96</td>
<td>0.010</td>
<td>0.002</td>
</tr>
<tr>
<td>Superior</td>
<td>0.73</td>
<td>0.97</td>
<td>0.85</td>
<td>0.040</td>
<td>0.005</td>
</tr>
<tr>
<td>Satisfaction</td>
<td>0.47</td>
<td>0.83</td>
<td>0.65</td>
<td>0.060</td>
<td>0.005</td>
</tr>
<tr>
<td>Inferior</td>
<td>0.27</td>
<td>0.63</td>
<td>0.45</td>
<td>0.060</td>
<td>0.005</td>
</tr>
<tr>
<td>Crisis</td>
<td>0.07</td>
<td>0.43</td>
<td>0.25</td>
<td>0.060</td>
<td>0.005</td>
</tr>
<tr>
<td>Disruption</td>
<td>0.02</td>
<td>0.18</td>
<td>0.10</td>
<td>0.026</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Based on the above presented generation algorithm of the cloud, the membership cloud for SCR evaluation can be obtained as shown in Fig. 3 representing the clustering and distribution of the memberships of the numerical values corresponding to different linguistic terms of SCR. In Fig. 3, the \(x\)-axis represents the definition domain, that is, different numerical values of \(S/S_0\) and the \(\mu(x)\)-axis represents membership corresponding to each linguistic evaluation of SCR.

Locate \(S/S_0 = 0.4125\) in the \(x\)-axis in Fig. 3. It can be seen from the \(\mu(x)\)-axis that the linguistic term ‘Inferior’ suit for expressing the current SCR when all data and eigenvalues are given in Table 1. In addition, the corresponding memberships in the \(\mu(x)\)-axis approximately fall into the scope of \([0.80, 0.90]\) reflecting that the numerical values of SCR evaluation are random variables and the linguistic evaluation of SCR accords with fuzzy rules.

The above analysis indicates that the randomness and fuzziness of SCR are integrated and shows that uncertain transformation is realized between a qualitative expression and corresponding quantitative values in the uncertainty evaluation process of SCR.

6. Conclusions

1. The fuzziness and randomness of SCR evaluation are analyzed in this paper. The cloud theory is applied to study the uncertainty evaluation of SCR. The classification rules of SCR and its influencing factors are brought forward and a comprehensive performance model is developed as the measurement approach. Uncertain transformation from a linguistic evaluation to its numerical representation is realized.

2. The paper has presented only a brief evaluation process. The classification approaches of some qualitative standards require further studies. The bilateral constraints of SCR come from the experiential estimation and need to be further examined considering practical requirements.

3. The evaluation method presented in this paper is not limited to the SCR field, and thus have a wide application in researches on the reliability of the transport system and other engineering systems. The idea of this paper provides a useful insight into the relevant areas.

References


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![Fig. 3. The membership cloud of SCR](image_url)


